Estruturas Tridimensionais

- Isostáticas:
  - grelhas
  - pórticos espaciais

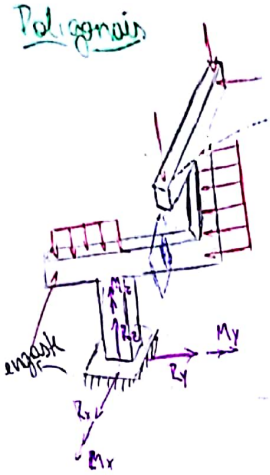
Grelhas

- Estrutura está contida no plano
- Carregamento está perpendicular ao plano da grelha

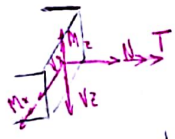
$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{cases}$

$\begin{cases} \Sigma M_{Ox} = 0 \\ \Sigma M_{O,y} = 0 \\ \Sigma M_{O,z} = 0 \end{cases}$

Estruturas Poligonais



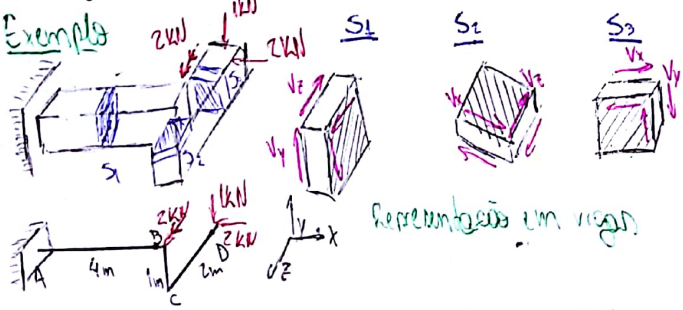
No corte:



Convenção de sinais:
$N > 0$: saindo da seção
$T > 0$: saindo da seção
Momentos → desenhar do lado tracionado

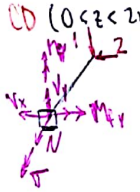
Contantes: positivas quando provocam um giro na seção transversal no sentido horário quando se está olhando no sentido contrário do eixo

Exemplo



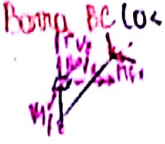
Representação em vigas

Barra CD ($0 < z < 2m$)



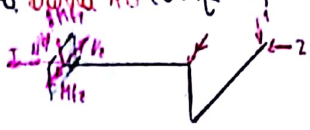
$\Sigma F_x = 0: -V_x - 2 = 0 \Rightarrow V_x = -2kN$
$\Sigma F_y = 0: +V_y - 1 = 0 \Rightarrow V_y = 1kN$
$\Sigma F_z = 0: N = 0$
$\Sigma M_{S,x} = 0: M_{fx} - 1z = 0 \Rightarrow M_{fx} = z$ ⊖ em cima
$\Sigma M_{S,y} = 0: M_{fy} + 2z = 0 \Rightarrow M_{fy} = 2z$ ⊖ para esquerda
$\Sigma M_{S,z} = 0: T = 0$

Barra BC ($0 < y < 1m$)

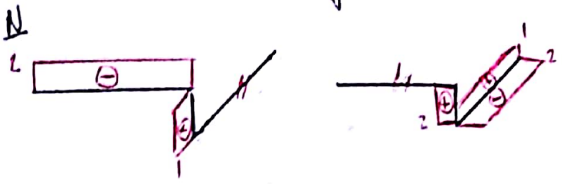


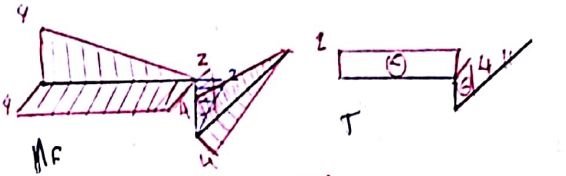
$\Sigma F_x = 0: V_x = 2kN$
$\Sigma F_y = 0: N = 1kN$
$\Sigma F_z = 0: V_z = 0$
$\Sigma M_{S,x} = 0: M_{fx} = 2kNm$ ⊖ atrás
$\Sigma M_{S,y} = 0: T - 2.2 = 0 \Rightarrow T = 4kNm$
$\Sigma M_{S,z} = 0: M_{fz} - 2y = 0 \Rightarrow M_{fz} = 2y$ ⊖ direita

Barra AB ($0 < \eta < 4m$)

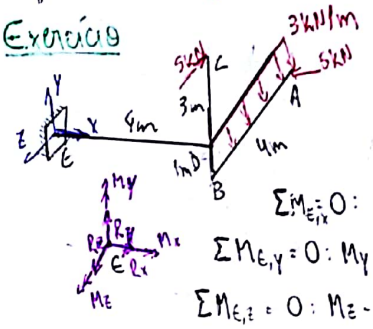


$N = -2kN$ $T = -1.2 = 2kNm$
$V_y = 1kN$ $M_{fy} = -4kNm$ ⊖ atrás
$V_z = 0$ M_{fz} ⊖ em cima



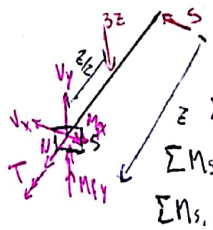


Exercício



$\Sigma F_x = 0: R_x = 5kN$
$\Sigma F_y = 0: R_y = 12kN$
$\Sigma F_z = 0: R_z = 5kN$
$\Sigma M_{E,x} = 0: M_{1x} - 5.3 - 3.4.2 = 0 \Rightarrow M_x = ?$
$\Sigma M_{E,y} = 0: M_y + 5.4 - 5.4 = 0 \Rightarrow M_y = -40kN$
$\Sigma M_{E,z} = 0: M_z - 3.4.4 - 5.1 = 0 \Rightarrow M_z = 53kNm$

