Exercícios de PMR

1) Módulo de Elasticidade do cobre = $110 \cdot 10^9$

$\sigma = 250 \cdot 10^6 Pa \Rightarrow 250 \cdot 10^6 = 110 \cdot 10^9 \cdot \varepsilon \Rightarrow$

$\Rightarrow \varepsilon = \frac{250}{110 \cdot 10^3} = 0,00227$

$\varepsilon = \frac{\Delta l}{l_0} \Rightarrow \Delta l = \varepsilon \cdot l_0 \Rightarrow \Delta l = 0,00227 \cdot 300 =$

$\approx 0,68 mm$

2) ν_c = coef. de Poisson do cobre = 0,34

E_c = módulo de elasticidade do cobre = $2,4 \cdot 10^9$

$\sigma = \frac{F}{A_0} \Rightarrow E \cdot \varepsilon_x = \frac{F}{A_0} \Rightarrow \varepsilon_x = \frac{F}{E \cdot A_0} \Rightarrow \varepsilon_x = \frac{F \cdot 10^6}{2,4 \cdot 10^9 \cdot 25\pi}$

$\varepsilon_y = \frac{\Delta d}{d_0} = \frac{2,5 \cdot 10^{-3}}{10} = 2,5 \cdot 10^{-4}$

$\nu_c = 0,34 \Rightarrow 0,34 = \frac{F \cdot 10^6}{2,4 \cdot 10^9 \cdot 25\pi} \cdot \frac{1}{2,5 \cdot 10^{-4}} \Rightarrow$

$\Rightarrow F = 0,34 \cdot 2,425\pi = 0,25 N$

3) a) $A_0 = \pi 6^2 = 36\pi$ e $A = \pi 5^2 = 25\pi$

$D_A = \frac{36\pi - 25\pi}{36\pi} \cdot 100\% = \frac{11}{36} \cdot 100\% \approx 30\%$

b) $\sigma_{eng} = 480 \cdot 10^6 Pa \Rightarrow 480 \cdot 10^6 = \frac{F}{36 \cdot 10^{-6} \pi} \Rightarrow$

$\Rightarrow F = 480 \cdot 36\pi \approx 54286,8 N$

$\sigma_{Real} = \frac{F}{A} \Rightarrow \sigma_{Real} = \frac{54286,8}{25 \cdot 10^{-6} \pi} = 0,69 GPa$

4) $\sigma_T = k \cdot \varepsilon_T^n \Rightarrow 400 = 900 \cdot 0,1^n \Rightarrow \frac{4}{9} = 0,1^n \Rightarrow$

$\Rightarrow n \cdot \ln 0,1 = \ln \frac{4}{9} \Rightarrow n \approx 0,35$

5) a) $\sigma_{calculado} < \sigma_{material} \therefore \sigma_{calculado} = \frac{\sigma_{material}}{FS} \Rightarrow$

$\Rightarrow \sigma_{calculado} = \frac{\sigma_e}{FS} \Rightarrow \frac{F}{A_0} = \frac{\sigma_e}{2} \Rightarrow \frac{300}{A_0} = \frac{250 \cdot 10^6}{2} \Rightarrow$

$\Rightarrow A_0 = \frac{600}{250 \cdot 10^6} \Rightarrow \pi \left(\frac{d_0}{2}\right)^2 = \frac{2,4}{10^6} \Rightarrow \left(\frac{d_0}{2}\right)^2 = \frac{2,4}{\pi 10^6} \Rightarrow$

$\Rightarrow d_0 \approx 0,00175 m = 1,75 mm$

b) $\sigma_{calculado} = E \cdot \varepsilon \Rightarrow \varepsilon = \frac{\sigma_e}{2} \frac{1}{E} \Rightarrow \varepsilon = \frac{250 \cdot 10^6}{2} \cdot \frac{1}{97 \cdot 10^9} \Rightarrow$

