RESOLUÇÃO LISTA 2 (Vítor Albuquerque M. Ribeiro)

CP 1)

a) $\boxed{\begin{array}{c} V_0 = 60\,m/s \\ \hline a = -0{,}4\,v^2 \end{array}}$ $\quad \boxed{a = \frac{dv}{dt} = -0{,}4\,v^2} \rightarrow \frac{1}{v^2}\,dv = -0{,}4\,dt \rightarrow$

$$\rightarrow \int_{V_0}^{V} \frac{1}{v^2}\,dv = -0{,}4\int_{t_0}^{t} dt \rightarrow -0{,}4(t - t_0) = -\left(\frac{1}{V} - \frac{1}{V_0}\right) \xrightarrow[V_0 = 60\,m/s]{t_0 = 0}$$

$$\rightarrow \frac{4}{10}t = \frac{1}{V} - \frac{1}{60} \rightarrow \frac{1}{V} = \frac{1+24t}{60} \rightarrow \boxed{V(t) = \frac{60}{1+24t}}$$

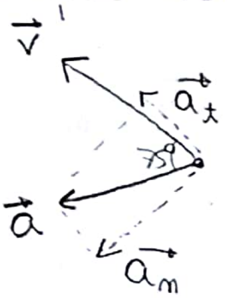
$$V(t=4) = \frac{60}{1+24.4} \Rightarrow \boxed{V(t=4) = 0{,}62\,m/s}$$

b) $$V(t) = \frac{dx}{dt} = \frac{60}{1+24t} \rightarrow dx = \left(\frac{60}{1+24t}\right)dt \rightarrow$$

$$\rightarrow \int_{x_0=0}^{x} dx = \int_{t_0=0}^{t}\left(\frac{60}{1+24t}\right)dt \xrightarrow[\boxed{\begin{array}{c} u = 1+24t \\ du = 24\,dt \end{array}}]{} x(t) = \frac{60}{24}\int_{1}^{24t+1}\frac{1}{u}\,du \rightarrow$$

$$\boxed{x(t) = \frac{60}{24}\cdot \ln(24t+1)} \rightarrow \boxed{x(t=4s) = 11{,}4\,m}$$

C.P. 2) a) A taxa de aumento da velocidade é a aceleração tangencial. Logo



$$|\vec{a_t}| = |\vec{a}|\cdot \cos 75° \rightarrow a_t = 1{,}4\cdot 0{,}258819 \rightarrow \boxed{a_t = 0{,}36\,m/s^2}$$

b) $$a_n = a\cdot \operatorname{sen} 75° \rightarrow a_n = 1{,}35\,m/s^2$$

Como $a_n = \frac{V^2}{\rho} \rightarrow \rho = \frac{V^2}{a_n} = \frac{20^2}{1{,}35} \rightarrow \boxed{\rho = 295{,}8\,m}$

C.P.3) Para A:

$$a = \frac{dv_A}{dt} = -0{,}042\ m/s^2\ (\text{cte}) \longrightarrow dv_A = -0{,}042\, dt \longrightarrow$$

$$\longrightarrow \int_{V_0=9{,}78}^{V_A} dv_A = \int_{t_0=0}^{t} -0{,}042\, dt \longrightarrow \boxed{V_A(t) = 9{,}78 - 0{,}042t}$$

OBS

$$35{,}2\frac{\cancel{km}}{\cancel{h}} \cdot \frac{1\cancel{h}}{60\cancel{min}} \cdot \frac{1\cancel{min}}{60s} = 9{,}78\ m/s \qquad 20{,}8\frac{km}{h} \div 3{,}6 = 5{,}78\ m/s$$

$$V_A(t) = \frac{dx_A}{dt} \longrightarrow (9{,}78 - 0{,}042t)\,dt = dx_A \longrightarrow \int_{t_0=0}^{t} (9{,}78 - 0{,}042t)\,dt = \int_{x_0=0}^{x_A} dx_A$$

$$\boxed{9{,}78t - 0{,}042 \cdot \frac{t^2}{2} = x_A(t)}$$

Para B: $a_B = 0 \longrightarrow V_B$ é cte.

$$x_B(t) = (x_0)_B + V_B \cdot t \longrightarrow \boxed{x_B(t) = d + 5{,}78 \cdot t}$$

Para A não cruzar B: $x_A(t) \leq x_B(t)$

$$9{,}78t - 0{,}021t^2 \leq d + 5{,}78t \longrightarrow \boxed{+0{,}021t^2 - 4t + d \geq 0}$$

Logo $\Delta \leq 0 \longrightarrow (-4)^2 - 4 \cdot 0{,}021 \cdot d \leq 0 \longrightarrow 0{,}084d \geq 16 \longrightarrow$

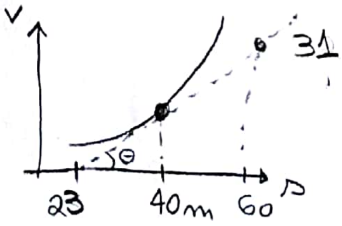
$$\longrightarrow \boxed{d \geq 190{,}5}$$