Barra AB ($0 < z < 4m$)



$\Sigma F_x = 0: -V_x - 5 = 0 \quad V_x = -5kN$
$\Sigma F_y = 0: V_y = 3z$
$\Sigma F_z = 0: N = 0$
$\Sigma M_{S,x} = 0: M_{fx} - 3z \cdot z/2 = 0 \quad M_{fx} = \frac{3}{2}z$ ⊖ em cima
$\Sigma M_{S,y} = 0: M_{fy} = -5z$ ⊖ esquerda
$\Sigma M_{S,z} = 0: T = 0$

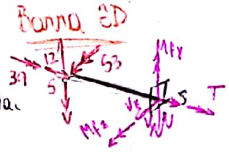
Barra BD ($0 < y < 1.m$)

$V_x = 5kN$
$V_y = 0$
$N = 12kN$
$M_{fx} = 24kNm$ ⊖ atrás
$T = -20kNm$
$M_{fz} = 5y$ ⊖ direita

Barra CD ($0 < \eta < 3m$)

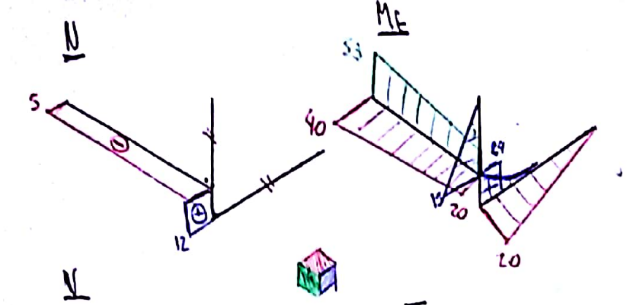
$V_x = 0$
$N = 0$
$V_z = 5kN$
$M_{fx} = 5\eta$ ⊖ frente
$T = 0$
$M_{fz} = 0$

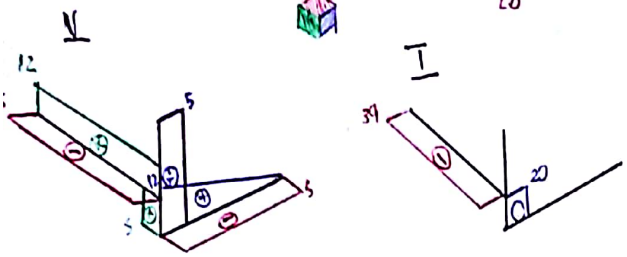
Barra ED



$N = -5kN$
$V_y = 12kN$
$V_z = -5kN$
$T = -39kNm$
$M_{fy} = 40 - 5x$ ⊖ frente
$M_{fz} = 12x - 53$ ⊖ embaixo

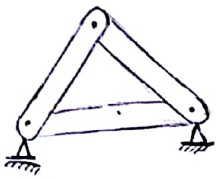
Scanned by CamScanner



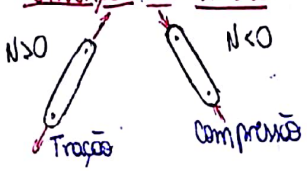


TRELIÇAS

- Barras ligadas por articulações
- Carregamentos exclusivamente radiais
- Nós: pontos de conexão entre as barras



Convenção de sinais:



Treliças Planas

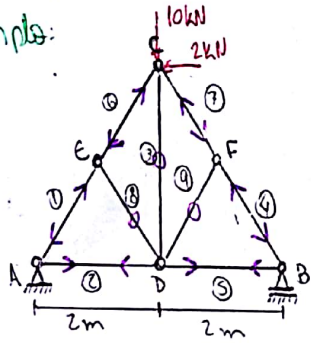
2 equações de equilíbrio ⇝ Para n nós temos 2n eqs. de equilíbrio

Incógnitas:
- b forças normais
- v reações vinculares

Em uma treliça isostática:

$2n = b + v$

Exemplo:



V_B	4
H_A	2
V_A	6
N_1	$-6\sqrt{2}$
N_2	4
N_3	0
N_4	$-4\sqrt{2}$
N_5	4
N_6	$-6\sqrt{2}$
N_7	$-4\sqrt{2}$
N_8	0
N_9	0

Reações de apoio

$\Sigma F_x = 0: H_A = 2\,kN$ $\quad \circlearrowleft \Sigma M_A = 0: -10.2 + 2.2 + V_B.4 = 0$

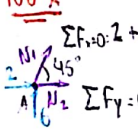
$\Sigma F_y = 0: V_A + V_B = 10$ $\quad V_B = 4\,kN$ $\quad V_A = 6\,kN$

Nó E

$N_1 = N_6$

$N_8 = 0$

Nó A

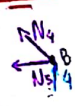


$\Sigma F_x = 0: 2 + N_2 + \frac{N_1}{\sqrt{2}} = 0$

$\Sigma F_y = 0: 6 + \frac{N_1}{\sqrt{2}} = 0$

$N_1 = -6\sqrt{2}\,kN$ $\quad N_2 = 4\,kN$

Nó B



$\Sigma F_x = 0: -N_5 - \frac{N_4}{\sqrt{2}} = 0$

$\Sigma F_y = 0: 4 + \frac{N_4}{\sqrt{2}} = 0$

$N_4 = -4\sqrt{2}\,kN$

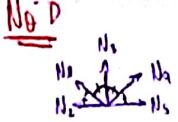
$N_5 = 4\,kN$

Nó F

$N_4 = N_7 = -4\sqrt{2}\,kN$

$N_9 = 0$

Nó D

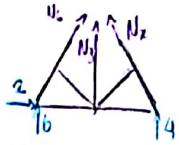


$N_3 = 0$

Corte de Ritter

↳ Corta-se a treliça em 2 partes

↳ Verifica-se o equilíbrio em uma das partes



$\Sigma F_x = 0: 2 + \frac{N_6}{\sqrt{2}} - \frac{N_7}{\sqrt{2}} = 0$

$\Sigma F_y = 0: 6 + 4 + N_3 + \frac{N_6}{\sqrt{2}} + \frac{N_7}{\sqrt{2}} = 0$

$\circlearrowleft \Sigma M_E = 0: 2.1 - 6.1 + N_3.1 + 4.3 + N_7\sqrt{2} = 0$

$8 + N_3 + \sqrt{2}N_7 = 0$ (1)