$\Rightarrow \varepsilon \approx 0,00129$

c) $\Delta h = l \cdot \varepsilon \Rightarrow \Delta h = 0,00129 m = 1,29 mm$

d) $A = \frac{\pi (1,75 \cdot 10^{-3})^2}{4} \approx 2,4 \cdot 10^{-6} m^2$

Considerando FS = 1:

$\sigma_{calculada} = \frac{\sigma_m}{FS} \Rightarrow \frac{F_{máx}}{A} = \sigma_m \Rightarrow F_{máx} = \sigma_m \cdot A \Rightarrow$

$\Rightarrow F_{máx} = 450 \cdot 10^6 \cdot 2,4 \cdot 10^{-6} \approx 1080 N$

e) $\varepsilon_{pot} = \varepsilon_{resiliência} \Rightarrow mgh_{máx} = Volume \cdot T \Rightarrow$

$\Rightarrow h_{máx} = \frac{Volume \cdot T}{mg} \Rightarrow h_{máx} = \frac{2,4 \cdot 10^{-6} \cdot 0,32 \cdot 10^6}{200} \Rightarrow$

$\Rightarrow h_{máx} = 2,56 \cdot 10^{-3} m = 2,56 mm$

6) $l_0 = 750 mm = 750 \cdot 10^{-3} m$

$T = 538°C$: $\dot{\varepsilon}_{propriedade} = 10^{-5,772727} = 1,687 \cdot 10^{-6}$

$\dot{\varepsilon}_{calculo} = \frac{\dot{\varepsilon}_{propriedade}}{FS} \Rightarrow \frac{\Delta l_{sec} / l_0}{\Delta t} = \dot{\varepsilon}_{propriedade} \Rightarrow$

$\Rightarrow \Delta l_{sec} = \dot{\varepsilon}_{propriedade} \cdot \Delta t \cdot l_0 = 1,687 \cdot 10^{-6} \cdot 5000 \cdot 750 \Rightarrow$

$\Rightarrow \Delta l_{sec} = 0,63 mm$

$\Delta l = \Delta l_{inst} + \Delta l_{primário} + \Delta l_{sec} = 3,63 mm$

Não usamos o $\Delta l_{terciário}$, pois, podemos perder o controle da influência ao utilizar este parâmetro.

7) k_{Ic} = tenacidade à fratura = $55 MPa\sqrt{m}$

$\sigma = 200 MPa$

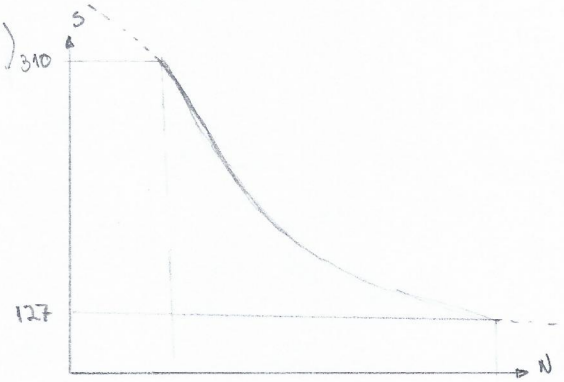
Como a chapa é larga temos que $Y = 1$

$55 = 200 \cdot 1 \cdot \sqrt{\pi a} \Rightarrow \sqrt{\pi a} = \frac{55}{200} \Rightarrow a = \frac{1}{\pi} \cdot \left(\frac{55}{200}\right)^2 \Rightarrow$

$\Rightarrow a \approx 0,024 m$

O comprimento mínimo será: $l = 2a = 0,048 m$

8) a)



b) De acordo com o gráfico temos que a resistência à fadiga para $5 \cdot 10^5$ ciclos será aproximadamente 260 MPa

c) $\sigma_a = \frac{\sigma_{máx} - \sigma_{mín}}{2} = 170$ e $\sigma_m = \frac{\sigma_{máx} + \sigma_{mín}}{2} = 18$

$\sigma_u = 450 MPa$

$$\frac{\sigma_a}{\sigma_{aeq}} + \frac{\sigma_m}{\sigma_u} = 1 \Rightarrow \frac{170}{\sigma_{aeq}} + \frac{180}{450} = 1 \Rightarrow$$

$$\Rightarrow \frac{1}{\sigma_{aeq}} = \frac{1}{170} \cdot \left(1 - \frac{180}{450}\right) \Rightarrow \sigma_{aeq} = 290\,MPa$$

$N \approx 3 \cdot 10^5$ ciclos

d) No gráfico vemos que não há limite de fadiga.