C.P.4) $v(t) = 0{,}2t^2 \longrightarrow v(t=3) = 0{,}2 \cdot 3^2 \longrightarrow \boxed{v(t=3) = 1{,}8\ m/s}$

$$a = \frac{dv}{dt} = \frac{d}{dt}(0{,}2t^2) \longrightarrow a(t) = 0{,}4t \longrightarrow \boxed{a(t=3) = 1{,}2\ m/s^2}$$

C.P.5) $\begin{cases} a = \dfrac{dv}{dt} & (1) \\ v = \dfrac{ds}{dt} & (2) \end{cases} \longrightarrow$ Dividindo (1) e (2): $\boxed{\dfrac{dv}{ds} = \dfrac{a}{v}}$ Logo, a inclinação do gráfico de $v \times s$ vale $\dfrac{a}{v}$

Logo em $s = 40m$



$$tg\theta \cong \frac{31}{60-23} = \frac{a}{v} \longrightarrow \frac{31}{37} = \frac{a}{15} \longrightarrow \boxed{a \approx 12{,}5\ \frac{m}{s^2}}$$

C.P.6) $\vec{r} = (x, y, z)$

a) $\boxed{\vec{v} = \frac{d\vec{r}}{dt} = (t^2, \sqrt{2}t, 1)}$ e $|\vec{v}| = \sqrt{(t^2)^2 + (\sqrt{2}t)^2 + 1^2} = \sqrt{t^4 + 2t^2 + 1}$

$|\vec{v}| = \sqrt{(t^2+1)^2} \longrightarrow \boxed{|\vec{v}| = t^2 + 1}$

b) $\vec{a} = \frac{d\vec{v}}{dt} \longrightarrow \boxed{\vec{a} = (2t, \sqrt{2}, 0)}$

c) Temos que o vetor tangente $\hat{t}$ é:

$$\boxed{\hat{t} = \frac{\vec{v}}{|\vec{v}|} = \left(\frac{t^2}{t^2+1}, \frac{\sqrt{2}t}{t^2+1}, \frac{1}{t^2+1}\right)}$$

O vetor binormal é $\hat{b} = \frac{\vec{v} \wedge \vec{a}}{|\vec{v} \wedge \vec{a}|}$

Calculando $\vec{v} \wedge \vec{a}$:

$$\vec{v} \wedge \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t^2 & \sqrt{2}t & 1 \\ 2t & \sqrt{2} & 0 \end{vmatrix} = (-\sqrt{2}, 2t, t^2(\sqrt{2} - 2\sqrt{2})) = (-\sqrt{2}, 2t, -\sqrt{2}t^2)$$

$$|\vec{v} \wedge \vec{a}| = \sqrt{2 + 4t^2 + 2t^4} = \sqrt{2} \cdot \sqrt{(t^2+1)^2} = \sqrt{2} \cdot (t^2+1)$$

$$\boxed{\hat{b} = \left(-\frac{1}{t^2+1}; \frac{\sqrt{2}t}{t^2+1}; \frac{-t^2}{t^2+1}\right)}$$

Finalmente, o vetor normal é $\hat{n} = \hat{b} \wedge \hat{t}$

$$\hat{b} \wedge \hat{t} = \left(\frac{1}{t^2+1}\right)^2 \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & \sqrt{2}t & -t^2 \\ t^2 & \sqrt{2}t & 1 \end{vmatrix} = \left(\frac{1}{t^2+1}\right)^2 \left(\sqrt{2}t + \sqrt{2}t^3; -t^4 + 1; -\sqrt{2}t - \sqrt{2}t^3\right) =$$

$$= \left(\frac{\sqrt{2}t(t^2+1)}{(t^2+1)^2}; \frac{-t^4+1}{(t^2+1)^2}; \frac{-\sqrt{2}t(t^2+1)}{(t^2+1)^2}\right) \longrightarrow \boxed{\hat{n} = \left(\sqrt{2}t; \frac{-t^4+1}{t^2+1}; -\sqrt{2}t\right) \cdot \frac{1}{t^2+1}}$$

OBS: Resolvi primeiro o item d pois acho mais fácil fazer assim. Agora, com o versor normal, podemos achar a aceleração normal.

$$|\vec{a_n}| = \vec{a} \cdot \hat{n} = (2t, \sqrt{2}, 0) \cdot \left(\frac{\sqrt{2}t}{t^2+1}, \frac{-t^4+1}{(t^2+1)^2}, \frac{-\sqrt{2}t}{(t^2+1)} \right) = \frac{2\sqrt{2}t^2}{t^2+1} + \sqrt{2}\left(\frac{-t^4+1}{(t^2+1)^2}\right)$$

$$= \frac{2\sqrt{2}t^4 + 2\sqrt{2}t^2 - \sqrt{2}t^4 + \sqrt{2}}{(t^2+1)^2} = \frac{\sqrt{2}(t^4+2t^2+1)}{(t^2+1)^2} = \frac{\sqrt{2}\cdot\cancel{(t^2+1)^2}}{\cancel{(t^2+1)^2}} = \sqrt{2}$$