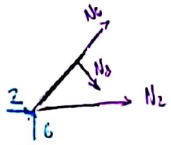
$2\sqrt{2} + N_6 - N_7 = 0$ (1)

$10\sqrt{2} + \sqrt{2}N_3 + N_6 + N_7 = 0$ (1)

$N_3 = 0$

$N_7 = -4\sqrt{2}\,kN$

$N_6 = -6\sqrt{2}\,kN$



$2 + \frac{N_6}{\sqrt{2}} + \frac{N_5}{\sqrt{2}} + N_2 = 0$

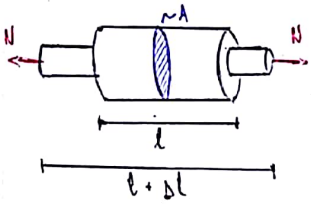
$\frac{6\sqrt{2}}{\sqrt{2}} - \frac{N_5}{\sqrt{2}} = 0$ $\quad \circlearrowleft \Sigma M_A = 0: N_5 = 0$

$N_2 = 4\,kN$ $\quad N_6 = -6\sqrt{2}\,kN$

TENSÕES E DEFORMAÇÕES

$\vec{t}$ e $\vec{n}$ são mecanicamente equivalentes às tensões

Lei de Hooke



Existe uma relação de proporcionalidade entre N/A e $\Delta l/l$, que depende do material

Definimos a tensão normal como:

$\sigma_n = \frac{N}{A}$ $\quad \left[\frac{N}{m^2}\right] = [Pa]$ (Pascal)

E deformação longitudinal como:

$\varepsilon_N = \frac{\Delta l}{l}$

A relação de proporcionalidade entre essas grandezas pode ser expressa como:

$\sigma_N = E\,\varepsilon_N$ $\quad E = \frac{\sigma_N}{\varepsilon_N}$ ⇝ módulo de elasticidade

Alguns módulos de elasticidade

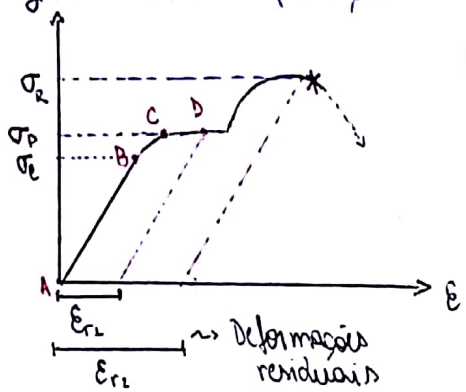
Material	E [GPa]	ν
Aço	210	0,3
Alumínio	70	0,25
Concreto	≈ 25	0,15
Madeira	≈ 10	?
Nylon	≈ ?	0,42

Scanned by CamScanner

ε_T ⇝ deformação transversal

$\varepsilon_T = \frac{\Delta\phi}{\phi} < 0$ $\qquad \varepsilon_T = -\nu \cdot \varepsilon_N$ $\qquad \nu$ ⇝ coeficiente de Poisson

Curva Tensão - Deformação



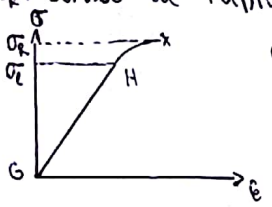
Material Dúctil

⇝ Deformações residuais

σ_E: tensão limite de proporcionalidade

σ_P: tensão de plastificação / escoamento

σ_R: tensão de ruptura



GH linear

Material Frágil

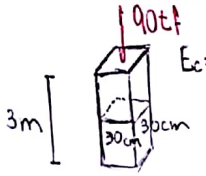
Retomando a formulação:

$\sigma_N = E \cdot \varepsilon_N \Rightarrow \varepsilon_N = \frac{\sigma_N}{E}$

Substituindo σ_N e ε_N:

$$\frac{\Delta l}{l} = \frac{1}{E} \cdot \frac{N}{A} \Rightarrow \Delta l = \frac{N l}{E A}$$

Exemplo. calcular o encurtamento do pilar de concreto



$E = 25 \cdot 10^9\ N/m^2$

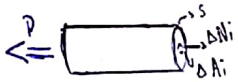
$N = -90 \cdot 10^4\ N$

$A = 0{,}3 \cdot 0{,}3 = 0{,}09\ m^2$

$l = 3\ m$

$$\Delta l = \frac{-90 \cdot 10^4 \cdot 3}{25 \cdot 10^9 \cdot 0{,}09} = -1{,}2 \cdot 10^{-3}\ m$$

A tensão σ_N é uma média!



