P1. 2019

1. c_1 c_2 c_3 V ou F

$P_i(\text{falha}) = q$ ensaio de Bernoulli; distribuição binomial

eventos independentes entre si

$P_T(\text{falha}) = ?$

2 ou mais falham:

se 2 falham, respectivamente: V F F / F V F / F F V

$C_2^3 = \frac{3!}{2!\,1!} = 3$ $P(e_1) = q^2 \cdot (1-q)$

se 3 falham

$C_3^3 = 1$ F F F $P(e_2) = q^3$

$P_T(\text{falha}) = P(e_1) \cup P(e_2) = 1 \cdot q^3 + 3 \cdot q^2(1-q)$

2. propriedade $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-1}^{2} (ab - ax^2)\,dx = 1 \rightarrow 3a(b-1) = 1$ ①

$P(x>0) = \frac{16}{27} \rightarrow \int_0^2 (ab - ax^2)\,dx = \frac{16}{27} \rightarrow$

$\rightarrow (3ab - 4a) = \frac{16}{18}$ ②

① com ②: $a = \frac{1}{9}$ e $b = 4$

3. $P(\text{acerto}) = p$

$P(Y \leq 3) = ?$

se $Y = 1$, $P(Y) = p$

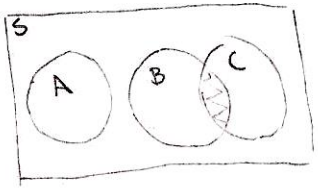
se $Y = 2$, $P(Y) = (1-p) \cdot p$

se $Y = 3$, $P(Y) = (1-p)^2 \cdot p$

$P(Y \leq 3) = p[1 + (1-p)(1+1-p)] = p[1 + (1-p)(2-p)] =$

$= p^3 - 3p^2 + 3p$

4.



a) $P(A|S) = 0$ F)

$P(A|S) = \frac{P(A \cap S)}{P(S)} = P(A) > 0$

b) requisito de independência:

$P(A \cap B) = P(A) \cdot P(B)$ F)

$0 = P(A) \cdot P(B)$, sendo que $P(A), P(B) > 0$

c) "

$P(B \cap C) = P(B) \cdot P(C)$ F)

pode ser verdadeiro

d) $P(A \cap (B \cup C)) = 0$

pelo diagrama de Venn, V)

5.

H $\rightarrow$ $P(2C) = \frac{1}{4}$ $P(2C \cap H) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$

D $\rightarrow$ $P(2C) = 1$ $P(2C \cap D) = 1 \cdot \frac{1}{2} = \frac{1}{2}$

$P(H|2C) = ? \rightarrow$ $P(2C) = \frac{1}{8} + \frac{1}{2} = \frac{5}{8}$

$\rightarrow P(H|2C) = \frac{P(H \cap 2C)}{P(2C)} = \frac{\frac{1}{8}}{\frac{5}{8}} = \frac{1}{5}$

6.

$f(t) = \begin{cases} kt^2, & 1 \leq t \leq 3 \\ 0, & \text{do contrário} \end{cases}$

$\int_{-\infty}^{\infty} f(t)\,dt = 1 \rightarrow \int_1^3 (kt^2)\,dt = 1 \rightarrow \frac{26}{3}k = 1 \rightarrow k = \frac{3}{26}$

$f(t) = \begin{cases} \frac{3}{26}t^2, & 1 \leq t \leq 3 \\ 0, & \text{do contrário} \end{cases}$

$P(\text{duro}) = \int_2^3 (\frac{3}{26}t^2)\,dt = \frac{19}{26}$

— — — $C_{3,2} = \frac{3!}{2!\,1!} = 3$ $P(e) = 3 \cdot \left(\frac{19}{26}\right)^2 \cdot \left(\frac{7}{26}\right)$

7. $P(Y=5) = ?$

1ª rodada:

não-S, $P(e) = \frac{9}{10}$

2ª rodada:

S, $P(e) = \frac{1}{9}$

$P_T = \frac{9}{10} \cdot \frac{1}{9} = \frac{1}{10}$

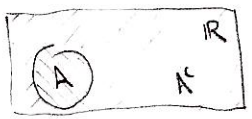
8. B $\rightarrow$ $P(P) = \frac{1}{2}$ $P(B \cap P) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

P $\rightarrow$ $P(P) = 1$ $P(P \cap P) = \frac{1}{2} \cdot 1 = \frac{1}{2}$

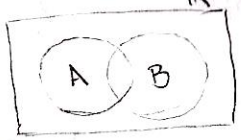
$P(R|P) = ?$ $P(P) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$P(R|P) = \frac{P(R \cap P)}{P(P)} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$

Diagrama de Venn

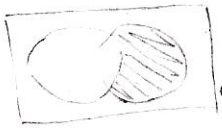


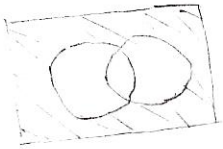
eventos
A e B

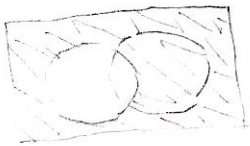


$A^c = \overline{A}$ = não-A

a) $A^c = (A^c \cap B) \cup (A^c \cap B^c)$

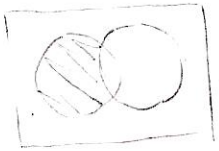
$A^c \cap B$:  ①

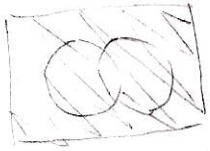
$A^c \cap B^c$:  ②

$(A^c \cap B) \cup (A^c \cap B^c)$ 

$= A^c$ V)