9) A tensão de compressão não tem grande influência na trinca:

$$a_i = \frac{2 \cdot 10^{-3}}{2} = 10^{-3}\,m$$

$$\Delta k_{ef} = 25 \Rightarrow 25 = 1 \cdot 100 \cdot \sqrt{\pi a_f} \Rightarrow a_f = \frac{1}{\pi} \cdot \left(\frac{1}{4}\right)^{2} \approx$$

$$\approx 0{,}0199\,m$$

$$\frac{da}{dN} = A(\Delta k_{ef})^m \Rightarrow dN = \frac{1}{A} \cdot (\Delta k_{ef})^{-m}\,da \Rightarrow$$

$$\Rightarrow N = \frac{1}{A} \int_{a_i}^{a_f} (\Delta k_{ef})^{-m}\,da \Rightarrow N = \frac{\sigma_{max}^{-3}\,\pi^{-3/2}}{A} \int_{a_i}^{a_f} a^{-3/2}\,da$$

Formulário PMR3100 - P2 de 2016

- Laminação:

$$\text{Avanço} = \frac{V_f - V_r}{V_r}, \quad \varepsilon = \ln\left(\frac{h_o}{h_f}\right), \quad d_{máx} = h_o - h_f = \mu^2 R, \quad F = L w \sigma_{médio},$$

$$\sigma_{médio} = \int_0^{\varepsilon_f} \sigma \, d\varepsilon = \frac{k \varepsilon^n}{n+1}, \quad P = 2\pi N F L, \quad L = \sqrt{R(h_o - h_f)}$$

- Forjamento:

$$F = \sigma_f \pi r^2 \left(1 + \frac{2\mu r}{3h}\right), \quad \varepsilon = \ln\left(\frac{h_o}{h_f}\right), \quad \sigma_f = k\varepsilon^n$$

- Extrusão:

$$p = \sigma_{media}\left(1 + \frac{tg\alpha}{\mu}\right)\left(T_x \mu \cos\alpha - 1\right), \quad F_Q = A_o k \cdot \ln\left(\frac{A_o}{A_f}\right), \quad F_E = 1100 A_o \sigma_{medio}$$

$$\varepsilon = \ln\left(\frac{A_o}{A_f}\right)$$

- Trefilação:

$$F = \sigma_{médio} A_f \cdot \left[\left(1 + \frac{\mu}{\alpha}\right) \ln\left(\frac{A_o}{A_f}\right) + \frac{2}{3}\alpha\right], \quad \varepsilon_n = \ln\left(\frac{A_o}{A_f}\right)$$

- Segunda Lei de Fick:

$$\frac{C_x - C_o}{C_s - C_o} = 1 - erf\left(\frac{x}{2\sqrt{Dt}}\right), \quad D = D_o \cdot e^{-\left(\frac{Q}{RT}\right)}$$

Exercícios P2:

1) $C_0 = 0{,}1\%C$, $C_s = 1{,}2\%C$, $C_x = 0{,}45\%C$, $x = 2mm$

$$\frac{C_x - C_0}{C_s - C_0} = 1 - erf\left(\frac{x}{2\sqrt{Dt}}\right) \Rightarrow \frac{0{,}45 - 0{,}1}{1{,}2 - 0{,}1} = 1 - erf\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$\Rightarrow erf\left(\frac{x}{2\sqrt{Dt}}\right) = 1 - \frac{0{,}35}{1{,}1} \simeq 0{,}6818 \therefore \frac{x}{2\sqrt{Dt}} = 0{,}71 \Rightarrow$$