$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta A_i = \int dA$$

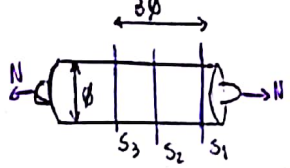
$$N = \int dN$$

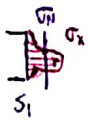
Para uma área infinitesimal dA

$\sigma_x = \frac{dN}{dA} \Rightarrow dN = \sigma_x\, dA \Rightarrow N = \int_S \sigma_x\, dA$

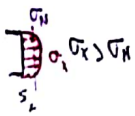
Se, por hipótese, $\sigma_x = \sigma_N$, $\forall P \in S$

$N = \sigma_N \int_S dA$ $\qquad N = \sigma_N A$

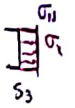




$\sigma_x >> \sigma_N$



$\sigma_x > \sigma_N$



$\sigma_x \approx \sigma_N$

Coeficiente de Segurança

No projeto, devemos evitar chegar perto das tensões limites

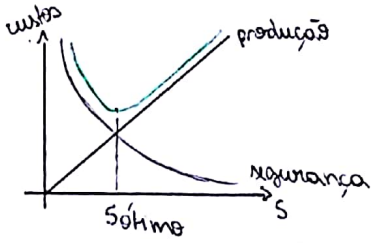
σ_{lim} { σ_E (dúcteis) / σ_R (frágeis) }

Definimos a tensão admissível como:

⇝ tensão admissível

$$\bar{\sigma} = \frac{\sigma_{lim}}{S}$$

↳ fator de segurança



+ caro para produzir

- gastos devido a falhas de segurança

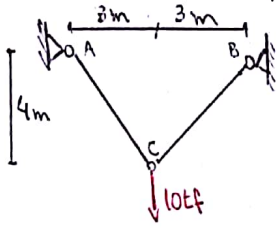
Alguns coeficientes de segurança mais utilizados

Aço: $S = 1{,}6$

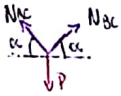
Concreto: $S \approx 2 \cdot 3$

Madeira: $S \approx 3 \cdot 4$

Exemplo: Determinar o diâmetro do fio para que o coeficiente de segurança seja 2. (σ_E=600 MPa, E=210GPa)



Nó C:



$-N_{AC}\cos\alpha + N_{BC}\cos\alpha$

$N_{AC} = N_{BC} = N$

$2N\sin\alpha = P$

$N = \frac{P}{2\sin\alpha} = 62{,}5\ kN$

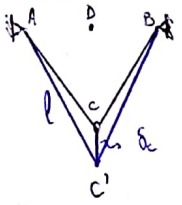
Calculando a tensão admissível

$\bar{\sigma} = \frac{\sigma_{lim}}{S} = \frac{600}{2} = 300\ MPa$

$\sigma_N \leq \bar{\sigma} \Rightarrow \frac{N}{A} \leq \bar{\sigma} \Rightarrow \frac{N}{\bar{\sigma}} \leq A$

$\frac{\pi\phi^2}{4} \geq \frac{N}{\bar{\sigma}} \Rightarrow \phi \geq \sqrt{\frac{4N}{\pi\bar{\sigma}}} \Rightarrow \phi_{min} = 1{,}63\ cm$

Calculando o deslocamento do ponto C:



$l = l_0 + \Delta l$

$l = l_0 + l_0 \varepsilon$

$l = l_0(1 + \varepsilon)$

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Calculando ε:

$$\varepsilon = \frac{\bar{\sigma}}{E} = \frac{300.10^6}{210.10^9} = 1{,}43.10^{-3}\ m/m$$

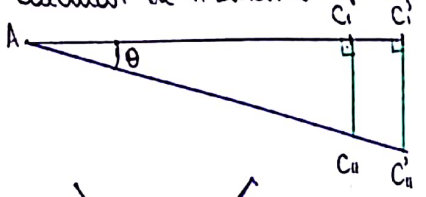
$$l = 5.(1 + 1{,}43.10^{-3}) = 5{,}0071$$

Considerando o $\Delta ADC'$

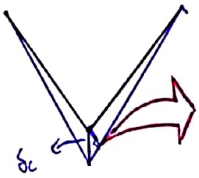
$$(AD)^2 + (DC')^2 = (AC')^2$$

$$3^2 + (4+\delta_c)^2 = 5{,}0071^2 \Rightarrow \delta_c = 0{,}0089\ m$$

Calcular de maneira aproximada:



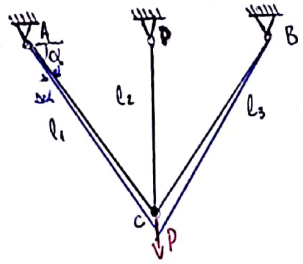
arco $C_1C_{11} \approx$ reta!



$$\sin\alpha = \frac{\Delta l}{\delta_c} \Rightarrow \delta_c = \frac{\Delta l}{\sin\alpha}$$

$$\delta_c = 0{,}0089$$

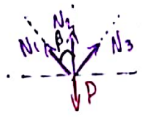
ESTRUTURAS HIPERESTÁTICAS



$\Delta\alpha \ll 1$

- Supor que a geometria permanece inalterada no equilíbrio
- → Permite a linearização do problema

Equilíbrio no nó C



$$\Sigma F_H = 0: -N_1\sin\beta + N_3\sin\beta = 0 \Rightarrow N_1 = N_3$$

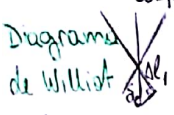
$$\Sigma F_V = 0: N_1\cos\beta + N_3\cos\beta + N_2 = P$$

$$2N_1\cos\beta + N_2 = P \quad (I)$$

- Compatibilidade de deslocamentos

$$l_1 = l_3 = \frac{h}{\cos\beta},\ l_2 = h$$

Diagrama de Williot



Logo: $\Delta l_1 = \delta_c\cos\beta$

$\Delta l_2 = \delta_c$

$\Delta l_3 = \delta_c\cos\beta$

Sabemos que:

$$\frac{N}{A} = E\frac{\Delta l}{l} \Rightarrow N = \frac{EA}{l}.\Delta l$$

Assim:

$$N_1 = \frac{EA}{l_1}.\Delta l_1 = \frac{EA\cos^2\beta}{h}.\delta_c = N_3 \qquad N_2 = \frac{EA}{l_2}.\Delta l_2 = \frac{EA}{h}.\delta_c \quad (II)$$

