P2. 2019

1. $E[X] = ?$

$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx =$

$= \frac{9}{10} \cdot \frac{8}{9} \cdot 4 + 2 \cdot \frac{1}{10} \cdot 1.52 = \frac{68}{5}$

2. $Cov(X,Y) = E[XY] - E[X] \cdot E[Y]$

$E[X] = \frac{1}{4} \cdot 0 + 2\frac{1}{4} 1 + \frac{1}{4} \cdot 2 = 1$

$E[Y] = E[X] = 1$

$E[XY] = ?$ H T T T H T T T H / T H H H T H H H T / T T T H H H

$E[XY] = \frac{1}{8} \cdot \cancel{1} \cdot 0 + \frac{1}{8} \cdot 1 \cdot 1 + \frac{1}{8} \cdot \cancel{2} \cdot 1 + \frac{1}{8} \cdot 1 \cdot 2 +$

$+ \frac{1}{8} \cdot 1 \cdot 1 + \frac{1}{8} \cdot 2 \cdot 1 + \frac{1}{8} \cdot \cancel{2} \cdot 0 + \frac{1}{8} \cdot 2 \cdot 2 =$

$= \frac{1}{8} \cdot (1 + 2 + 1 + 2 + 4) = \frac{10}{8}$

$Cov(X,Y) = \frac{10}{8} - 1 = \frac{1}{4}$

3. $P(prego) = 10\%$ distribuição binomial

$C_{10,1} = \frac{10!}{8!\,2!} = 45$

$P(e) = 45 \cdot (0,1)^2 \cdot (0,9)^8$

4. 25/viagem $P(encontrar) = \frac{1}{4}$

$E[X] = ?$ independentes

distribuição geométrica $E[X] = \frac{1-p}{p}$

$E[X] = \frac{1-\frac{1}{4}}{\frac{1}{4}} = 3$ fracassos

4 viagens → custo = 100

5. $E[X] = ?$

$\int_{-\infty}^{\infty} x \cdot f(x)\,dx = E[X]$

$\int_0^1 x \cdot x\,dx + \int_1^2 x\,(2-x)\,dx =$

$= \left[\frac{x^3}{3}\right]_0^1 + \left[x^2 - \frac{x^3}{3}\right]_1^2 =$

$= \frac{1}{3} + \left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right) =$

$= 1$

6. $E[X] = 2$ distribuição poisson

$E[X] = \mu$ independentes

$\mu = 2 \quad P(X=x) = \begin{cases} \frac{e^{-\mu}\,\mu^x}{x!}, & x \in \mathbb{N} \\ 0, & x \in \mathbb{N} \end{cases}$

$P(X=0) = e^{-2} \quad P(inc.) = 1 - e^{-2}$

$C_{5,1} = \frac{5!}{1!\,4!} = 5 \quad P(e) = 5 \cdot (e^{-2})^4 \cdot (1-e^{-2})$

7. $f(x) = \left(\frac{1}{t}\right) e^{-\frac{x}{t}}, \quad x \geq 0 \text{ e } t > 0$

$E[X] = ? \quad \int_{-\infty}^{\infty} x \cdot f(x)\,dx = E[X]$

$E[X] = \frac{1}{t} \cdot \int_0^{\infty} x \cdot e^{-\frac{x}{t}}\,dx = \frac{1}{t} \cdot t^2 = t$

8. $V[X] = E[X^2] - E^2[X]$

$E[X] = \frac{1}{36} \cdot 2 + 2 \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot 1 + \frac{5}{6} \cdot \cancel{\frac{5}{6}} \cdot 0 =$

$= \frac{1}{3}$

$E[X^2] = \frac{1}{36} \cdot 2^2 + 2 \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot 1^2 + \frac{5}{6} \cdot \cancel{\frac{5}{6}} \cdot 0^2 =$

$= \frac{7}{18}$

$V[X] = \frac{7}{18} - \left(\frac{1}{3}\right)^2 = \frac{5}{18}$

9. $P(abrir) = 0,1 \quad / \ 2 \text{ min}$

$P(dentro) = 0,1 + 0,9 \cdot 0,1 + (0,9)^2 \cdot 0,1 =$

$= 0,271$

$P(livre) = 0,271 \cdot 0,9 = 0,2439$

$P(preso) = 1 - 0,2439 = 0,7561$

10. X e Y independentes

$P(X=5, Y=2) = a = P(X=5) \cdot P(Y=2) =$

$= (0,1+a) \cdot (0,25+a+b) \rightarrow$

$\rightarrow a^2 + ab - 0,65a + 0,1b + 0,025 = 0$ ①

$P_{total} = 0,4 + (0,25+a+b) + 0,1 = 1 \rightarrow$

$\rightarrow a + b = 0,25$ ②

de ① e ②: $(0,1+a) \cdot 0,5 = a \rightarrow$

$\rightarrow a = 0,1$ e $b = 0,15$

para que $X - Y = 4$, $X = 6$ e $Y = 2$ ou

$X = 5$ e $Y = 1$ $\quad P(X=6, Y=2) = 0,15$

$P(X=5, Y=1) = 0,08 \quad P(e) = 0,23$

11. pares: $(-1,0); (0,0); (1,0); (0,-1); (0,0); (0,1)$

$E[Y|X=1] = ?$ se $X=1$, $Y=0$

$\therefore E[Y|X=1] = 0$

12.

	1	2	3	4	5	6
P(e)	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{9}$

$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \frac{1}{9} \cdot [1+2+3] +$

$+ \frac{2}{9} \cdot [4+5+6] = \frac{6}{9} + \frac{30}{9} = 4$

13. $S_X = \sqrt{V[X]} = \sqrt{E[X^2] - E^2[X]}$

$E[X] = n\,p = 25 \cdot 0,36 = 9$

$S_X = \sqrt{np(1-p)} = \sqrt{9 \cdot 0,64} = 2,4$

$5 \cdot S_X = 12$

$P(X > 21) = ? = P(X=22) + P(X=23) + P(X=24) + P(X=25)$

14. $E[X^2 + Y^2] = E[X^2] + E[Y^2]$ aleatórias = indep. e distr. uniformemente $\quad S_X^2 = E[X^2] - E^2[X] \rightarrow$

$\rightarrow 3 = E[X^2] - 3^2 \rightarrow E[X^2] = 3$

$S_Y^2 = E[Y^2] - E^2[Y] \rightarrow 5 = E[Y^2] - 4^2 \rightarrow$

$\rightarrow E[Y^2] = 21 \qquad E[X^2 + Y^2] = 24$

15. $E[X|Y=3] = ?$

$P(X|Y=3) = \frac{P(X \cap Y=3)}{P(Y=3)}$

	Y=3
X=1	0,25
X=2	0,5
X=4	0,25

$E[X|Y=3] = 1 \cdot 0,25 + 2 \cdot 0,5 + 4 \cdot 0,25 = 2\frac{1}{4} = \frac{9}{4}$

Marcus Vinícius Roso

Aulas particulares de Probabilidade

Valores a combinar

Contato via Facebook