

1.2) a) $\frac{dy}{dx} = y^2 \Rightarrow \frac{dy}{y^2} = dx \Rightarrow -\frac{1}{y} = x + C_1 \Rightarrow$

$\Rightarrow y = \frac{1}{C_2 - x}, C_2 \neq 0$ para todo $x \neq C_2 \in \mathbb{R}$

Se $y=0: y'=0: y(x)=0$ é solução para todo $x \in \mathbb{R}$

b) $x \frac{dy}{dx} = y \Rightarrow \frac{dy}{y} = \frac{dx}{x} \Rightarrow \ln|y| = \ln|x| + C_1$

$\Rightarrow |y| = e^{\ln|x| + C_1} \Rightarrow |y| = C_2 |x|, C_2 > 0 \Rightarrow$

$\Rightarrow y = C_3 |x| \Rightarrow y = Cx, \forall x \in \mathbb{R} - \text{holo} \text{ e } C \neq 0$

$x=0 \Rightarrow y=0: y(x)=0$ é solução

$y(x) = Cx, \forall x \in \mathbb{R} \text{ e } C \in \mathbb{R}$

c) $y \frac{dy}{dx} = x \Rightarrow y dy = x dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C_1 \Rightarrow$

$\Rightarrow y^2 = x^2 + C_2 \Rightarrow y(x) = \pm \sqrt{x^2 + C_2}$, com

$\begin{cases} x^2 + C_2 > 0 \Rightarrow x^2 > -C_2, \text{ se } C_2 < 0 \\ x \in \mathbb{R}, \text{ se } C_2 \geq 0 \end{cases}$

e) $\frac{dy}{dx} + \left(-\frac{3}{x}y\right) = 1, \text{ se } x \neq 0$

Note que temos uma equação linear de primeira ordem. Portanto, o fator integrante será dado por

$g(x) = e^{-3 \int \frac{1}{x} dx} = e^{-3 \ln|x| + C_1}$. Tomamos

$C_1 = 1: g(x) = e^{-3 \ln|x|} = e^{\ln|x|^{-3}} = |x|^{-3}$

Do fator integrante na equação

$|x|^{-3} \frac{dy}{dx} - \frac{3|x|^{-3}}{x} y = |x|^{-3} \Rightarrow$

$\Rightarrow x^{-3} \frac{dy}{dx} - 3x^{-4} y = x^{-3} \Rightarrow \frac{d}{dx}(x^{-3} y) = x^{-3} \Rightarrow$

regra do produto

$\Rightarrow x^{-3} y = \int x^{-3} dx \Rightarrow x^{-3} y = -\frac{1}{2x^2} + C \Rightarrow$

$\Rightarrow y(x) = x^3 C - \frac{x}{2}, x \in \mathbb{R} - \text{holo} \text{ e } C \in \mathbb{R}$

Se $x=0 \Rightarrow y=0: y(x) = x^3 C - \frac{x}{2}, x \in \mathbb{R} \text{ e } C \in \mathbb{R}$

1) $\frac{dy}{dx} = 2y + e^x \Rightarrow \frac{dy}{dx} - 2y = e^x$

Note que novamente temos uma equação linear de primeira ordem. Portanto, o fator integrante será dado por:

$g(x) = e^{\int -2 dx} = e^{-2x} + C_1$

Tomando $C_1 = 1: g(x) = e^{-2x}$

Do fator integrante na equação:

$e^{-2x} \frac{dy}{dx} - 2e^{-2x} y = e^x \cdot e^{-2x} \Rightarrow$

$\Rightarrow \frac{d}{dx}(e^{-2x} y) = e^{-x} \Rightarrow e^{-2x} y = \int e^{-x} dx \Rightarrow$

$\Rightarrow \frac{y}{e^{2x}} = -e^{-x} + C \Rightarrow y = -e^x + e^{2x} C,$

$\forall x \in \mathbb{R} \text{ e } C \in \mathbb{R}$.

3) a) $\begin{cases} \frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x \text{ (I)} \\ y(0) = 1 \end{cases}$

(I) é uma EDO Linear de 1ª ordem:

$g(x) = e^{\int -1 dx} = e^{-x} + C_1 \xrightarrow{C_1=1} g(x) = e^{-x} \text{ (II)}$

Multiplicando (I) por (II):

$e^{-x} \frac{dy}{dx} - e^{-x} y = x e^{-x} \Rightarrow$

$\Rightarrow \frac{d}{dx}(e^{-x} y) = x e^{-x} \Rightarrow \frac{y}{e^x} = \int x e^{-x} dx \Rightarrow$

$\begin{cases} \alpha(x) = x \Rightarrow \alpha'(x) = 1 \\ \beta'(x) = e^{-x} \Rightarrow \beta(x) = -e^{-x} \end{cases}$

$\Rightarrow \frac{y}{e^x} = x e^{-x} - \int -e^{-x} dx = x e^{-x} - e^{-x} + C \Rightarrow$

$\Rightarrow y = -x - 1 + C e^x$

Da condição inicial:

$y(0) = 1 \Rightarrow 1 = 0 - 1 + C \Rightarrow C = 2$.

