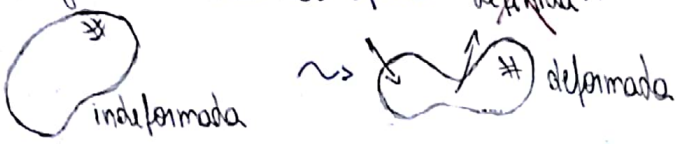
MEC SÓLIDOS

Introdução

Def: **Sólido** ~ é o conjunto conexo de material que possui forma definida quando NÃO sujeito a carregamentos externos (fluido ~ forma definida).

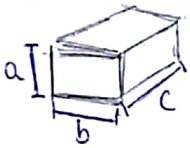


Def.: **Estrutura** ~ é o conjunto de partes resistentes de um objeto, dispositivo ou construção. Partes resistentes são caracterizadas por sua capacidade de transmitir esforços, geralmente sofrendo pequenas deformações.

Def.: **Elementos Estruturais** ~ são partes resistentes com geometria características bem definidas. Classificamos elementos estruturais em:

- Blocos / Estruturas 3D
- Folhas (2D)
- Linhas

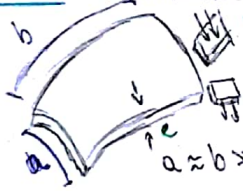
Blocos: 3 dimensões com mesma ordem de grandeza



$a \approx b \approx c$

Ex: barragem de gravidade

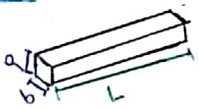
Folhas: 2 dimensões predominantes



- placas: esforços perpendiculares ao plano médio
- chapas: esforços no plano médio
- cascas: curvas

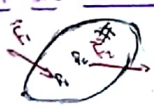
$a \approx b \gg e$

Linhas: 1 dimensão predominante



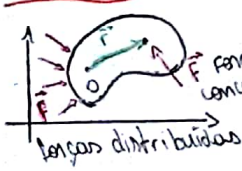
$L \gg a, b$ Ex: treliça, viga, eixo

2ª Lei de Newton: no Equilíbrio:



$$\begin{cases} \vec{R} = \sum \vec{F}_i = \vec{0} \\ \vec{M}_b = \sum (P_i - O) \wedge \vec{F}_i = \vec{0} \end{cases}$$

Problemas Planos



$\{\vec{f}, \vec{F}, \vec{r}\} \in O_{xyz}$

Isso implica que $\vec{M}_O = M_O \vec{k}$

Equilíbrio (3D)

$$\begin{cases} \vec{R} = R_x \vec{i} + R_y \vec{j} + R_z \vec{k} = \vec{0} \\ \vec{M}_O = M_{xO} \vec{i} + M_{yO} \vec{j} + M_{zO} \vec{k} = \vec{0} \end{cases}$$

$$\begin{cases} R_x = \sum F_{i,x} = 0 \\ R_y = \sum F_{i,y} = 0 \\ R_z = \sum F_{i,z} = 0 \end{cases} \qquad \begin{cases} M_{xO} = \sum M^i_{x,O} = 0 \\ M_{yO} = \sum M^i_{y,O} = 0 \\ M_{zO} = \sum M^i_{z,O} = 0 \end{cases}$$

No plano: $\sum F_x = 0$; $\sum F_y = 0$, $\sum M_{Oz} = 0$

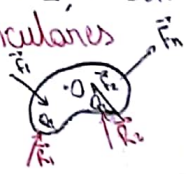
convenção de sinal do momento $\circlearrowleft +$

Def.: **Vínculos ou Apoios** ~ são elementos que conectam a estrutura a outros sistemas estruturais ou ao meio externo.

F_1, F_2, F_n ~ forças externas ativas

Diagrama de corpo livre (DCL)

No DCL, substituem-se os vínculos por reações vinculares



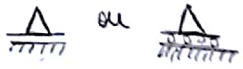
R_1, R_2 ~ forças externas reativas

$$\sum \vec{F}_i + \sum \vec{R}_i = \vec{0}$$

$$\sum (P_i - O) \wedge \vec{F}_i + \sum (Q_i - O) \wedge \vec{R}_i = \vec{0}$$

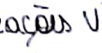
Tipos de Vínculos:

- **Apoio simples**: impede o movimento de translação em 1 direção
  - Reação vincular na direção travada



Ex: anel, mancal

- **Articulações** (apoio duplo): trava o movimento nas duas direções (rotação livre)
  - Reações vinculares nas duas direções



Ex: pino

- **Engaste**: trava translações e rotações
  - Reações vinculares nas direções de translação e o momento no giro

Graus de Estaticidade

↳ relação entre o nº de incógnitas e as equações de equilíbrio

3 incógnitas

3 equações

Isostática

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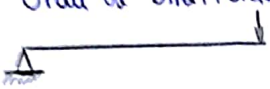
4 incógnitas
3 equações

Hiperestática

Grau de estaticidade: G = nº incóg. − nº eqs. G = 1

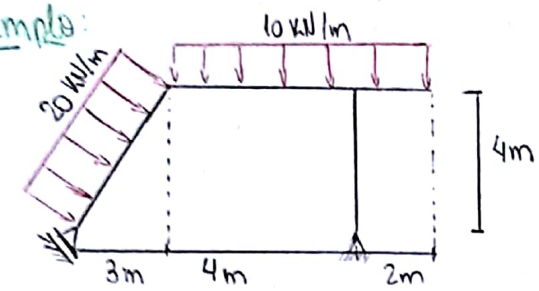


2 incógnitas
3 equações

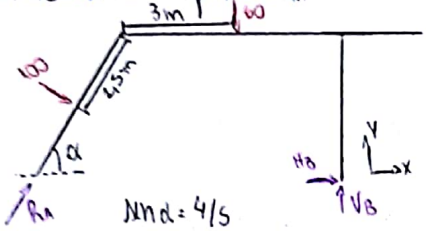
- movimento de corpo rígido

Hipostática

Exemplo:



Mecanicamente equivalente



$\sin\alpha = 4/5$

$\cos\alpha = 3/5$

$\sum F_x = 0: H_B + 100\cos\alpha + R_A\sin\alpha = 0$

$\frac{3R_A}{5} + H_B + 80 = 0$ (I)

$\sum F_y = 0: R_A\sin\alpha - 100\cos\alpha - 60 + V_B = 0$

$\frac{4R_A}{5} + V_B = 120$ (II)

$\curvearrowleft \sum M_A = 0: -100.2{,}5 - 60.6 + V_B.7 = 0$

$7V_B = 250 + 360$

$V_B \cong 87{,}1$ kN

De (II):

$\frac{4R_A}{5} = 120 - V_B$

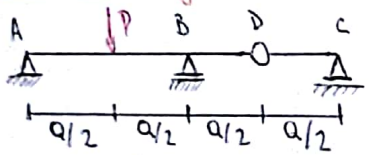
$R_A \cong 41{,}1$ kN

Substituindo em (I)

$H_B = -80 - \frac{3}{5}R_A$

$H_B \cong -104{,}6$ kN

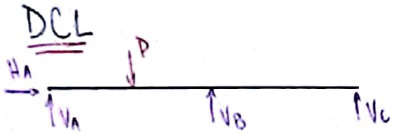
Exemplo: Viga Gerber



4 incógnitas
? equações

Em D temos uma articulação ⇝ não transmite momento

DCL



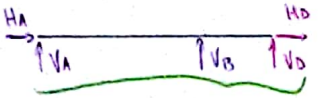
$\sum F_x = 0 \quad H_A = 0$

$\sum F_y = 0 \quad V_A + V_B + V_C = P$ (I)

$\curvearrowleft \sum M_A = 0 \quad -P\frac{a}{2} + V_B a + V_C 2a = 0$

$V_B + 2V_C = P/2$ (II)

Cortando em D

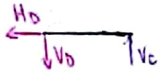


$\sum F_x = 0: -H_A = H_D \quad H_D = 0$

$\sum F_y = 0: V_A + V_B + V_D = P$ (III)

$\curvearrowleft \sum M_A = 0: -P\frac{a}{2} + V_B a + V_D\frac{3}{2}a = 0$

$V_B + \frac{3}{2}V_D = P/2$ (IV)



Comparando (IV) com (II)

$2V_C = \frac{3}{2}V_B$

Comparando (I) com (III):

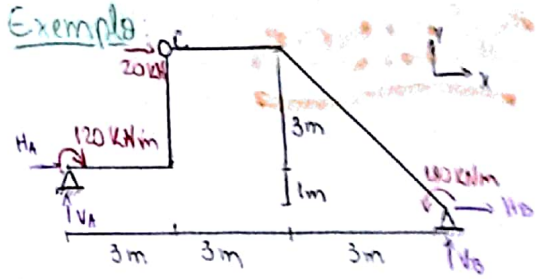
$V_C = V_D$

Logo:

$V_B = P/2$

$V_C = V_D = 0 \quad V_A = P/2$

Exemplo:



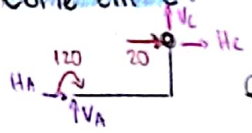
$\sum F_x = 0: H_A + H_B + 20 = 0$ (I)

$\sum F_y = 0: V_A + V_B = 0$ (II)

$\curvearrowleft \sum M_A = 0: -120 + 180 - 20.3 + H_B.1 + V_B.9 = 0$

$H_B + 9V_B = 0$ (III)

Corte em C:



$\sum F_x = 0: H_A + H_C + 20 = 0$ (IV)

$\sum F_y = 0: V_A + V_C = 0$ (V)

$\curvearrowleft \sum M_C = 0: -120 + H_A.3 - V_A.3 = 0$

$H_A - V_A = 40$ (VI)

(I) e (IV): $H_B = H_C \quad H_B = -20 - H_A$

(II) e (V): $V_B = V_C \quad V_B = -V_A$

$H_B + 9V_B = 0 \Rightarrow -20 - H_A - 9V_A = 0$

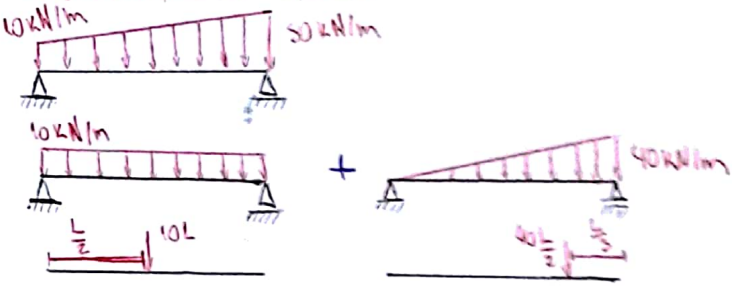
$H_B = -54$ kN

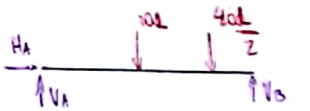
$H_A = 34$ kN

$V_A = -6$ kN

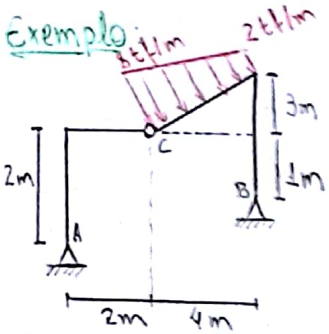
$V_B = 6$ kN

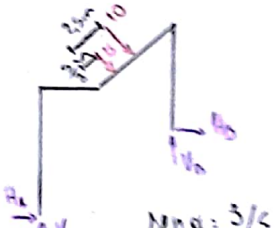
Superposição de efeitos:





Exemplo:





$\sin\alpha = 3/5$

$\cos\alpha = 4/5$

$\sum F_x = 0: H_A + H_B + 15\sin\alpha + 10\sin\alpha = 0$

$\sum F_y = 0: V_A + V_B - 15\cos\alpha - 10\cos\alpha = 0$

$\sum M_A = 0: -15\cos\alpha(2 + 4/3) - 15\sin\alpha.3 - 10\cos\alpha.4 - 10\sin\alpha.3{,}5 + V_B.6 - H_B.1 = 0$