$$\Rightarrow \frac{2\cdot 10^{-3}}{2\sqrt{2\cdot 10^{-11} t}} = 0{,}71 \Rightarrow \frac{1}{\sqrt{2\cdot 10^{-11} t}} = 710 \Rightarrow \frac{1}{710^2} = 2\cdot 10^{-11} t$$

$$\Rightarrow t \simeq 10000\,s \simeq 27{,}7h$$

2) As profundidades são as mesmas. Da Segunda Lei de Fick:

$$\frac{x}{2\sqrt{D_{1073} t_{1073}}} = \frac{x}{2\sqrt{D_{1173} t_{1173}}} \Rightarrow t_{1173} = \frac{D_{1073}\cdot t_{1073}}{D_{1173}} \Rightarrow$$

$$\Rightarrow t_{1173} = \frac{e^{\left(\frac{-137859}{1073\cdot 8{,}314}\right)}}{e^{\left(\frac{-137859}{1173\cdot 8{,}314}\right)}} \cdot 10 \simeq 2{,}7h$$

3) $T_1 = 473$, $T_2 = 773$, $R = 8{,}314$

$$\frac{D_{200°C}}{D_{500°C}} = \frac{D_0 \cdot e^{\left(\frac{-Q}{8{,}314\cdot 473}\right)}}{D_0 \cdot e^{\left(\frac{-Q}{8{,}314\cdot 773}\right)}} \Rightarrow \frac{Q}{8{,}314\cdot 773} - \frac{Q}{8{,}314\cdot 473} = \ln\left(\frac{2{,}5\,10^{-24}}{3{,}1\cdot 10^{-17}}\right)$$

$$\Rightarrow Q\left(\frac{1}{8{,}314\cdot 773} - \frac{1}{8{,}314\cdot 473}\right) = \ln\left(\frac{2{,}5\,10^{-24}}{3{,}1\ \ 10^{-17}}\right) \Rightarrow$$

$$\Rightarrow 9{,}8689\cdot 10^{-5} Q = 16{,}3332 \Rightarrow Q = \frac{16{,}3332}{9{,}8689}\cdot 10^5 \Rightarrow$$

$$\Rightarrow Q \simeq 165501\ J/mol$$

4) $C_0 = 0{,}2\%C$, $C_x = 0{,}8\%C$, $C_s = 1{,}1\%C$, $x = 500\cdot 10^{-6} m$

$D_{930°C} = 1{,}51\cdot 10^{-11} m^2/s$

$$\frac{C_x - C_0}{C_s - C_0} = 1 - erf\left(\frac{x}{2\sqrt{D_{930°C}\, t}}\right) \Rightarrow \frac{0{,}6}{0{,}9} = 1 - erf\left(\frac{x}{2\sqrt{Dt}}\right) \Rightarrow$$

$$\Rightarrow erf(y) = 1 - \frac{0{,}6}{0{,}9} \Rightarrow erf(y) = 0{,}33 \therefore y \simeq 0{,}3 \Rightarrow$$

$$\Rightarrow \frac{500\cdot 10^{-6}}{2\cdot\sqrt{1{,}5\,10^{-11} t}} = 0{,}3 \Rightarrow \sqrt{1{,}5\,10^{-11} t} = \frac{500\cdot 10^{-6}}{0{,}6} \Rightarrow$$

$$\Rightarrow t \simeq 46296{,}3\,s \simeq 12{,}86h$$

5) $C_0 = 0{,}25\%C$, $C_s = 1{,}2\%C$, $C_x = 0{,}8\%C$, $x = 0{,}5\cdot 10^{-3} m$, $D = 1{,}6\cdot 10^{-11} m^2/s$

$$\frac{C_x - C_0}{C_s - C_0} = 1 - erf\left(\frac{x}{2\sqrt{Dt}}\right) \Rightarrow$$

$$\Rightarrow \frac{0{,}55}{0{,}95} = 1 - erf(y) \Rightarrow erf(y) \simeq 0{,}42$$

$$\therefore y = 0{,}4 \Rightarrow \frac{5\,10^{-4}}{2\cdot\sqrt{1{,}6\cdot 10^{-11} t}} = 0{,}4 \Rightarrow$$