A solução particular será: $y(x) = 2e^x - x - 1$

$$b) (\cos t) \frac{dx}{dt} - \sin t \cdot x = 1 \rightarrow$$

$$\rightarrow \frac{dx}{dt} - \frac{\sin t}{\cos t} x = \frac{1}{\cos t} \quad (I)$$

Note que x depende de t e que temos uma EDO Linear de Primeira Ordem.

$$g(x) = e^{-\int \frac{\sin t}{\cos t} dt} = e^{\int \frac{1}{u} du} = e^{\ln|\cos t| + C_1}$$

$u = \cos t \Rightarrow \frac{du}{dt} = -\sin t \Rightarrow -du = \sin t dt$

Tomando $C_1 = 1$: $g(x) = e^{\ln|\cos t|} = |\cos t|$

Multiplicando (I) por (II):

$$|\cos t| \frac{dx}{dt} - \frac{|\cos t| \sin t}{\cos t} x = \frac{|\cos t|}{\cos t} \rightarrow$$

$$\Rightarrow \cos t \frac{dx}{dt} - \sin t x = 1 \rightarrow \frac{d}{dt} (\cos t \cdot x) = 1 \rightarrow$$

$$\rightarrow \cos t \cdot x = t + C \rightarrow x = \frac{t+C}{\cos t}$$

Da condição inicial:

$$x(2\pi) = \pi \rightarrow \pi = \frac{2\pi + C}{\cos 2\pi} \rightarrow C = -\pi$$

$$\therefore x(t) = \frac{t - \pi}{\cos t}$$

$$c) \begin{cases} \frac{dy}{dx} = x(1+y) \Rightarrow \frac{dy}{1+y} = x dx \quad (I) \\ y(0) = -1 \end{cases}$$

De (I): $\int \frac{dy}{1+y} = \int x dx \rightarrow \ln|1+y| = \frac{x^2}{2} + C_1$

$$\Rightarrow |1+y| = e^{\frac{x^2}{2} + C_1} \Rightarrow |1+y| = C_2 e^{x^2/2}, C_2 > 0$$

$$\rightarrow 1+y = C_3 e^{x^2/2}, C_3 \neq 0 \rightarrow$$

$$\rightarrow y = C_3 e^{x^2/2} - 1$$

Da condição inicial:

$$y(0) = -1 \rightarrow -1 = C_3 - 1 \rightarrow C_3 = 0$$

A solução particular é $y(x) = -1$.

$$4) a) \begin{cases} \frac{dy}{dx} = 5y^{4/5} \quad (I) \\ y(0) = 0 \end{cases}$$

De (I): $y \neq 0$

$$\frac{dy}{5y^{4/5}} = dx \rightarrow \int \frac{dy}{5y^{4/5}} = \int dx \Rightarrow \frac{5}{5} y^{1/5} = x + C$$

$$\rightarrow y = (x+C)^5$$

Da condição inicial:

$$y(0) = 0 \rightarrow 0 = (0+C)^5 \rightarrow C = 0$$

As soluções particulares são $y(x) = x^5$ e $y(x) = 0$

$$f(x,y) = 5y^{4/5} \rightarrow \frac{\partial f}{\partial y} = 5 \cdot \frac{4}{5} y^{-1/5} = 4y^{-1/5}$$

não é contínua em $y=0$: não respeita a hipótese do Teorema de Cauchy \rightarrow pode haver mais de uma solução.

$$b) \begin{cases} \frac{dy}{dx} = 3y^{2/3}(3x^2+1) = 0 \quad (I) \\ y(0) = 0 \end{cases}$$

De (I):

$$\frac{dy}{3y^{2/3}} = (3x^2+1) dx \rightarrow \int \frac{dy}{3y^{2/3}} = \int (3x^2+1) dx$$

$$\rightarrow \frac{1}{3} \cdot 3 y^{1/3} = x^3 + x + C \rightarrow y = (x^3 + x + C)^3$$

Da condição inicial:

$$y(0) = 0 \rightarrow 0 = C^3 \rightarrow C = 0$$

As soluções particulares são: $\begin{cases} y(x) = (x^3 + x)^3 \\ y(x) = 0 \end{cases}$

$$f(x,y) = 3y^{2/3}(3x^2+1) \rightarrow$$

$$\rightarrow \frac{\partial f}{\partial y} = 2y^{-1/3}(3x^2+1) \text{ não é contínua em } y=0$$