Substituindo (II) em (I):

$$2\cos\beta\left(\frac{EA}{h}\cos^2\beta.\delta_c\right) + \frac{EA}{h}.\delta_c = P$$

$$\frac{EA}{h}\delta_c.\left(2\cos^3\beta + 1\right) = P$$

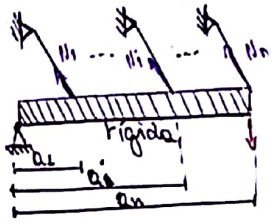
$$\delta_c = \frac{Ph}{EA(2\cos^3\beta+1)}$$

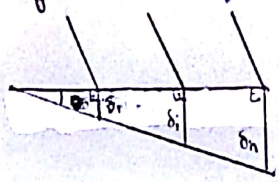
$$N_3 = N_1 = \frac{P\cos^2\beta}{2\cos^3\beta+1}$$

$$N_2 = \frac{P}{2\cos^3\beta+1}$$

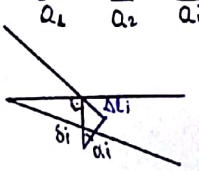
Estruturas Estaiadas (n-1) vezes hiperestático

Fazendo a aproximação





$$tg\theta = \frac{\delta_1}{a_1} = \frac{\delta_2}{a_2} = \frac{\delta_i}{a_i} = \frac{\delta_n}{a_n} = cte\ (I)$$



$$\sin\alpha_i = \frac{\Delta l_i}{\delta_i} \Rightarrow \Delta l_i = \delta_i\sin\alpha_i \Rightarrow N_i = \frac{E_iA_i\Delta l_i}{l_i}$$

$$N_i = \frac{E_iA_i\delta_i\sin\alpha_i}{l_i}$$

$$\delta_i = \frac{N_il_i}{E_iA_i\sin\alpha_i} \quad (II)$$

Em (I): $\dfrac{N_il_i}{E_iA_ia_i\sin\alpha_i} = tg\theta$ — **(n-1) equações de compatibilidade**

Exemplo: Calcular o diâmetro mínimo dos estais, considerando:

$\bar{\delta} = 1cm$

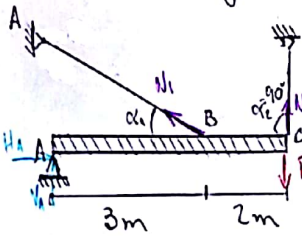
$\bar{\sigma} = 60MPa = 600\ kgf/cm^2$

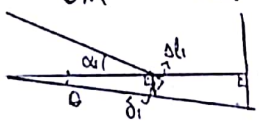
$E = 200GPa = 2.10^6\ kgf/cm^2$

$\Sigma F_H = 0: H_A - N_1\cos\alpha_1 = 0$

$\Sigma F_V = 0: V_A + N_1\sin\alpha_1 + N_2 = P$

$\Sigma M_{(A)} = 0: N_1\sin\alpha_1.3 + 5N_2 = 5P$





$$\sin\alpha_1 = \frac{\Delta l_1}{\delta_1};\ \delta_1 = \frac{\Delta l_1}{\sin\alpha_1}$$

$$tg\theta = \frac{\delta_1}{3} = \frac{\delta_2}{5}$$

$$\frac{\Delta l_1}{3\sin\alpha_1} = \frac{\Delta l_2}{5}$$

$$\Delta l_i = \frac{N_il_i}{EA} \Rightarrow \frac{N_1l_1}{3\sin\alpha_1} = \frac{N_2l_2}{5} \qquad N_1 = 0{,}12N_2$$

$N_1 = 232{,}5\ kgf$

$N_2 = 1937{,}6\ kgf$

$H_1 = 207{,}9\ kgf$

$V_A = -41{,}6\ kgf$

Critério de Tensão máxima:

$\sigma \leq \bar{\sigma} \qquad \sigma = N/A$

$\sigma_2 \leq \bar{\sigma}$

$A \geq \dfrac{N_2}{\bar{\sigma}} \Rightarrow A \geq 3{,}23\ cm^2$

Critério de Deslocamento máximo:

$\delta \leq \bar{\delta}_c$

$\delta_2 \leq \bar{\delta}_c$

$\delta_2 = \Delta l_2 = \dfrac{N_2l_2}{EA} \Rightarrow A \geq \dfrac{N_2l_2}{E\bar{\delta}_c} = 0{,}14\ cm^2$

Logo, $A_{mín} = 3{,}23\ cm^2$

$$\frac{\pi\phi^2}{4} = A_{mín} \Rightarrow \phi_{mín} = 2{,}03cm$$

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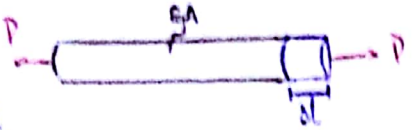
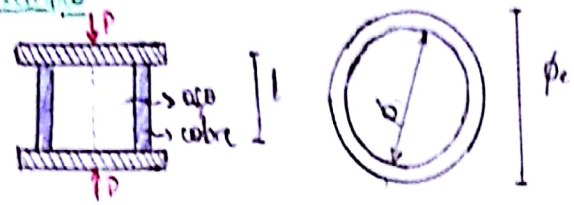
Forças normais em estruturas hiperestáticas



$\Delta l = \frac{Pl}{E.A}$

Exemplo:



Pede-se Δl ($\sigma_{aço} = \sigma_a$, $\sigma_{cobre} = \sigma_c$)

para o cilindro composto da figura
Tratar as tensões em módulos

$A_a = \frac{\pi \phi_i^2}{4}$, $A_c = \frac{\pi \phi_e^2}{4} - A_a$

Condição de compatibilidade:

$\varepsilon_a = \varepsilon_c = \frac{\Delta l}{l} = \varepsilon$

Podemos escrever:

$\sigma_a = E_a . \varepsilon$ (I) $\quad \sigma_c = E_c . \varepsilon$ (II)