$H_A + H_B = -15$ (I)

$V_A + V_B = 20$ (II)

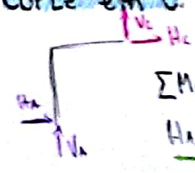
$6V_B - H_B = 120$ (III)

$V_B = 17$ tf

$H_B = -18$ tf

$V_A = H_A = 3$ tf

Corte em C:



$\sum M_C = 0 \quad -V_A.2 + H_A.2 = 0$

$H_A = V_A$

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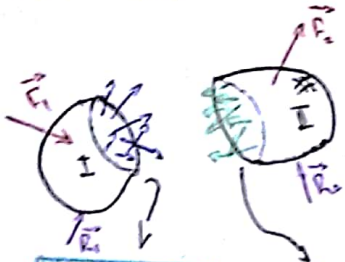
Esforços Internos

$\vec{F}_1, \vec{F}_2 \rightsquigarrow$ forças ativas

$\vec{R}_1, \vec{R}_2 \rightsquigarrow$ forças reativas



Ação de I em II	$= -$	Ação de II em I

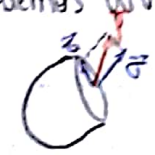
Tensão média

$\hookrightarrow \vec{p}_m(P, \Delta A, \vec{n}) = \frac{\Delta \vec{F}}{\Delta A}$, $\vec{n}$ normal ao plano

Admitindo uma distribuição contínua dos esforços internos, definimos tensão como:

$$\vec{p}(P, \vec{n}) = \lim_{\Delta A \to 0} \frac{\Delta \vec{F}}{\Delta A} \quad \left[\frac{N}{m^2}\right] = [Pa]$$

Podemos dividir a tensão em 2 componentes:



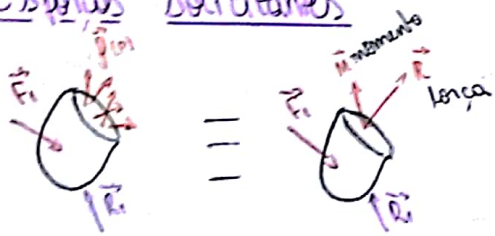
$\vec{p} = \vec{\sigma} + \vec{\tau}$

$\vec{\sigma}$: normal — $\vec{\tau}$: tangencial ou cisalhamento

Onde $\vec{\sigma} = \sigma \vec{n}$, $\vec{n} \perp S$

$\vec{\tau} = \vec{p} - \vec{\sigma}$

Esforços Solicitantes



$\{\vec{R}, \vec{M}\}$ é mecanicamente equivalente a $\{\vec{p}(P), P \in S\}$

$$\vec{R} = \int_S \vec{p}(P)\, dA$$

$$\vec{M} = \int_S (P - O) \wedge \vec{p}(P)\, dA$$

Esforços Solicitantes

O equilíbrio exige que

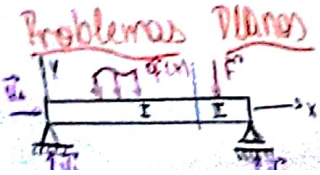
$$\sum \vec{F}_i + \sum \vec{R}_i + \vec{R} = 0$$

$$\sum (P_i - O) \wedge \vec{F}_i + \sum (Q_i - O) \wedge \vec{R}_i + \vec{M} = 0$$

$$\vec{R} = -\sum \vec{F}_i - \sum \vec{R}_i$$

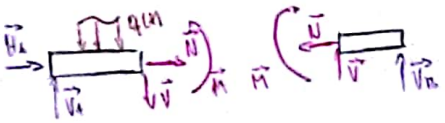
$$\vec{M} = -\sum (P_i - O) \wedge \vec{F}_i - \sum (Q_i - O) \wedge \vec{R}_i$$

Problemas Planos

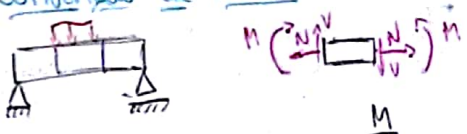


N, M, V $\rightsquigarrow$ esforços solicitantes

- N: força normal
- V: força cortante
- M: momento fletor



Convenção de sinais:



N saindo da seção

$N > 0 \rightsquigarrow$ tração

$N < 0 \rightsquigarrow$ compressão

M

$M > 0 \rightsquigarrow$ tração embaixo

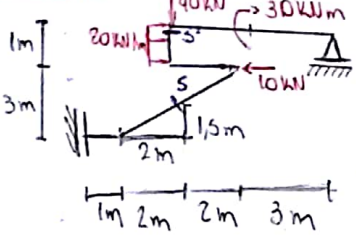
$M < 0 \rightsquigarrow$ tração em cima

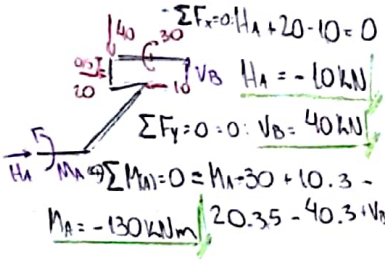
V

$V > 0 \rightsquigarrow$ giro no sentido horário

$V < 0 \rightsquigarrow$ giro no sentido anti-horário

Exemplo: Determinar os esforços solicitantes em S e S'





$\sum F_x = 0: H_A + 20 - 10 = 0$

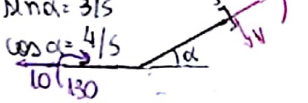
$H_A = -10\,kN$

$\sum F_y = 0: V_B = 40\,kN$

$\sum M_{(A)} = 0: M_A - 30 + 10 \cdot 3 - 20 \cdot 3 \cdot 5 - 40 \cdot 3 + V_B \cdot 5 = 0$

$M_A = -130\,kN.m$

Seção S:



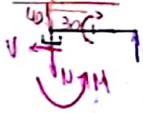
$\sum F_x = 0: -10 + N\cos\alpha + V\sin\alpha = 0$