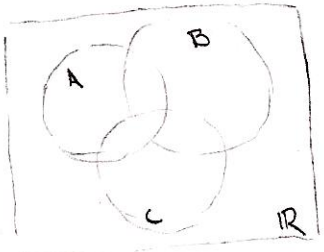
b) $(A \cap B)^c = (A^c \cap B) \cup (A^c \cap B^c) \cup (A \cap B^c)$
① ②

$A \cap B^c$: 

$(A^c \cap B) \cup (A^c \cap B^c) \cup (A \cap B^c)$ 

$= (A \cap B)^c$ V)

c) $(B \cup A)^c \cap C = A^c \cap B^c \cap C^c$

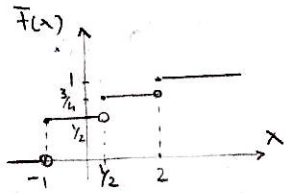


(*) Tq. P1 2019

P1. 2019

9. (*)

10.



a) $P(X \geq -1) = 1$ V

por definição

b) $P(X > -1) = \frac{1}{2}$ V

$P(X = -1) = \frac{1}{2}$ pelo gráfico

c) $P(X > -1 \mid X < 0) = 0$

pelo gráfico F

11. $A = P(X=2)$; $B = P(X+Y=7)$; $C = P(Y=3)$

por definição: A e B são independentes se

$$P(A \cap B) = P(A) \cdot P(B)$$

soma dos dados

	1	2	3	4	5	6
1	2	3	4	5	6	[7]
2	3	4	5	6	[7]	8
3	4	5	6	[7]	8	9
4	5	6	[7]	8	9	10
5	6	[7]	8	9	10	11
6	[7]	8	9	10	11	12

$P(X+Y=7) = \frac{6}{36} = \frac{1}{6}$

se $X=2$, $Y=5$ para que $X+Y=7$

$\therefore P(A \cap B) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$P(A \cap B) = P(A) \cdot P(B)$ $\quad$ $P([A \cap B] \cap C) = P(A \cap B) \cdot P(C)$

$\frac{1}{36} = \frac{1}{6} \cdot \frac{1}{6}$ V $\quad$ $0 \neq \frac{1}{36} \cdot \frac{1}{6}$ F

Analogamente, A e C são independentes V

12. eventos independentes

$P(A \cap B) = P(A) \cdot P(B) = 0{,}4 \cdot p$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0{,}7 = 0{,}4 + p - 0{,}4p \rightarrow$

$\rightarrow 0{,}3 = 0{,}6p \rightarrow$

$\rightarrow p = \frac{1}{2}$

13.

$P(\overline{A}) = \frac{4}{5}$ $\quad$ $P(\overline{N}) = \frac{44}{45} \cdot \frac{43}{44} \cdot \frac{42}{43} \cdot \frac{41}{42} \cdot \frac{40}{41} \cdot \frac{39}{40} \cdot \frac{38}{39} \cdot \frac{37}{38} \cdot \frac{36}{37} \cdot \frac{35}{36}$

$P(c) = P(\overline{A}) \cdot P(\overline{N}) = \frac{4}{5} \cdot \frac{35}{45} = \frac{28}{45}$

14.

	1	2	3	4	5	6
P(e)	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$

$>3 \rightarrow [1, \ldots, 9]$ 5 imp, 4 par

$<4 \rightarrow [1, \ldots, 5]$ 3 imp, 2 par

$P(e \text{ par}) = ?$

$P(>3) = \frac{1}{21} \cdot (4+5+6) = \frac{15}{21} = \frac{5}{7}$

$P(<4) = 1 - \frac{5}{7} = \frac{2}{7}$

$P(e \text{ par}) = \frac{5}{7} \cdot \frac{4}{9} + \frac{2}{7} \cdot \frac{2}{5} = \frac{20}{63} + \frac{4}{35} = \frac{100+36}{315} = \frac{136}{315}$

15. $P(\text{palavras} \mid \text{spam}) = 0{,}7$ $\quad$ $P(\text{spam}) = ?$

$P(\text{spam} \mid \text{palavras}) = 0{,}7 \rightarrow P(\overline{\text{spam}} \mid \text{palavras}) = 0{,}3$ ①

$P(\text{palavras} \mid \overline{\text{spam}}) = 0{,}2$

$P(p \mid \overset{0,7}{\text{spam}}) \cdot P(\text{spam}) = P(p \cap \text{spam})$

$P(\text{spam} \mid \overset{0,7}{p}) \cdot P(p) = P(p \cap \text{spam})$ $\rightarrow$

$\rightarrow 0{,}7 \cdot P(\text{spam}) = 0{,}7 \cdot P(p) \rightarrow P(\text{spam}) = P(p)$

$P(p \mid \overset{0,2}{\overline{\text{spam}}}) \cdot P(\overline{\text{spam}}) = P(p \cap \overline{\text{spam}})$

$0{,}2 \cdot (1 - P(\text{spam})) = P(p \cap \overline{\text{spam}})$ ②

de ① $P(\overline{\text{spam}} \mid \overset{0,3}{\text{palavras}}) \cdot P(\text{palavras}) = P(p \cap \overline{\text{spam}})$

de ① e ②: $0{,}2 \cdot (1 - P(\text{spam})) = 0{,}3 \cdot P(\text{spam}) \rightarrow$

$\rightarrow 0{,}2 = 0{,}5\, P(\text{spam}) \rightarrow P(\text{spam}) = 0{,}4$