P3 2019

1. 3 comp. ind.

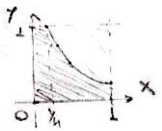
$X \sim Exp(\lambda)$  $f(x) = \lambda . e^{-\lambda x}$

$P(X<T) = \int_0^T f(x)dx = \left[-e^{-\lambda x}\right]_0^T = 1 - e^{-\lambda T}$

2 falharem: $P_1(e) = (1-e^{-\lambda T})^2 . [1-(1-e^{-\lambda T})]$

3° falhar: $P_2(e) = 1 - e^{-\lambda T}$

$P(X>T) = 1 - (P_1(e) + P_2(e)) = e^{-\lambda T}\left[1-(1-e^{-\lambda T})^2\right]$

2. 

$X z^2 + z + Y = 0$

$\Delta = b^2 - 4ac$

$\Delta = 1^2 - 4XY >= 0$ para ter raízes reais

$XY <= \frac{1}{4} \to Y <= \frac{1}{4X}$ se $X > 0$

$P(XY <= \frac{1}{4}) = \int_0^{1/4} 1 \, dx + \int_{1/4}^1 \frac{1}{4x} dx =$

$= \frac{1}{4} + \frac{1}{4}\left[\ln x\right]_{1/4}^1 = \frac{1}{4}(1 + \ln 1 - \ln(\frac{1}{4})) =$

$= \frac{1}{4} + \frac{1}{4} . \ln 4$

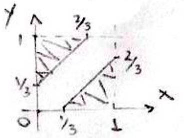
3. $Coef(X,Y) = \frac{Cov(X,Y)}{\delta_X . \delta_Y} = \rho$  X e Y aleatórios

$Cov(X,Y) = 0 \to \rho = 0$   $Coef(X - 0Y, Y) =$

$= Coef(X, Y) = 0$

4. $[0,1]$ X e Y, $|X-Y| > \frac{1}{3} \to X - Y > \frac{1}{3}$

ou $Y - X > \frac{1}{3}$



$Y > \frac{1}{3} + X$ ou $Y < X - \frac{1}{3}$

$P(|X-Y| > \frac{1}{3}) = \int_0^{2/3} (1 - \frac{1}{3} - x)dx + \int_{1/3}^1 (x - \frac{1}{3})dx =$

$= \left[\frac{2x}{3} - \frac{x^2}{2}\right]_0^{2/3} + \left[\frac{x^2}{2} - \frac{x}{3}\right]_{1/3}^1 = \frac{4}{9}$

5. $X \sim N(\mu, \delta^2)$  $P(X<30) = 15,87\%$

$\to$ com $\mu+2$  $P(X<30) = 2,28\%$

$\mu+2+x$ : $P(X<30) = 0,5\%$ ; $x = ?$

$z = \frac{x-\mu}{\delta} \to Z \sim N(0,1)$

(?)

6. $E[T] = \frac{a+b}{2} = 20$   $V[T] = \frac{(b-a)^2}{12} = \frac{64}{3}$

$b = 40 - a \to (40-2a)^2 = 256 \to$

$\to 1600 - 160a + 4a^2 = 256 \to 400 - 40a + a^2 = 64 \to$

$\to a^2 - 40a + 336 = 0 \to (a-12)(a-28) = 0 \to$

$\to a = 12$ ou $\cancel{a = 28}$ $\to a = 12$ e $b = 28$

não convém

$f(x) = \frac{1}{b-a} = \frac{1}{16}$   $P(X>24) = \int_{24}^{28} (\frac{1}{16}) dx = \frac{1}{4}$