$\sum F_y = 0: N\sin\alpha = V\cos\alpha$

$N = 8\,kN$, $V = 6\,kN$

$\sum M_S = 0: M - 130 - 10 \cdot 1{,}5 = 0$ $\quad M = 145\,kN.m$

Seção S':



$\sum F_x = 0: \quad V = 0$

$\sum F_y = 0: \quad -N - 40 + 40 = 0 \Rightarrow N = 0$

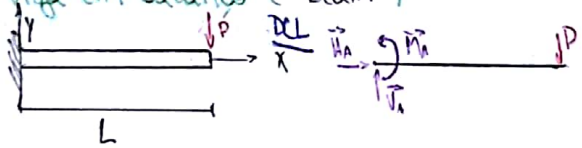
$\sum M_{S'} = 0: M - 30 + 40 \cdot 5 = 0 \Rightarrow M = -170\,kN.m$

Diagrama de esforços solicitantes

↳ cortar em uma posição genérica

↳ equilibrar

Viga em balanço (cantilever beam)

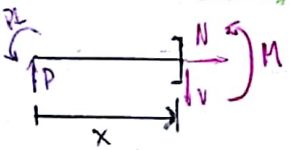


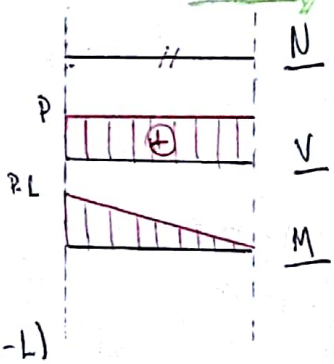
$\sum F_x = 0 \Rightarrow H_A = 0$

$\sum F_y = 0 \Rightarrow V_A = P$

$\sum M_{(A)} = 0: M_A - P \cdot L = 0$

$M_A = P \cdot L$





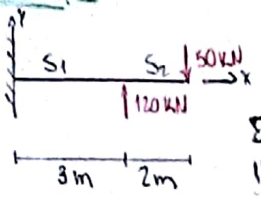
$\sum F_x = 0: N = 0$

$\sum F_y = 0: V_A = P$

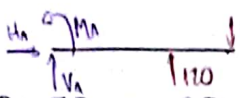
$\sum M_S = 0: M = -P.(x - L)$

Scanned by CamScanner

Exemplo



DCL:

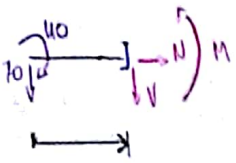


$\Sigma F_x = 0$: $H_A = 0$

$\Sigma F_y = 0$: $V_A = -70\,kN$

$\Sigma M_{(A)} = 0$: $M_A = -110\,kN.m$

Seção S_1: $0 < x < 3m$



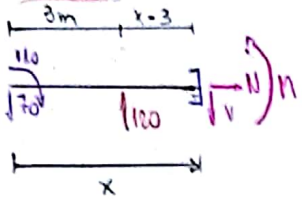
$\Sigma F_x = 0$: $N = 0$

$\Sigma F_y = 0$: $V = -70\,kN$

$\Sigma M_{(S1)} = 0$: $M - 110 + 70x = 0$

$M = 110 - 70x$

Seção S_2: $3 < x < 5m$

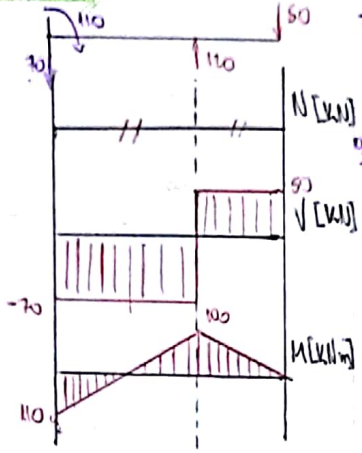


$\Sigma F_x = 0$: $N = 0$

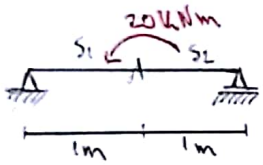
$\Sigma F_y = 0$: $V = 50\,kN$

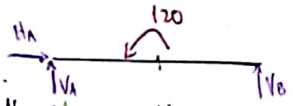
$\Sigma M_{(S2)} = 0$: $M = 50x - 250$

$M + 70x - 110 - 120(x-3) = 0$



Exemplo





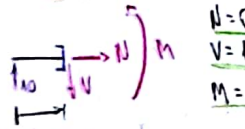
$H_A = 0$

$V_B \cdot 2 = -20 \Rightarrow V_B = -10$

$V_A = -V_B$

$V_A = 10$

Trecho S_1: $0 < x < 1m$

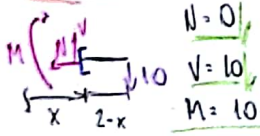


$N = 0$

$V = 10$

$M = 10x$

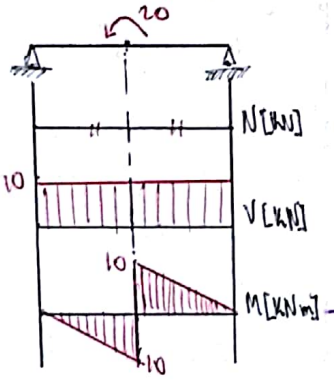
Trecho S_2: $1 < x < 2m$



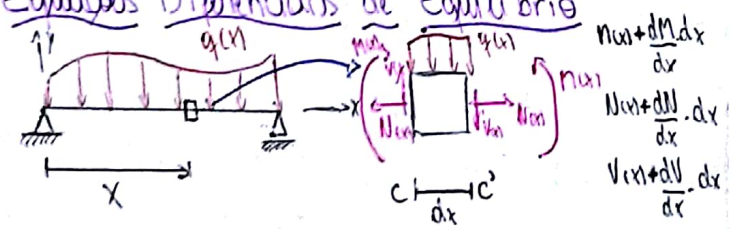
$N = 0$

$V = 10$

$M = 10x - 20$



Equações Diferenciais de Equilíbrio



Fazendo o equilíbrio:

$\Sigma F_y = 0$: $V(x) - q(x)dx - \left(V(x) + \frac{dV}{dx}dx\right) = 0$

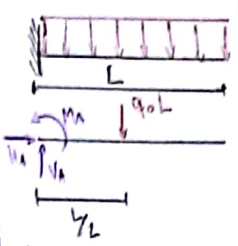
$\frac{dV}{dx}dx = -q(x)dx \Rightarrow \frac{dV}{dx} = -q(x)$ (I)

$\Sigma M_C = 0$: $-M(x) + \left(M(x) + \frac{dM}{dx}dx\right) - V(x)dx + q(x)dx\frac{dx}{2} = 0$

$\frac{dM}{dx}dx = V(x)dx \Rightarrow \frac{dM}{dx} = V(x)$ (II)

$\frac{d(II)}{dx} \sim \frac{d}{dx}\left(\frac{dM}{dx}\right) = \frac{d(V)}{dx} \Rightarrow \frac{d^2M}{dx^2} = -q(x)$

Exemplo

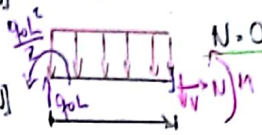


$q(x) = q_0$

$H_A = 0$

$V_A = q_0 L$

$M_A = \frac{q_0 L^2}{2}$



$\frac{dV}{dx} = -q(x) \Rightarrow \frac{dV}{dx} = -q_0$

$V(x) = \int -q_0 dx = -q_0 x + C_1$

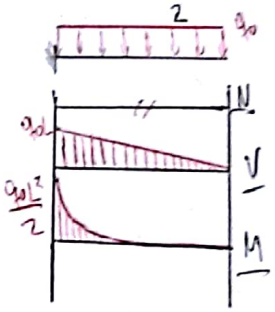
Para $x = 0$, $V(0) = q_0 L \rightsquigarrow C_1 = q_0 L$

$V(x) = q_0 L - q_0 x$

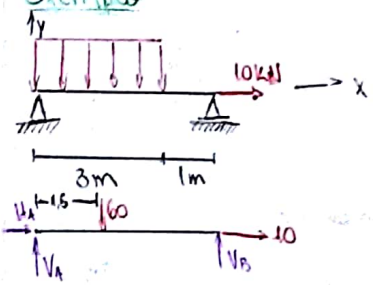
$\frac{dM}{dx} = V(x) \Rightarrow \frac{dM}{dx} = q_0 L - q_0 x \Rightarrow M(x) = q_0 L - \frac{q_0 x^2}{2} + C_2$

Para $x = 0$, $M(0) = -\frac{qL^2}{2}$

$M(x) = -\frac{q_0 x^2}{2} + q_0 L x - \frac{q_0 L^2}{2}$



Exemplo



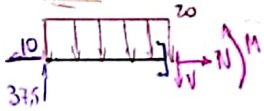
$H_A = -10\,kN$

$V_A + V_B = 60$

$60 \cdot 1,5 = V_B \cdot 4$

$V_B = 22,5\,kN$, $V_A = 37,5\,kN$

Trecho 1: $(0 < x < 3m)$



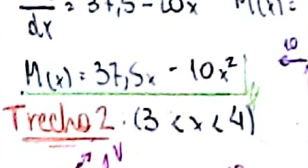
$N = 10\,kN$, $q(x) = 20$, $\frac{dV}{dx} = -20 \Rightarrow V(x) = -20x + C_1$

$V(x) = 37,5 - 20x$ $(V(0) = 37,5)$

$\frac{dM}{dx} = 37,5 - 20x$ $M(x) = 37,5x - 10x^2 + C_2$ $(M(0) = 0)$

$M(x) = 37,5x - 10x^2$

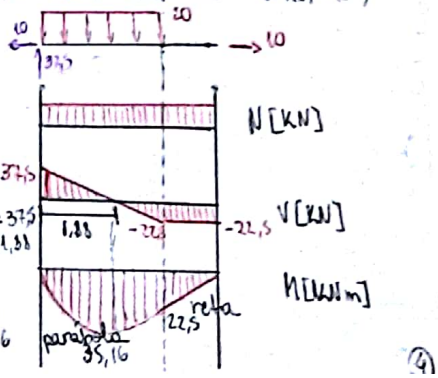
Trecho 2: $(3 < x < 4)$



$N = 10\,kN$, $V = -22,5$

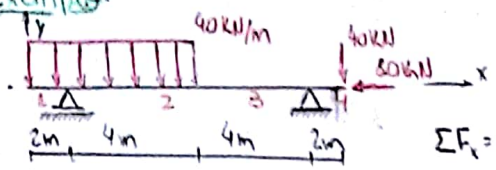
$M = 22,5(4-x)$

$M = -22,5x + 90$



Scanned by CamScanner

Exemplo:



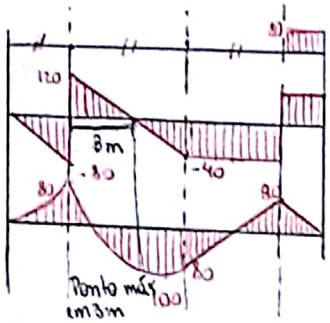
$\Sigma F_x = 0$: $H_A = 80\,kN$

$\Sigma F_y = 0$: $V_A + V_B = 280$

$\circlearrowleft \Sigma M_A = 0$: $-240 \cdot 1 + V_B \cdot 8 - 40 \cdot 10 = 0$

$V_B = 80\,kN$

$V_A = 200\,kN$



Utilizando apenas os valores de forças aplicadas e montando o gráfico

Utilizando as equações:

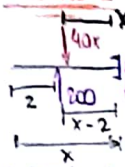
Trecho 1: $(0 < x < 2m)$

$N = 0$

$V = -40x$

$M = -20x^2$

Trecho 2 $(2 < x < 6m)$



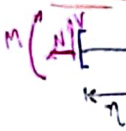
$N = 0$

$V = 200 - 40x$

$M = -20x^2 + 200x - 400$

$M = 200(x-2) - 20x^2$ $\quad \bar{x} = 5m \Rightarrow M = 100\,kN.m$

Trecho 4 $(0 < \eta < 2m)$

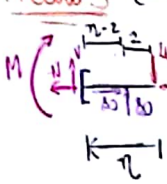


$N = 80\,kN$

$V = 40\,kN$

$M = -40\eta$

Trecho 3: $(2 < \eta < 6m)$



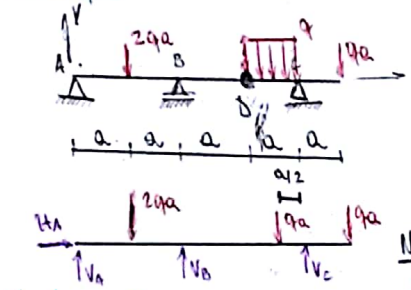
$N = 0$

$V = -40\,kN$

$M = 40\eta - 160$

$M = 80(\eta - 2) - 40\eta$

Exemplo:



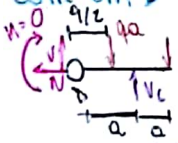
$\Sigma F_x = 0$: $H_A = 0$

$\Sigma F_y = 0$: $V_A + V_B + V_C = 4qa$

$\circlearrowleft \Sigma M_A = 0$: $-2qa \cdot a + V_B \cdot 2a - qa \cdot 3{,}5a + V_C \cdot 4a - qa \cdot 5a = 0$

$2V_B + 4V_C = 10{,}5\,qa$

Corte em D



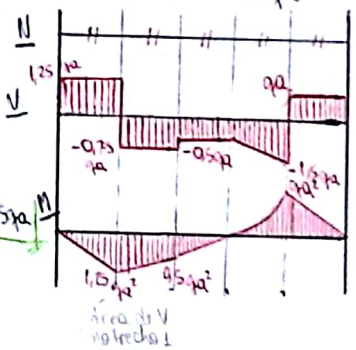
$\circlearrowleft \Sigma M_{(D)} = 0$:

$-qa \cdot 0{,}5a + V_C \cdot a - qa \cdot 2a = 0$

$V_C = 2{,}5\,qa$

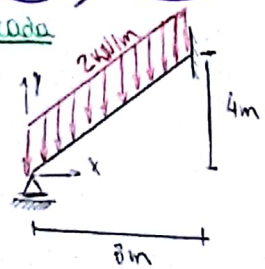
$V_B = 0{,}25qa$

$V_A = 1{,}25qa$

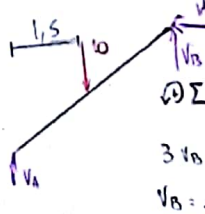


Vigas Inclinadas

Escada



$\operatorname{sen}\alpha = 0{,}8$ $\quad \cos\alpha = 0{,}6$



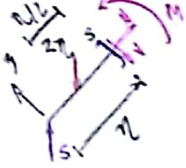
$\Sigma F_x = 0$: $H_B = 0$

$\Sigma F_y = 0$: $V_A + V_B = 10$

$\circlearrowleft \Sigma M_A = 0$: $-10 \cdot 1{,}5 + V_B \cdot 3 + H_B \cdot 4 = 0$

$3V_B = 15$

$V_B = 5\,kN$ $\quad V_A = 5\,kN$



$\Sigma F_x = 0$: $N + 5\operatorname{sen}\alpha - 2\eta \operatorname{sen}\alpha = 0$

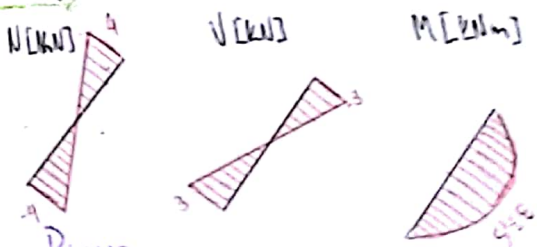
$N = 1{,}6\eta - 4$

$\Sigma_y = 0$: $-V + 5\cos\alpha - 2\eta\cos\alpha = 0$

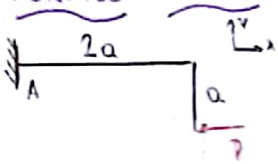
$V = 3 - 1{,}2\eta$

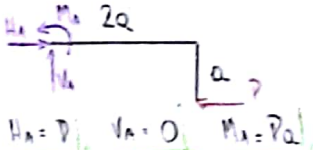
$\circlearrowleft \Sigma M_3 = 0$: $-5\cos\alpha\,\eta + 2\eta\cos\alpha\,\frac{\eta}{2} + M = 0$