$y=0$: não satisfaz as hipóteses do Teorema de Cauchy.

$$15) a) \frac{dy}{dx} = e^x - 2y = \frac{e^x}{e^{2y}} \rightarrow e^{2y} dy = e^x dx \rightarrow$$

$$\rightarrow \int e^{2y} dy = \int e^x dx \rightarrow \frac{e^{2y}}{2} = e^x + C_1 \rightarrow$$

$$\rightarrow e^{2y} = 2e^x + 2C_1 \rightarrow 2y = \ln|2e^x + C_2| \rightarrow$$

$$\rightarrow y = \ln(\sqrt{2e^x + C_2})$$

$$b) x^2 y^2 dy = (1+x^2) dx \rightarrow y^2 dy = \left(\frac{1}{x^2} + 1\right) dx \rightarrow$$

$$\rightarrow \int y^2 dy = \int 1 + \frac{1}{x^2} dx \Rightarrow \frac{y^3}{3} = x - \frac{1}{x} + C_1 \rightarrow$$

$$\rightarrow y^3 = 3x - \frac{3}{x} + C_1 \rightarrow y = \sqrt[3]{3x - \frac{3}{x} + C_1} \rightarrow$$

$$\rightarrow y = \sqrt{\frac{3x^2 - 3 + C_1 x}{x}}, C_1 \in \mathbb{R}$$

c) $y' \sin x + y \cos x = 1 \rightarrow y' + \frac{\cos x}{\sin x} y = \frac{1}{\sin x}$

$$g(x) = e^{\int \frac{\cos x}{\sin x} dx} = e^{\int \frac{1}{u} du} = e^{\ln|\sin x|} + C_1 \rightarrow$$

$$u = \sin x \Rightarrow du = \cos x \cdot dx$$

$$\Rightarrow g(x) = C_2 \cdot e^{\ln|\sin x|} \rightarrow g(x) = C_2 |\sin x|$$

Para $C_2 = 1: g(x) = |\sin x|$

$$|\sin x| \cdot \frac{dy}{dx} + \frac{\cos x}{\sin x} \cdot |\sin x| y = \frac{|\sin x|}{\sin x} \Rightarrow$$

$$\rightarrow \sin x \frac{dy}{dx} + \cos x y = 1 \Rightarrow \frac{d}{dx} (\sin x \cdot y) = 1 \rightarrow$$

$$\Rightarrow \sin x \cdot y = x + C \rightarrow y(x) = \frac{x+C}{\sin x}$$

d) $\frac{dy}{dx} = x^3 - 2xy \Rightarrow \frac{dy}{dx} + 2xy = x^3$

$$g(x) = e^{\int 2x dx} = e^{x^2} = C_1 \cdot e^{x^2} - C_2 \cdot e^{x^2}$$

Para $C_2 = 1: g(x) = e^{x^2}$

$$e^{x^2} \frac{dy}{dx} + 2x e^{x^2} y = x^3 e^{x^2} \rightarrow \frac{d}{dx} (e^{x^2} y) = x^3 e^{x^2} \rightarrow$$

$$\rightarrow e^{x^2} y = \int x^3 e^{x^2} dx$$

$$\textcircled{*} \int x^3 e^{x^2} dx = \int \frac{x^3}{2x} e^u du = \frac{1}{2} \int u e^u du =$$

$$u = x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \quad \alpha(u) = u \quad \alpha'(u) = 1$$

$$= \frac{(u e^u - \int e^u du)}{2} = \frac{u e^u - e^u}{2} = \frac{x^2 e^{x^2} - e^{x^2}}{2}$$

Validando a eq principal:

$$e^{x^2} y = \frac{e^{x^2}(x^2 - 1)}{2} + C \rightarrow y(x) = \frac{(x^2 - 1)}{2} + \frac{C}{e^{x^2}}$$

f) $(1 - xy) dx + (xy - x^2) dy = 0$ (1)

$$\frac{\partial P}{\partial y} = -x \quad ; \quad \frac{\partial Q}{\partial x} = y - 2x \quad \left\{ \begin{array}{l} \text{A condição de Euler} \\ \text{não é respeitada} \end{array} \right. \Rightarrow \text{EDO não é exata}$$

Devemos procurar um fator integrante $g(x)$ ou $h(y)$

$$\frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{Q} = \frac{-x - y + 2x}{xy - x^2} = \frac{x - y}{-x(-y + x)} = -\frac{1}{x}$$

Como o coeficiente depende só de x então:

$$|g(x)| = e^{-\int \frac{1}{x} dx} = e^{-\ln|x|} + C_1 = C_2 e^{-\ln|x|}$$

$$\rightarrow |g(x)| = \frac{C_2}{|x|} \rightarrow g(x) = \frac{C_2}{x}$$

Para $C_2 = 1: g(x) = \frac{1}{x}$ (2)