7. $E[X] = E[N] = 0$   $Coef(Y,X) = \frac{Cov(Y,X)}{\delta_Y . \delta_X}$

$Y = X + N$ , $Cov(Y,X) = E[XY] - E[X].E[Y] \to$

$\to Cov(Y,X) = E[(X+N).X] - E[X].E[Y] \to$

$\to Cov(Y,X) = E[X^2] + E[NX] = E[X^2] + E[X].E[N]$

$Var[X] = E[X^2] - E^2[X] = \delta_X^2 \to Cov(Y,X) = \delta_X^2$

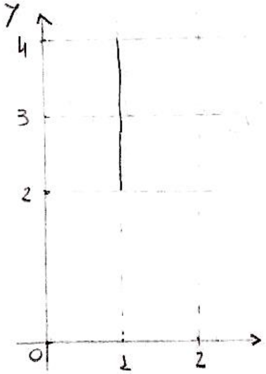
$Var[Y] = \delta_X^2 + \delta_N^2 \to \delta_Y = \sqrt{\delta_X^2 + \delta_N^2}$ comb. linear

$Coef(Y,X) = \frac{\delta_X^2}{\sqrt{\delta_X^2 + \delta_Y^2} . \delta_X} = \frac{\delta_X}{\sqrt{\delta_X^2 + \delta_Y^2}}$

8. $\lambda = 2$ , $X \sim Exp(2)$ , $f(x) = 2 . e^{-2x}$

$E[X] = \frac{1}{\lambda} = \frac{1}{2}$   $P(X<1) = \int_0^1 f(x)dx = 1 - e^{-2x}$

9. $P(Y>3 | X=1) = \frac{P(Y>3, X=1)}{P(X=1)}$



$f(1,y) = \frac{1}{8}(5-y)$   $P(Y>3) = \int_3^4 f(1,y)dy =$

$= \frac{1}{8}\left[5y - \frac{y^2}{2}\right]_3^4 = \frac{3}{16}$

$P(X=1) = \int_2^4 f(1,y)dy = \frac{1}{8}\left[5y - \frac{y^2}{2}\right]_2^4 = \frac{1}{2}$

$P(Y>3 | X=1) = \frac{3/16}{1/2} = \frac{3}{8}$

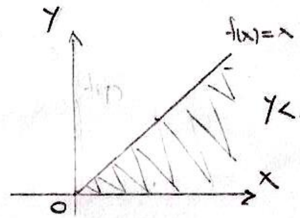
10. $S_Y = 2.S_X$   $W \in [395, 410]$ (1) sendo W comb. linear de $Y_i$

pelo Teorema do Limite Central: $Z_n = \frac{2.S_X - n.\mu}{\delta \sqrt{n}} \to$

$\to Z_{n1} = \frac{205 - 400 . \frac{1}{2}}{\frac{1}{4} . \sqrt{400}} = 1$ , $Z_{n2} = \frac{\frac{395}{2} - 200}{5} = -\frac{1}{2}$   pela tabela: $Z_n \sim N(0,1)$

$P(W \in [395, 410]) = 0,3413 + 0,1915 = 0,5328$

(1) ERRATA NA PROVA !!

11. 

$Y < X$

$P(X>Y) = ?$

$f(x,y) = f_X(x) . f_Y(y)$ independentes

para $x \geq 0$ e $y \geq 0$, $f(x,y) = \frac{1}{6} . e^{-\frac{2x+3y}{6}}$

$P(Y<X) = \int_0^\infty \int_0^x f(x,y)\,dy\,dx = \int_0^\infty \left[-\frac{1}{3} . e^{-\frac{2x+3y}{6}}\right]_0^x dx = \frac{1}{3}\int_0^\infty (e^{-\frac{x}{3}} - e^{-\frac{5x}{6}})dx =$

$= +\frac{1}{3}\left[-3 . e^{-\frac{x}{3}} + \frac{6}{5} . e^{-\frac{5x}{6}}\right]_0^\infty = +\frac{1}{3} . \left[(0+0) - (-3 + \frac{6}{5})\right] = \frac{1}{3} . \frac{9}{5} = \frac{3}{5}$


Marcus Vinícius Roso

Aulas particulares de Probabilidade

Contato via Facebook