$M = 3\eta - 0{,}6\eta^2$



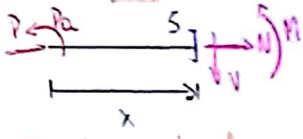
Pórtico Plano





$H_A = P$ $\quad V_A = 0$ $\quad M_A = Pa$

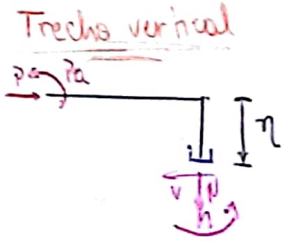
Trecho horizontal



$N = -P$ $\quad M = -Pa$

$V = 0$

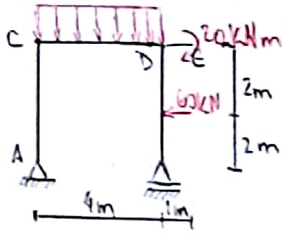
Trecho vertical

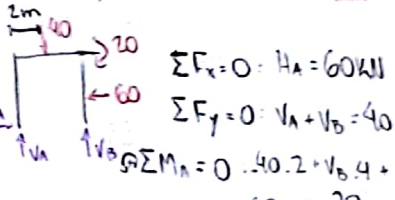


$V = P$ $\quad M + Pa - P\eta = 0$

$N = 0$ $\quad M = P(\eta - a)$

Exemplo





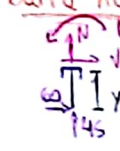
$\Sigma F_x = 0$: $H_A = 60\,kN$

$\Sigma F_y = 0$: $V_A + V_B = 40$

$\circlearrowleft \Sigma M_A = 0$: $-40 \cdot 2 + V_B \cdot 4 + 60 \cdot 2 = 20$

$V_B = -5\,kN$ $\quad V_A = 45\,kN$

Barra AC $(0 < y < 4m)$



$N = -45\,kN$

$V = -60\,kN$

$M = -60y$

Barra CD $(0 < x < 4m)$

$N = -60$

$V = 45 - 10x$

$M = 45x - 5x^2 - 240$

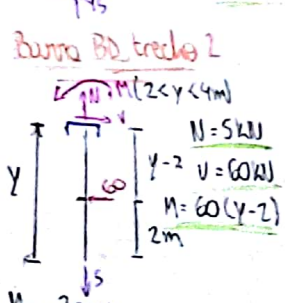
Barra BD trecho 1 $(0 < y < 2m)$

$N = 5\,kN$

$V = 0$

$M = 0$

Barra BD trecho 2 $(2 < y < 4m)$



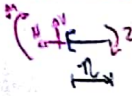
$N = 5\,kN$

$V = 60\,kN$

$M = 60(y-2)$

$M = -20\,kN.m$

Barra DE



$N = 0$

$V = 0$

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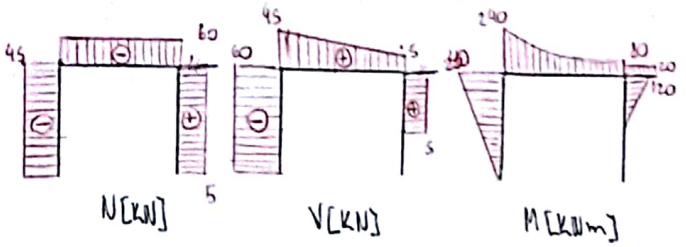
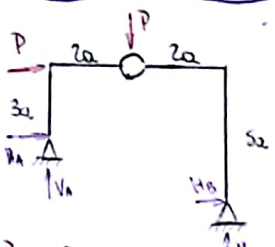


PÓRTICO ARTICULADO



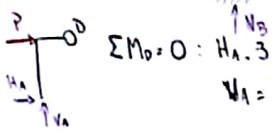
$H_A + H_B = -P$

$V_A + V_B = P$

$-H_A \cdot 2a - V_B \cdot 4a - P \cdot 3a + P \cdot 2a = 0$

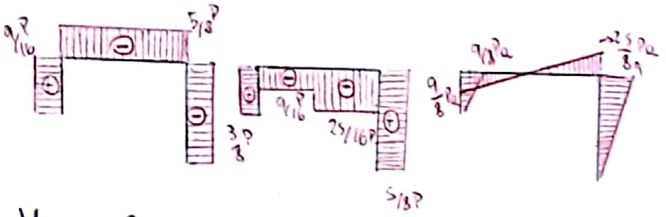
$4V_B + 2H_A = -3P$

$H_A = -3/8\,P$ $\quad V_A = -9/16\,P$



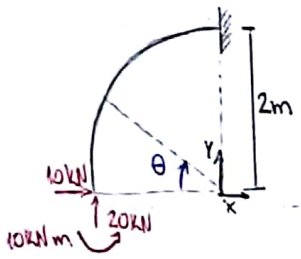
$\Sigma M_D = 0: \; H_A \cdot 3a = V \cdot 2a$

$V_A = 1.5\,H_A$ $\quad V_B = 25/16\,P$ $\quad H_B = -3/8\,P$

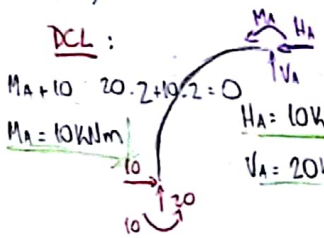


VIGAS CURVAS

Esforços solicitantes em funções de θ.



DCL:



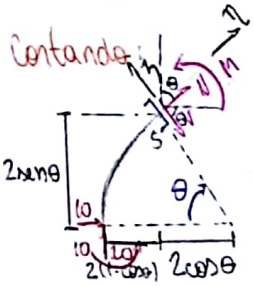
$M_A + 10 \cdot 2 + 10 \cdot 2 = 0$

$M_A = 10\,kN.m$

$H_A = 10\,kN$

$V_A = 20\,kN$

Contando:



$\Sigma F_\eta = 0: \; N + 10\sin\theta + 20\cos\theta = 0$

$N = -10\sin\theta - 20\cos\theta$

$\Sigma F_\zeta = 0: \; -V - 10\cos\theta + 20\sin\theta = 0$

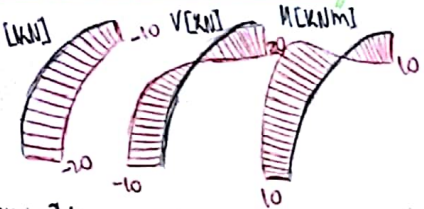
$V = -10\cos\theta + 20\sin\theta$

$\Sigma M_S = 0: \; M + 10 + 10 \cdot 2\sin\theta - 20 \cdot 2(1-\cos\theta) = 0$

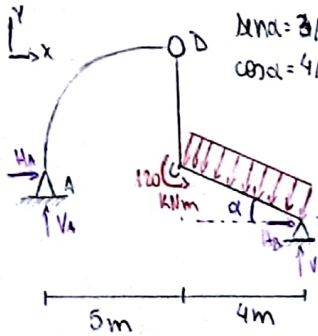
$M = -40\cos\theta - 20\sin\theta + 30$

Note que:

$$\frac{1}{R}\cdot\frac{dM}{d\theta} = V$$



Exercício:



$\sin\alpha = 3/5$

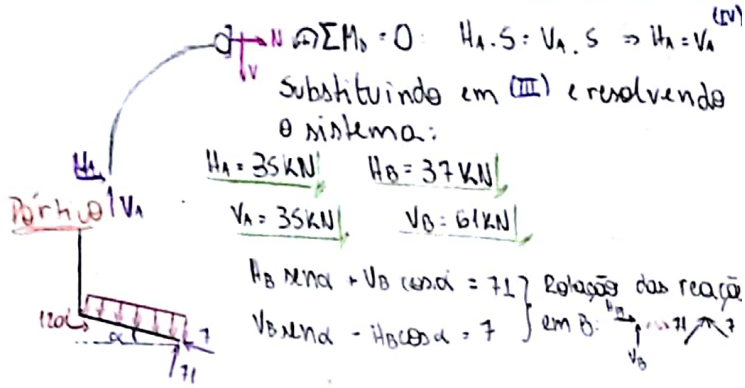
$\cos\alpha = 4/5$

(I) $\Sigma F_x = 0: \; H_A + H_B = 120\sin\alpha$

(II) $\Sigma F_y = 0: \; V_A + V_B = 120\cos\alpha$

(III) $\Sigma M_B = 0: \; 120 \cdot 2{,}5 + 120 - V_A \cdot 9 - H_A \cdot 3 = 0$

$H_A + 3V_A = 140$ (III)



$\Sigma M_D = 0: \; H_A \cdot 5 = V_A \cdot 5 \Rightarrow H_A = V_A$ (IV)

Substituindo em (III) e resolvendo o sistema:

$H_A = 35\,kN$ $\quad H_B = 37\,kN$

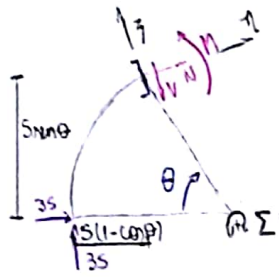
$V_A = 35\,kN$ $\quad V_B = 61\,kN$

Pórtico

$H_B \sin\alpha + V_B \cos\alpha = 71$

$V_B \sin\alpha - H_B \cos\alpha = 7$

} Relação das reações em B:



$\Sigma F_\eta = 0: \; N + 35\cos\theta + 35\sin\theta = 0$

$N = -35(\sin\theta + \cos\theta)$

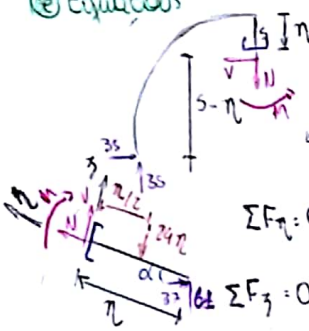
$\Sigma F_\zeta = 0: \; -V + 35\sin\theta - 35\cos\theta = 0$

$V = 35(\sin\theta - \cos\theta)$

$\circlearrowleft \Sigma M_S = 0: \; M + 35 \cdot 5\sin\theta - 35 \cdot 5(1-\cos\theta) = 0$

$M = 175(1 - \cos\theta - \sin\theta)$

(IV) Equações



$\Sigma F_\eta = 0: \; V = 35\,kN$

$\Sigma F_\zeta = 0: \; N = 35\,kN$

$\circlearrowleft \Sigma M_S = 0: \; M - 35 \cdot 5 + (5-\eta)35 = 0$

$M = 35\eta$

$\Sigma F_\eta = 0: \; N + 61\sin\alpha - 37\cos\alpha = 0$

$N = -7\,kN$

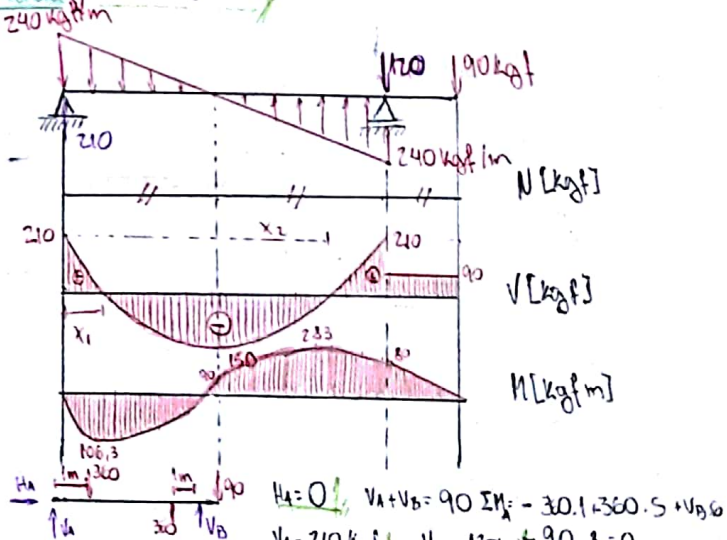
$\Sigma F_\zeta = 0: \; V - 24\eta + 37\sin\alpha + 61\cos\alpha = 0$

$V = 24\eta - 71$

$\circlearrowleft \Sigma M_S = -12\eta^2 - M + 37\sin\alpha\,\eta + 61\cos\alpha\,\eta = 0$

$M = -12\eta^2 + 71\eta$

Exercício



$q(x) = 240 - 80x$

$\frac{dV}{dx} = -q(x) = 80x - 240$

$V(x) = 40x^2 - 240x + C_1$

$V(0) = 210$ $\quad V(x) = 0 \Rightarrow x_1 = 1{,}0635$ ou $x_2 = 4{,}9365$

$V(x) = 40x^2 - 240x + 210$

$H_A = 0$ $\quad V_A + V_B = 90$ $\quad \Sigma M_A = -30 \cdot 1 + 360 \cdot 5 + V_B \cdot 6 - 90 \cdot 8 = 0$

$V_A = 210\,kgf$ $\quad V_B = -120\,kgf$

$\frac{dM}{dx} = V(x) = 40x^2 - 240x + 210$

$M(x) = \frac{40}{3}x^3 - 120x^2 + 210x + C_2$

$M(0) = 0$ $\quad M(x_1) = 103{,}6$ $\quad M(x_2) = -283$

$M(x) = \frac{40}{3}x^3 - 120x^2 + 210x$

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