Multiplicando (1) por (2):

$$\frac{1}{x} (1 - xy) dx + \frac{1}{x} (xy - x^2) dy = 0 \rightarrow$$

$$\rightarrow \left(\frac{1}{x} - y\right) dx + (y - x) dy = 0$$

$$\left\{ \begin{array}{l} \frac{\partial \Phi}{\partial x} = \frac{1}{x} - y \\ \frac{\partial \Phi}{\partial y} = y - x \end{array} \right. \Rightarrow \Phi(x, y) = \ln x - xy + f(y)$$

$$\frac{\partial \Phi}{\partial y} = -x + f'(y) = y - x \Rightarrow f'(y) = y \rightarrow$$

$$\rightarrow f(y) = \frac{y^2}{2}$$

$$\Phi(x, y) = \ln x - xy + \frac{y^2}{2}$$

Solução Geral: $\ln x^2 - 2xy + y^2 = C$

g) $xy' = y + \sqrt{x^2 + y^2} \Rightarrow y' = \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}, x > 0$

$$z = \frac{y}{x} \Rightarrow y = xz \Rightarrow \frac{dy}{dx} = z + xz'$$

$$\therefore z + xz' = z + \sqrt{1 + z^2} \rightarrow$$

$$\rightarrow x \frac{dz}{dx} = \sqrt{1 + z^2} \rightarrow \frac{dz}{\sqrt{1 + z^2}} = \frac{dx}{x} \rightarrow$$

$$\rightarrow \int \frac{dz}{\sqrt{1 + z^2}} = \int \frac{dx}{x} \quad (1)$$

$$\textcircled{*} \int \frac{1}{\sqrt{1 + z^2}} dz = \int \frac{\sec^2 \theta}{\sec \theta} d\theta = \int \sec \theta d\theta =$$

$$z = \frac{1}{\tan \theta} \Rightarrow \frac{dz}{d\theta} = \frac{\cos \theta \cos \theta + \sin \theta \cos \theta}{\cos^2 \theta} = \sec \theta$$

$$= \ln |z \sec \theta + \sec \theta| = \ln |z + \sec(\arctan z)| = \ln |z + \sqrt{1 + z^2}|$$

Note que $\sec^2(\arctg z) = 1 + z^2 \Rightarrow$
 $\rightarrow \sec^2(\arctg z) = 1 + z^2 \Rightarrow$
 $\rightarrow \sec(\arctg z) = \sqrt{1 + z^2}$

Voltando para a eq (1):

$\ln|z + \sqrt{1+z^2}| = \ln|x| + C_1 \Rightarrow$
 $\rightarrow z + \sqrt{1+z^2} = C_2 x \Rightarrow$
 $\rightarrow \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = C_2 x \Rightarrow y + \sqrt{x^2 + y^2} = C_2 x^2,$
 com $C_2 \in \mathbb{R} / \{0\}$

16) a) $(x+y)dx + xdy = 0$

$\frac{\partial P}{\partial y} = 1$
 $\frac{\partial Q}{\partial x} = 1$ } Condição de Euler é respeitada:
 a equação é exata

$\begin{cases} \frac{\partial \Phi}{\partial x} = x+y \\ \frac{\partial \Phi}{\partial y} = x \end{cases} \therefore \Phi(x,y) = \frac{x^2}{2} + xy + f(y)$

$\frac{\partial \Phi}{\partial y} = x + f'(y) = x \Rightarrow f'(y) = 0 \therefore f(y) = C_1$

$\Phi(x,y) = \frac{x^2}{2} + xy + C_1$

Solução Geral: $\frac{x^2}{2} + xy = C$

c) $(1-y-\sin x)dx - \cos x dy = 0$

$\frac{\partial P}{\partial y} = -1$
 $\frac{\partial Q}{\partial x} = \sin x$ } Condição de Euler não é respeitada:
 logo a equação não é exata

$\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} = \frac{-1 - \sin x}{- \cos x} = \frac{1 + \sin x}{\cos x}$ } só depende de x

$g(x) = e^{\int \frac{1 + \sin x}{\cos x} dx} = e^{\int \sec x dx + \int \frac{\sin x}{\cos x} dx} =$
 $= e^{\ln| \sec x + \tan x | - \ln|\cos x|} + C_1 =$

$= C_2 \cdot \frac{e^{\ln| \sec x + \tan x |}}{e^{\ln|\cos x|}} = C_2 \cdot \frac{|\sec x + \tan x|}{\cos x}$

Para $C_2 = 1$: $g(x) = \frac{\sec x + \tan x}{\cos x} = \frac{\sin x + 1}{\cos^2 x}$