E, por equilíbrio:

$\sigma_a . A_a + \sigma_c A_c = P$ (III)

Substituindo (I) e (II) em (III):

$E_a \varepsilon A_a + E_c \varepsilon A_c = P \qquad \Delta l = \frac{Pl}{E_a A_a + E_c A_c}$

$(E_a A_a + E_c A_c) \frac{\Delta l}{l} = P$

$\Delta l = \frac{Pl}{(EA)_{equivalente}} \qquad \sigma_a = \frac{P E_a}{E_a A_a + E_c A_c} \qquad \sigma_c = \frac{P E_c}{E_a A_a + E_c A_c}$

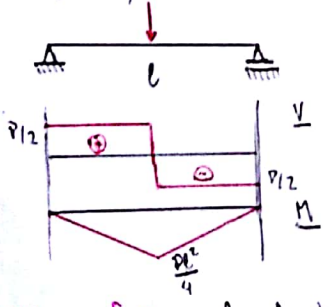
Sabemos que:

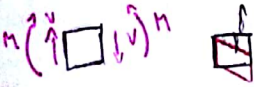
$A_{total} = A = A_a + A_c$

$E_{eq} = \frac{E_a A_a + E_c A_c}{A_a + A_c} = \frac{E_a A_a + E_c A_c}{A}$

TORÇÃO

→ Deformações / tensão: cisalhamento





γ: distorção [rad]

$\gamma \ll 1$

$\gamma \approx tg\gamma = \frac{dy}{dx}$, cte

Tensão média de cisalhamento: $\tau = \frac{V}{A}$

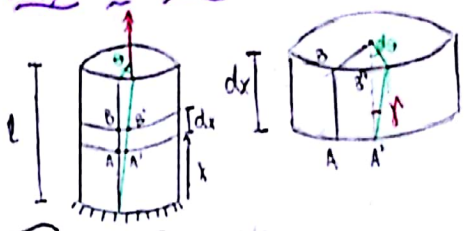
Lei de Hooke: $\tau = G . \gamma$

G: módulo de elasticidade transversal

Material elástico e isotrópico:

$G = \frac{E}{2(1+\nu)}$

Torção de eixos



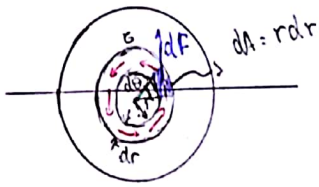
$\left.\begin{array}{l} \widehat{B'B''} = r\,d\theta \\ \widehat{B'B''} = \gamma\,dx \end{array}\right\} \; r\,d\theta = \gamma\,dx$

$\gamma = r\frac{d\theta}{dx} = \psi . r$ se $\frac{d\theta}{dx}$ é constante

Lei de Hooke: $\tau = G . \gamma \Rightarrow \tau = G.\psi . r$

$\tau_{máx} = G\psi R$

Distribuição das tensões de cisalhamento



$dF = \tau\,dA$

$dT = r\,dF$

$dT = \tau r\,dA = \tau r^2 dr d\theta$

$dT_{(r)} = \int_0^{2\pi} dT = \int_0^{2\pi} \tau r^2 dr d\theta$

$dT_{(r)} = 2\pi \tau r^2 dr$

$T = \int_0^R dT_{(r)} = \int_0^R 2\pi G \psi r^3 dr$

$T = \frac{\pi G \psi R^4}{2} \Rightarrow \psi = \frac{2T}{\pi G R^4}$

Substituindo na Lei de Hooke

$\tau = G\psi r = G\left(\frac{2T}{\pi G R^4}\right) r$

$\tau = \frac{T}{J} r \qquad J = \frac{\pi R^4}{2} = \frac{\pi D^4}{32}$

J: momento polar de inércia de um círculo cheio de raio extremo R

$\gamma = \psi r$

$\psi = \frac{T}{GJ}$ ⇝ módulo de rigidez à torção

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$\widehat{BB'} = R\theta = \gamma l$

$\gamma = \frac{TR}{GJ} = \frac{R\theta}{l}$ $\quad \theta = \frac{Tl}{GJ}$

Eixo vazado

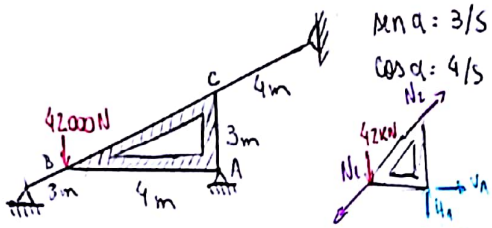
$\tau_{máx} = \frac{T}{J}.R_e$, $\quad J = \frac{\pi(R_e^4 - R_i^4)}{2}$

Exercícios

1) Mesma Seção Transversal e Área = ?

$E = 10^3$ MPa, $\bar{\sigma} = 40$ MPa $\quad V_B \leq 0{,}1$ m

Chapa triangular rígida



$\tan\alpha = 3/5$

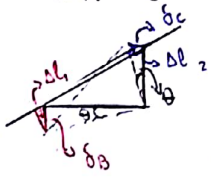
$\cos\alpha = 4/5$

$\Sigma F_H = 0 : -N_1\cos\alpha + H_A + N_2\cos\alpha = 0$

$\Sigma F_V = 0 : -42 - N_1\operatorname{sen}\alpha + V_A + N_2\operatorname{sen}\alpha = 0$

$\Sigma M_B = 0 : V_A = 0$, $\quad H_A = 56$ kN

$N_2 - N_1 = \frac{42}{\operatorname{sen}\alpha} = \frac{42}{3}.5$ $\quad N_2 - N_1 = 70$ (I)



$\cos\alpha = \frac{|\Delta l_2|}{\delta_C}$ $\quad \tan\theta = \frac{\delta_C}{3} = \frac{\delta_B}{4}$