$$\Rightarrow \sqrt{1{,}6\,10^{-11} t} = \frac{5\cdot 10^{-4}}{0{,}8} \Rightarrow t \simeq 24414\,s \simeq$$

$$\simeq 6{,}78h$$

6) $T_1 = 773k$, $T_2 = 873k$, $D_1 = 4{,}8\cdot 10^{-14} m^2/s$

$D_2 = 5{,}3\cdot 10^{-13} m^2/s$

Para a mesma profundidade:

$$D_1 \cdot t_1 = D_2 t_2 \Rightarrow D_1 t_1 = 10 D_2 \Rightarrow$$

$$\Rightarrow t_1 = \frac{10 D_2}{D_1} \Rightarrow t_1 = 10\ \frac{5{,}3\ 10}{4{,}8} \Rightarrow$$

$$\Rightarrow t_1 \simeq 110{,}4h.$$

7) $w = 300mm$, $r = 250mm$, $h_0 = 25mm$,

$h_f = 22mm$, $N = 500\ rev/min$, $n = 0{,}15$, $\mu = 0{,}12$

$d = h_0 - h_f = 3mm$

$d_{máx} = \mu^2 r = (0{,}12)^2\cdot 250 = 3{,}6mm$

Como $d < d_{máx}$, o coeficiente de atrito é suficiente para fazer a laminação.

$$\overline{\sigma_f} = \frac{k\cdot \varepsilon^n}{1+n} \Rightarrow \overline{\sigma_f} = \frac{275}{1{,}15}\cdot\left[\ln\left(\frac{25}{22}\right)\right]^{0{,}15} \Rightarrow$$

$$\Rightarrow \overline{\sigma_f} \simeq 175{,}6\,MPa$$

$$L = \sqrt{r(h_0 - h_f)} = \sqrt{rd} = \sqrt{250\cdot 3} = 27{,}4mm$$

$$F = L\,w\,\overline{\sigma_f} = 27{,}4\cdot 10^{-3}\cdot 300\cdot 10^{-3}\cdot 175{,}6\cdot 10^6 \Rightarrow$$

$$\Rightarrow F = 1443925{,}2N$$

$$T = 0{,}5\,F\cdot L = 0{,}5\cdot 1443925{,}2\cdot 27{,}4\cdot 10^{-3} \Rightarrow$$

$$\Rightarrow T \simeq 19781{,}8\,Nm$$

$$P = 2\pi F L N = 2\pi\ \frac{1443925{,}2\cdot 27{,}4\cdot 10^{-3}\cdot 50}{60000} \Rightarrow$$

$$\Rightarrow P = 207{,}154{,}3\,kW$$

8) $w = 228mm$, $h_o = 25mm$, $h_f = 20mm$, $r = 300mm$, $N = 100 rpm$

$\varepsilon = \ln\left(\frac{25}{20}\right) = 0,223 \therefore \sigma_m = \frac{\sigma_{def} + \sigma_{idef}}{2} \Rightarrow$

$\Rightarrow \sigma_m = \frac{280 + 80}{2} = 180 MPa$

$L = \sqrt{r(h_o - h_f)} = \sqrt{300 \cdot 5} \simeq 38,7 \cdot 10^{-3} m$

$F = L w \sigma_m = 38,7 \cdot 10^{-3} \cdot 228 \cdot 10^{-3} \cdot 180 \cdot 10^6 \Rightarrow$

$\Rightarrow F = 1588248 N$

$P = 2\pi N F L = 2\pi \cdot 100 \cdot \frac{1588248}{60000} \cdot 38,7 \cdot 10^{-3} =$

$= 643 kW$

9) $w = 200mm$, $h_o = 10 mm$, $h_f = 7mm$, $r = 200mm$, $N = 200 rpm$

$L = \sqrt{r(h_o - h_f)} = \sqrt{200 \cdot 3} = \sqrt{600} \simeq 24,5 \cdot 10^{-3} m$