$(\frac{\sec x + \tan x}{\cos x})(1-y-\sin x)dx - (\sec x + \tan x)dy = 0$

$\begin{cases} \frac{\partial \Phi}{\partial x} = (\frac{\sec x + \tan x}{\cos x})(1-y-\sin x) \\ \frac{\partial \Phi}{\partial y} = -\sec x - \tan x \end{cases}$

$\Phi(x,y) = \ln|\cos x| - \ln|\sec x + \tan x| + f(x) \Rightarrow$

$\Rightarrow \Phi(x,y) = \ln\left(\frac{\cos x}{\sec x + \tan x}\right) + f(x)$

(condas)

b) $(2xy - e^y - x)dx + (-xe^y - y + x^2)dy = 0$

$\frac{\partial P}{\partial y} = 2x - e^y$
 $\frac{\partial Q}{\partial x} = -e^y + 2x$ } A condição de Euler é
 respeitada logo a EDO
 é exata

$\begin{cases} \frac{\partial \Phi}{\partial x} = 2xy - e^y - x \\ \frac{\partial \Phi}{\partial y} = -xe^y - y + x^2 \end{cases}$

$\Phi(x,y) = x^2y - xe^y - \frac{x^2}{2} + f(y)$

$\frac{\partial \Phi}{\partial y} = x^2 - xe^y + f'(y) = -xe^y - y + x^2 \Rightarrow$

$\Rightarrow f'(y) = -y \therefore f(y) = -\frac{y^2}{2}$

$\Phi(x,y) = x^2y - xe^y - \frac{x^2}{2} - \frac{y^2}{2}$

$x^2y - xe^y - \frac{x^2}{2} - \frac{y^2}{2} = C$

17) b) $u = 3x - 2y \therefore du = 3dx - 2dy \Rightarrow$

$\rightarrow 2dy = 3dx - du \Rightarrow dy = \frac{3dx - du}{2}$

$(u+1)dx - (u+3)\left(\frac{3dx - du}{2}\right) = 0 \Rightarrow$

$\Rightarrow (u+1)dx - \frac{3(u+3)dx}{2} + \frac{(u+3)du}{2} = 0 \Rightarrow$

$\Rightarrow \left[\frac{2u+2-3u-9}{2}\right]dx + \frac{(u+3)du}{2} = 0 \Rightarrow$

$\Rightarrow \left[\frac{-u-7}{2}\right]dx + \frac{u+3}{2}du = 0 \Rightarrow$

$\Rightarrow (u+3)du = (u+7)dx \Rightarrow \left(\frac{u+3}{u+7}\right)du = dx \Rightarrow$

$$\Rightarrow \int \frac{u+3}{u+7} du = \int dx \quad (1)$$

$$\int \frac{u+3}{u+7} du = \int 1 - \frac{4}{u+7} du = u - 4 \ln|u+7|$$

Volto para a equação (1):

$$u - 4 \ln|u+7| = x + C_1 \Rightarrow 3x - 2y - 4 \ln|3x - 2y + 7| = x + C_1$$

$$\Rightarrow 2x - 2y - 4 \ln|3x - 2y + 7| = C_1 \Rightarrow 2x + 2y + 4 \ln|3x - 2y + 7| = C$$

a) $\cos y = w \Rightarrow \frac{dw}{dy} = -\sin y \Rightarrow dw = -\sin y dy$

$$(1 + 3xw) dx + (-x^2) dw = 0 \quad (1)$$

$$\left. \begin{array}{l} \frac{\partial P}{\partial w} = 3x \\ \frac{\partial Q}{\partial x} = -2x \end{array} \right\} \begin{array}{l} \text{Condição de Euler não é respeitada} \\ \text{Logo a EDO não é exata} \end{array}$$

$$\frac{\frac{\partial P}{\partial w} - \frac{\partial Q}{\partial x}}{Q} = \frac{5x}{-x^2} = -\frac{5}{x} \Rightarrow |g(x)| = e^{-\int \frac{5}{x} dx} = e^{-5 \ln|x| + C_1} = C_2 e^{-5 \ln|x|} = \frac{C_2}{|x|^5} \Rightarrow$$

$$\Rightarrow g(x) = \frac{C_2}{x^5}$$

Para $C_2 = 1: g(x) = \frac{1}{x^5} \quad (2)$

Multiplicando (1) por (2):

$$\left(\frac{1}{x^5} + \frac{3w}{x^4}\right) dx + \left(-\frac{1}{x^3}\right) dw = 0$$

$$\left\{ \begin{array}{l} \frac{\partial \Phi}{\partial x} = \frac{1}{x^5} + \frac{3w}{x^4} \\ \frac{\partial \Phi}{\partial w} = -\frac{1}{x^3} \end{array} \right. \Rightarrow \Phi(x, w) = \frac{1}{2x^2} + f(x)$$