$\operatorname{sen}\alpha = \frac{|\Delta l_1|}{\delta_B} \rightarrow \frac{1}{3}.\frac{\Delta l_2}{\cos\alpha} = \frac{1}{4}\frac{|\Delta l_1|}{\operatorname{sen}\alpha}$

$\frac{\Delta l_2}{3}.\frac{5}{4} = \frac{|\Delta l_1|}{4}.\frac{5}{3}$

$|\Delta l_1| = \Delta l_2$

$\Delta l_1 = -\Delta l_2$

$N_i = \frac{EA\Delta l_i}{l_i} \Rightarrow \Delta l_i = \frac{N_i l_i}{EA}$

$\frac{N_2 l_2}{EA} = -\frac{N_1 l_1}{EA}$ $\quad 4N_2 = -3N_1$ $\quad N_1 = -40$ kN

$N_1 = -4/3\,N_2$ (II) $\quad N_2 = 30$ kN

Critérios

① $\sigma \leq \bar{\sigma}$ $\quad A \geq \frac{|N_1|}{\bar{\sigma}} = 10^{-3}\,m^2$

$|\sigma_1| > |\sigma_2|$

$\frac{|N_1|}{A} \leq \bar{\sigma}$

② $\delta_B \leq V_B$ $\quad A \geq 2.10^{-3}\,m^2$

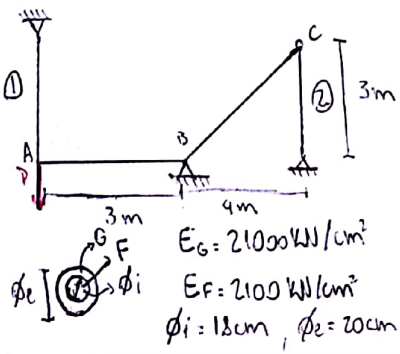
$\frac{|\Delta l_1|}{\operatorname{sen}\alpha} \leq V_B$

$\frac{|N_1| l_1}{EA}.\frac{1}{\operatorname{sen}\alpha} \leq V_B$

$A \geq \frac{|N_1| l_1}{E V_B \operatorname{sen}\alpha}$

de ① e ②: $A \geq 2.10^{-3}$ m

x)



Barra ABC rígida

Barras ① e ② compostas de materiais G e F. $P_{máx} = ?$

$E_G = 21000$ kN/cm²

$E_F = 2100$ kN/cm²

$\Phi_i = 18$cm, $\quad \Phi_e = 20$cm

$\delta_C \leq \bar{\delta} = 1$cm

$\sigma_G \leq \bar{\sigma}_G = 15$ kN/cm²

$\sigma_F \leq \bar{\sigma}_F = 5$ kN/m²

$E A_{eq}$

$A_F = 81\pi$ cm², $\quad A_G = 19\pi$ cm²

$EA_{eq} = E_F A_F + E_G A_G$

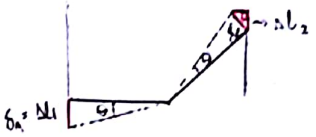
$EA_{eq} = 1\,780\,320$ kN

$H_B = 0$

$N_1 - P + V_B - N_2 = 0$

$\circlearrowleft \Sigma M_B = 0 : P.3 - N_1.3 - N_2.4 = 0$

$N_1 + 4/3\,N_2 = P$



$\Delta l_1 = \frac{3}{5}\delta_C$

$\Delta l_2 = \frac{4}{5}\delta_C$

$\tan\theta = \frac{\delta_C}{3} = \frac{\delta_C}{5}$ $\quad \delta_A = \Delta l_1$

$\delta_C = \frac{\Delta l_2}{\cos\alpha}$

$\varepsilon_1 = \frac{\Delta l_1}{l_1} = \frac{1}{500}\delta_C$ $\quad \varepsilon_2 = \frac{\Delta l_2}{l_2} = \frac{4}{1500}\delta_C$

Condições de deformação máxima

$\sigma_F = E_F.\varepsilon_F \leq \bar{\sigma}_F \Rightarrow \varepsilon_F \leq \bar{\sigma}_F / E_F$

$\sigma_G = E_G.\varepsilon_G \leq \bar{\sigma}_G \Rightarrow \varepsilon_G \leq \bar{\sigma}_G / E_G$

Substituindo

$\varepsilon_G \leq 7{,}143.10^{-4}$ cm/cm $\quad \varepsilon_F \leq 2{,}38.10^{-5}$ cm/cm

Usando como limite $\bar{\varepsilon} = 7{,}143.10^{-4}$ cm/cm

$máx(\varepsilon_1, \varepsilon_2) \leq \bar{\varepsilon}$

$\varepsilon_2 \leq \bar{\varepsilon}$

$\frac{4\delta_C}{1500} \leq \bar{\varepsilon}$ $\quad \delta_C \leq 0{,}27$ cm, satisfaz critério $\delta_{máx}$

$\varepsilon = \frac{N}{EA}$ $\quad$ como $\frac{4}{3}\varepsilon_1 = \varepsilon_2$ $\quad$ Assim: $N_2 > N_1$

$\varepsilon_1 = \frac{N_1}{EA_{eq}}$ $\quad$ então: $\frac{4}{3}N_1 = N_2$ $\quad \varepsilon_2 \leq \bar{\varepsilon}$

$\varepsilon_2 = \frac{N_2}{EA_{eq}}$ $\quad \frac{N_2}{EA_{eq}} \leq \bar{\varepsilon}$

$N_2 \leq 7{,}143.10^{-4}.1\,787\,880$

$N_2 \leq 1277$ kN, $\quad N_1 \leq 958$ kN

Usando o equilíbrio:

$P \leq 2660$ kN

$P_{máx} = 2660$ kN

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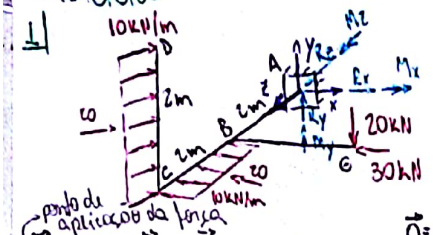
Exercícios

1)



$\vec{r_1} = \vec{j} + 4\vec{k}$, $\vec{F_1} = -20\vec{k}$

$\vec{r_2} = 3\vec{k}$, $\vec{F_2} = -20\vec{i}$

$\vec{r_3} = 2\vec{i} + 2\vec{k}$, $\vec{F_3} = -30\vec{i} - 20\vec{j}$

$R_x\vec{i} + R_y\vec{j} + R_z\vec{k} + \Sigma\vec{F} = \vec{0}$

$R_x = 50\,kN$ | $R_y = 20\,kN$ | $R_z = 20\,kN$

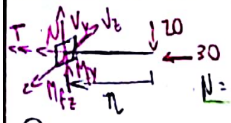
$\vec{0} = M_x\vec{i} + M_y\vec{j} + M_z\vec{k} + \Sigma\vec{r_i}\times\vec{F_i}$

$\vec{M} + (\vec{j}+4\vec{k})\times(-20\vec{k}) + (3\vec{k})\times(-20\vec{i}) + (2\vec{i}+2\vec{k})\times(-30\vec{i}-20\vec{j}) = \vec{0}$

$\vec{M} = 20\vec{i} - 60\vec{j} + 40\vec{k} - 60\vec{j} + 40\vec{i} = \vec{0}$

$M_x = -20\,kNm$ | $M_y = 120\,kNm$ | $M_z = 40\,kNm$

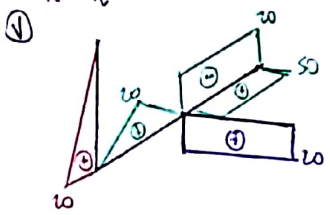
Barra BE $(0 < \eta < 2m)$:



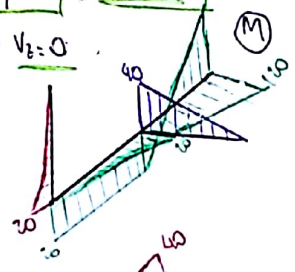
$T = 0$ | $M_{fy} = 0$ | $M_{fz} = 20\eta$ | ⊕ em cima

$N = -30$ | $V_y = 20$ | $V_z = 0$

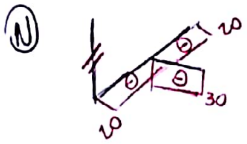
Ⓥ



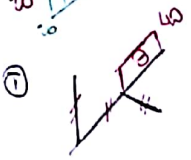
Ⓜ



Ⓝ



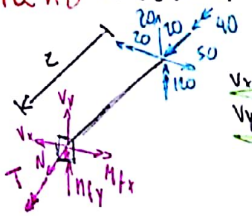
Ⓣ



Barra CD $(0 < \eta < 2m)$:

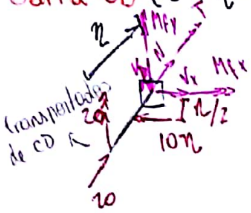
$N = 0$ | $V_x = 0$ | $V_z = 10\eta$ | $M_{fx} = 5\eta^2$ | ⊖ na frente | $T = 0$ | $M_{fz} = 0$

Barra AB $(0 < z < 2m)$:



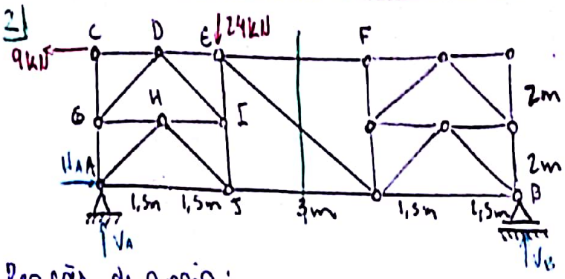
$N = -20$ | $V_x = 50$ | $V_y = 20$ | $M_{fx} = 20 - 20z$ | $M_{fy} = 50z - 120$ | $T = -40$

Barra CB $(0 \le \eta < 2m)$:



$N = -20$ | $V_y = 0$ | $V_x = 10\eta$ | $M_{fx} = 20$ | ⊖ embaixo | $M_{fy} = 5\eta^2$ | ⊖ na frente | $T = 0$

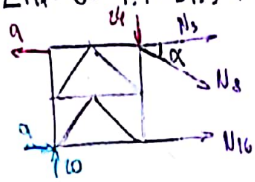
2)



Reações de apoio:

$H_A = 9\,kN$ | $V_A + V_B = 24$ | $V_B = 4\,kN$

$\Sigma M_A = 0$: $9.4 - 24.3 + V_B.9 = 0$ | $V_A = 20\,kN$



$\cos\alpha = 3/5$ | $N_3 + N_8\cos\alpha + N_{16} = 0$

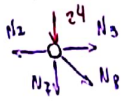
$\operatorname{sen}\alpha = 4/5$ | $-24 + 20 - N_8\operatorname{sen}\alpha = 0$

$\frac{4}{5}N_8 = -4$ | $N_8 = -5$

$\Sigma M_E = 0$: $9.4 + N_{16}.4 - 20.3 = 0$

$N_{16} = 6\,kN$ | $N_3 = -3\,kN$

Equilíbrio no nó E:



$-N_2 + N_3 + N_8\cos\alpha = 0$

$N_2 = 3 - 5.(3/5) \Rightarrow N_2 = -6\,kN$

$-N_2 - N_8\operatorname{sen}\alpha - 24 = 0$

$N_7 = -24 + 5(4/5) \Rightarrow N_7 = -20\,kN$

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