$\varepsilon = \ln\left(\frac{h_o}{h_f}\right) = \ln\left(\frac{10}{7}\right) \simeq 0,36 \therefore \sigma_m = \frac{480 + 190}{2} \Rightarrow$

$\Rightarrow \sigma_m = 335 MPa$

$F = wL\sigma_m = 200 \cdot 10^{-3} \cdot 24,5 \cdot 10^{-3} \cdot 335 \cdot 10^6 = 1,64 \cdot 10^6 N$

$T = FL = 1,64 \cdot 10^6 \cdot 24,5 \cdot 10^{-3} = 40,18 \cdot 10^3 Nm$

10) $r_o = 75mm$, $h_o = 100mm$, $h_f = 50mm$, $\mu = 0,2$

$V = cte \Rightarrow \pi 75^2 \cdot 100 = \pi r_f^2 \cdot 50 \Rightarrow r_f^2 = 11250 \Rightarrow$

$\Rightarrow r_f \simeq 106 mm$

$\varepsilon = \ln\left(\frac{100}{50}\right) \simeq 0,6 \therefore \sigma_f = 1000 MPa$

$F = \sigma_f \pi r_f^2 \left(1 + \frac{2\mu r_f}{3h_f}\right) = 1000\pi (0,106)^2 \left(1 + \frac{2 \cdot 0,2 \cdot 0,106}{3 \cdot 0,075}\right)$

$F \simeq 45 \cdot 10^6 N$

11) $r_o = 62,5 mm$, $r_f = 25mm$, $k = 250 MPa$

$F = A_o \cdot k \cdot \ln\left(\frac{A_o}{A_f}\right) \Rightarrow F = \pi (62,5)^2 250 \cdot \ln\left(\frac{62,5^2}{25^2}\right) \Rightarrow$

$\Rightarrow F = \pi (62,5)^2 \cdot 250 \cdot 1,8 \Rightarrow F \simeq 5,5 \cdot 10^6 N$

12) $r_o = 150mm$, $r_f = 35mm$, $k = 400 Mpa$

$F = A_o \cdot k \cdot \ln\left(\frac{A_o}{A_f}\right) = \pi 150^2 \cdot 400 \ln\left(\frac{150^2}{35^2}\right) \Rightarrow$

$\Rightarrow F = \pi 150^2 \cdot 400 \cdot 2,9 \Rightarrow F \simeq 82 \cdot 10^6 N$

13) $\alpha = \frac{15\pi}{180} - \frac{5\pi}{60} = \frac{\pi}{12}$, $k = 205 MPa$, $n = 0,2$, $\mu = 0,07$, $r_o = 1,25mm$, $r_f = 1mm$, L ...

$\overline{\sigma_f} = \frac{k \cdot \varepsilon^n}{n+1} \Rightarrow \overline{\sigma_f} = \frac{205}{1,2}\left[\ln\left(\frac{1,25^2}{1^2}\right)\right]^{0,2} \simeq 145,4 MPa$

$F = \overline{\sigma_f} \cdot A_f \left[\left(1 + \frac{\mu}{\alpha}\right)\ln\left(\frac{A_o}{A_f}\right) + \frac{2}{3}\alpha\right] \Rightarrow$

$\Rightarrow F = 145\pi \left[\left(1 + \frac{0,07 \cdot 12}{\pi}\right)\ln\left(\frac{1,25^2}{1^2}\right) + \frac{\pi}{18}\right] \Rightarrow$

$\Rightarrow F \simeq 295,5 N$

14) $r_o = 1,27mm$, $r_f = 0,889mm$, $\alpha = \frac{\pi}{12}$, $\mu = 0,1$,

$\varepsilon = \ln\left(\frac{1,27^2}{0,889^2}\right) \simeq 0,71 \therefore \overline{\sigma_f} = \frac{1275 \cdot 0,71^{0,45}}{1,45} = 755 MPa$

$F = 750 \cdot 0,889^2 \pi \left[\left(1 + \frac{0,1 \cdot 12}{\pi}\right) 0,71 + \frac{\pi}{18}\right] = 2,176 kN$