$$\frac{\partial \Phi}{\partial x} = -\frac{1}{x^3} + f'(x) = \frac{1}{x^5} + \frac{3w}{x^4} \Rightarrow$$

$$\Rightarrow f'(x) = \frac{1}{x^5} + \frac{3w}{x^4} + \frac{1}{x^3} \Rightarrow f(x) = -\frac{1}{4x^4} - \frac{w}{x^3} - \frac{1}{2x^2}$$

$$\Phi(x, w(y)) = \frac{1}{2x^2} - \frac{1}{4x^4} - \frac{w}{x^3} - \frac{1}{2x^2} \Rightarrow$$

$$\Rightarrow \Phi(x, y) = \frac{1}{2x^2} - \frac{1}{4x^4} - \frac{\cos y}{x^3} - \frac{1}{2x^2}$$

$$\frac{1}{2x^2} - \frac{1}{4x^4} - \frac{\cos y}{x^3} - \frac{1}{2x^2} = C$$

d) $u = x + 2y \Rightarrow du = dx + 2dy \Rightarrow dx = du - 2dy$

$$(u-1)(du - 2dy) + (2u-3)dy = 0 \Rightarrow$$

$$\Rightarrow (u-1)du - 2(u-1)dy + (2u-3)dy = 0 \Rightarrow$$

$$\Rightarrow (u-1)du + (-2u+2+2u-3)dy = 0 \Rightarrow$$

$$\Rightarrow (u-1)du - dy = 0 \Rightarrow \frac{u^2}{2} - u - y + C = 0$$

$$\Rightarrow \frac{(x+2y)^2}{2} - (x+2y) - y = C$$

9) a) $2xy' + (1-x)y = -6x^2y^3 \Rightarrow$

$$\Rightarrow y' + \left(\frac{1-x}{2x}\right)y = -3xy^3$$

$$p(x) = \frac{1-x}{2x}; q(x) = -3x; n=3$$

Seja $z = y^{-2}$

$$\frac{dz}{dx} + (-2) \left(\frac{1-x}{2x}\right)z = (-2)(-3x) \Rightarrow$$

$$\Rightarrow \frac{dz}{dx} + \frac{x-1}{x}z = 6x$$

$$g(x) = e^{\int \frac{x-1}{x} dx} = e^{x - \ln|x| + C_1} = \frac{C_2 e^x}{|x|} = \frac{C_3 e^x}{x}$$

Para $C_3 = 1: g(x) = \frac{e^x}{x}$

$$\frac{e^x}{x} \frac{dz}{dx} + \frac{e^x}{x} \cdot \frac{x-1}{x} z = 6e^x \Rightarrow$$

$$\Rightarrow \frac{d}{dx}(e^x z) = 6e^x \Rightarrow e^x z = 6e^x + C \Rightarrow$$

$$\Rightarrow y^{-2} = \frac{6e^x + C}{e^x} \Rightarrow y^2 = \frac{e^x}{6e^x + C}$$

b) $\frac{dy}{dx} - y = e^{-3x} y^4$

$$p(x) = -1, q(x) = e^{-3x}, n=4$$

Seja $z = y^{-3}$

$$\frac{dz}{dx} + (-3)(-1)z = -3e^{-3x} \Rightarrow \frac{dz}{dx} + 3z = -3e^{-3x}$$

$$g(x) = e^{\int 3 dx} = e^{3x + C_1} = C_2 e^{3x}$$

Para $C_2 = 1: g(x) = e^{3x}$

$$e^{3x} \frac{dy}{dx} + 3e^{3x} y = -3e^{-3x} e^{3x} \rightarrow$$

$$\rightarrow \frac{d}{dx} (e^{3x} y) = -3 \rightarrow e^{3x} y = -3x + C \rightarrow$$

$$\rightarrow y = -e^{-3x} 3x + e^{-3x} C \rightarrow$$

$$\rightarrow y = \frac{1}{\sqrt{-e^{-3x} 3x + e^{-3x} C}} = \frac{x}{\sqrt{C - 3x}}$$

25)

a) $y'' + 2y' + y = 0$

Eq Característica: $\lambda^2 + 2\lambda + 1 = 0 \rightarrow (\lambda + 1)^2 = 0$

Base = $\{e^{-x}, xe^{-x}\}$

Soluggö: $y(x) = Ae^{-x} + Bxe^{-x}, A, B \in \mathbb{R}$

b) $y'' - 4y' + 4y = 0$

Eq Característica: $\lambda^2 - 4\lambda + 4 = 0 \rightarrow (\lambda - 2)^2 = 0$

Base = $\{e^{2x}, xe^{2x}\}$

Soluggö: $y(x) = Ae^{2x} + Bxe^{2x}, A, B \in \mathbb{R}$

c) $y''' - y'' + y' - y = 0$

Eq Característica: $\lambda^3 - \lambda^2 + \lambda - 1 = 0$

$\lambda = 1$ é raiz

1	1	-1	1	-1
1	0	1	0	

$\rightarrow \lambda^2 + 1 = 0$

$(\lambda - 1)(\lambda^2 + 1) = 0 \rightarrow \lambda = 1$ ou $\lambda = \pm i$

Base = $\{e^x, \cos x, \sin x\}$

Soluggö: $y(x) = Ae^x + B\cos x + C\sin x, A, B, C \in \mathbb{R}$

e) $y'' - 9y' + 20y = 0$

Eq Característica: $\lambda^2 - 9\lambda + 20 = 0 \rightarrow$

$\rightarrow (\lambda - 5)(\lambda - 4) = 0$

Base = $\{e^{5x}, e^{4x}\}$

Soluggö: $y(x) = Ae^{4x} + Be^{5x}, A, B \in \mathbb{R}$

f) $y''' - 3y'' + 3y' - y = 0$

Eq Característica: $\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$

$\lambda = 1$ é raiz

1	1	-3	3	-1
1	1	-2	1	0

$\rightarrow \lambda^2 - 2\lambda + 1 = 0$

$(\lambda - 1) \cdot (\lambda - 1) \cdot (\lambda - 1) = 0 \rightarrow (\lambda - 1)^3 = 0$

Base = $\{e^x, xe^x, x^2e^x\}$

Soluggö: $y(x) = Ae^x + Bxe^x + Cx^2e^x, A, B, C \in \mathbb{R}$

h) $y^{(4)} + y = 0$

Eq Característica: $\lambda^4 + 1 = 0 \rightarrow$

$\rightarrow (\lambda - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i) (\lambda - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i) (\lambda + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i) (\lambda + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i) = 0$

Base = $\{e^{\frac{\sqrt{2}}{2}x} \cos \frac{\sqrt{2}}{2}x, e^{\frac{\sqrt{2}}{2}x} \sin \frac{\sqrt{2}}{2}x, e^{-\frac{\sqrt{2}}{2}x} \cos \frac{\sqrt{2}}{2}x, e^{-\frac{\sqrt{2}}{2}x} \sin \frac{\sqrt{2}}{2}x\}$

Soluggö: $y(x) = Ae^{\frac{\sqrt{2}}{2}x} \cos \frac{\sqrt{2}}{2}x + Be^{\frac{\sqrt{2}}{2}x} \sin \frac{\sqrt{2}}{2}x + Ce^{-\frac{\sqrt{2}}{2}x} \cos \frac{\sqrt{2}}{2}x + De^{-\frac{\sqrt{2}}{2}x} \sin \frac{\sqrt{2}}{2}x$

27) a) $y'' + 3y' + 2y = e^x + e^{2x}$

Homogênea: $y'' + 3y' + 2y = 0$

Eq Característica: $\lambda^2 + 3\lambda + 2 = 0 \rightarrow$

$\rightarrow (\lambda + 1)(\lambda + 2) = 0$

Base = $\{e^{-x}, e^{-2x}\}$

$y_H(x) = Ae^{-x} + Be^{-2x}, A, B \in \mathbb{R}$

Particular: (método dos coeficientes a determinar)

- $y_1(x) = e^x$
- $y_{p1}(x) = Ae^x$
- $y_{p2}(x) = Ae^x$
- $y_{p3}(x) = Ae^x$

$\rightarrow Ae^x + 3Ae^x + 2Ae^x = e^x \rightarrow$

$\rightarrow 6Ae^x = e^x \therefore A = 1/6$

- $y_2(x) = e^{2x}$
- $y_{p4}(x) = Be^{2x}$
- $y_{p5}(x) = 2Be^{2x}$
- $y_{p6}(x) = 4Be^{2x}$

$$4Be^{2x} + 6Be^{2x} + 2Be^{2x} = e^{2x} \Rightarrow 12Be^{2x} = e^{2x}$$

$$\therefore B = \frac{1}{12}$$

Solução geral:

$$y(x) = Ae^{-x} + Be^{-2x} + \frac{e^x}{6} + \frac{e^{2x}}{12}; A, B \in \mathbb{R}$$

b) $y'' + y' - 6y = \sin x + xe^{2x}$

Homogênea: $y'' + y' - 6y = 0$

Eq Característica: $\lambda^2 + \lambda - 6 = 0 \Rightarrow$

$$\Rightarrow (\lambda - 2)(\lambda + 3) = 0$$

Base = $\{e^{2x}, e^{-3x}\}$

$$y_h(x) = Ae^{2x} + Be^{-3x}; A, B \in \mathbb{R}$$

Particular:

$\varphi_1(x) = \sin x$

$$y_{p1}(x) = C \sin x + D \cos x$$

$$y'_{p1}(x) = C \cos x - D \sin x$$

$$y''_{p1}(x) = -C \sin x - D \cos x$$

$$-C \sin x - D \cos x + C \cos x - D \sin x - 6C \sin x - 6D \cos x = \sin x$$

$$\Rightarrow \sin x (-C - D - 6C) + \cos x (-D + C - 6D) = \sin x \Rightarrow$$

$$\Rightarrow (-7C - D) \sin x + (C - 7D) \cos x = \sin x$$

$$\begin{cases} -7C - D = 1 \Rightarrow -49D - D = 1 \Rightarrow D = -\frac{1}{50} \\ C - 7D = 0 \Rightarrow C = 7D \Rightarrow C = -\frac{7}{50} \end{cases}$$

$\varphi_2(x) = xe^{2x}$

$$y_{p2}(x) = x(Ex + F)e^{2x} = (Ex^2 + Fx)e^{2x}$$

$$y'_{p2}(x) = (2Ex + F)e^{2x} + 2(Ex^2 + Fx)e^{2x}$$

$$y''_{p2}(x) = 2Ee^{2x} + 2(2Ex + F)e^{2x} + 2(2Ex + F)e^{2x} +$$

$$+ 4(Ex^2 + Fx)e^{2x}$$

$$y_{p2}''(x) \cdot e^{2x} (2E + 4F) + xe^{2x} (4E + 4E + 4F) +$$

$$+ x^2 e^{2x} (E)$$

$$-6y_{p2}(x) = (-6Ex^2 - 6Fx)e^{2x}$$

Na eq principal:

$$e^{2x}(2E + 5F) + xe^{2x}(10E) = xe^{2x} \Rightarrow$$

$$\Rightarrow (2E + 5F) + x(10E) = x$$

$$\begin{cases} 2E + 5F = 0 \Rightarrow \frac{1}{5} + F = 0 \Rightarrow F = -\frac{1}{5} \\ 10E = 1 \Rightarrow E = \frac{1}{10} \end{cases}$$

Solução geral:

$$y(x) = Ae^{2x} + Be^{-3x} - \frac{7 \sin x}{50} - \frac{\cos x}{50} - \frac{xe^{2x}}{25} + \frac{x^2 e^{2x}}{10}$$

$$A, B \in \mathbb{R}$$

28) c) $y''' + y' = \sec x$

Homogênea: $y''' + y' = 0$

$$\lambda^3 + \lambda = 0 \Rightarrow \lambda(\lambda^2 + 1) = 0 \Rightarrow$$

$$\Rightarrow \lambda = 0 \text{ ou } \lambda = \pm i$$

Base = $\{1, \cos x, \sin x\}$

Particular: (método da variação de parâmetros)

$$y_p = U_1 y_1 + U_2 y_2 + U_3 y_3 \Rightarrow$$

$$\Rightarrow y_p = U_1 + U_2 \cos x + U_3 \sin x$$

$$\begin{cases} U_1' + U_2' \cos x + U_3' \sin x = 0 \\ -U_2' \sin x + U_3' \cos x = 0 \\ -U_2' \cos x - U_3' \sin x = \sec x \end{cases}$$

$$W = \begin{vmatrix} 1 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ 0 & -\cos x & -\sin x \end{vmatrix} = \sin^2 x + \cos^2 x = 1$$

$$W_1 = \begin{vmatrix} 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ \sec x & -\cos x & -\sin x \end{vmatrix} = \cos^2 x \sec x + \sin^2 x \sec x = \sec x$$

$$U_1' = \frac{W_1}{W} = \sec x \therefore U_1 = \int \sec x dx = \ln | \tan x + \sec x |$$

$$W_2 = \begin{vmatrix} 1 & 0 & \sin x \\ 0 & 0 & \cos x \\ 0 & \sec x & -\sin x \end{vmatrix} = -\cos x \cdot \sec x = -1$$

$$U_2' = \frac{W_2}{W} = -1 \therefore U_2 = \int dx = -x$$

$$W_3 = \begin{vmatrix} 1 & \cos x & 0 \\ 0 & -\sin x & 0 \\ 0 & -\cos x & \sin x \end{vmatrix} = -\sin x \sin x = -\sin^2 x$$

$$U_3' = \frac{W_3}{W} = -\sin^2 x \quad U_3 = \int -\sin^2 x dx = \ln|\cos x|$$

$$y_p(x) = \ln|\cos x + \sin x| - x \cos x + \ln|\cos x| \sin x$$

Solució general:

$$y_p(x) = A \cdot \cos x + B \sin x + \ln|\cos x + \sin x| + \ln|\cos x| \sin x - x \cos x, \quad A, B, C \in \mathbb{R}$$