Como $|\vec{a_n}| = \frac{|\vec{v}|^2}{\rho} \longrightarrow \boxed{\rho = \frac{(t^2+1)^2}{\sqrt{2}}}$

d) Já calculado

e) $|\vec{a_n}| = \sqrt{2}$

$$|\vec{a_t}| = \vec{a} \cdot \hat{t} = (2t, \sqrt{2}, 0) \cdot \frac{1}{t^2+1} \cdot (t^2, \sqrt{2}t, 1) =$$

$$= \frac{1}{t^2+1}(2t^3+2t) = \frac{2t(t^2+1)}{(t^2+1)} \longrightarrow \boxed{|\vec{a_t}| = 2t}$$

OBS: Nesse exercício, eu poderia ter seguido várias ordens. Por exemplo achar $\vec{v} \rightarrow$ achar $\hat{t} \rightarrow$ achar $\vec{a_t}$ fazendo $|\vec{a_t}| = \vec{a} \cdot \hat{t}$, $\rightarrow$ achar $\vec{a_n} = \vec{a} - \vec{a_t} \rightarrow$ achar $|\vec{a_n}| \rightarrow$ $\rightarrow$ achar ρ.

C.P.7)

$\rho = \frac{|\vec{v}|^2}{|\vec{a_n}|} \rightarrow c = \frac{|\vec{a_n}|}{v^2}$. Então, tenho que provar que

$|\vec{a_n}| = \frac{|\vec{v} \wedge \vec{a}|}{v}$. Temos que: $\vec{v} = v\,\hat{t}$ e $\vec{a} = a_t \cdot \hat{t} + a_n \cdot \hat{n}$

$\vec{v} \wedge \vec{a} = (v\,\hat{t}) \wedge (a_t \cdot \hat{t} + a_n \cdot \hat{n})$

$\vec{v} \wedge \vec{a} = (v \cdot a_n)\,\hat{b} \rightarrow |\vec{v} \wedge \vec{a}| = v \cdot a_n$, pois $|\hat{b}| = 1$

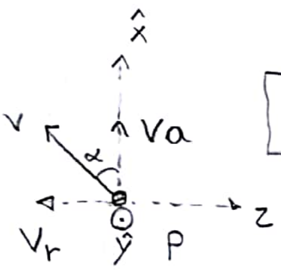
Logo $\boxed{a_n = \frac{|\vec{v} \wedge \vec{a}|}{v}}$ e $c = \frac{|\vec{v} \wedge \vec{a}|}{v^3}$. c.q.d //

C.P.8)

d) Pela vista superior, vemos que

$V_a \rightarrow$ velocidade de avanço

$V_r \rightarrow$ velocidade "rotacional"

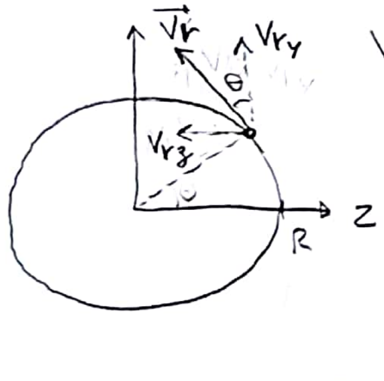


$\boxed{\vec{V_a} = V\cos\alpha\,\hat{i}}$

$|\vec{V_r}|$ é constante, mas sua direção varia de acordo com o θ.

$\boxed{|\vec{V_r}| = V \cdot \text{sen}\,\alpha}$

Vendo a vista traseira:



$V_{ry} = |\vec{V_r}|\cos\theta\,\hat{j} = V\text{sen}\,\alpha\cos(\theta - \theta_0)\,\hat{j}$

$V_{rz} = -|\vec{V_r}|\,\text{sen}\,\theta\,\hat{k} = V\text{sen}\,\alpha\,\text{sen}(\theta - \theta_0)\,\hat{k}$

Logo

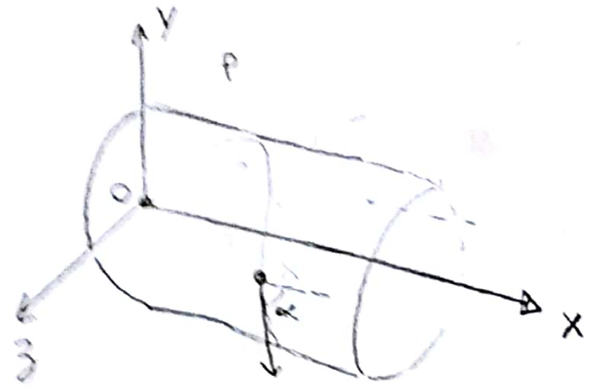
$$\boxed{\vec{V} = V\cos\alpha\,\hat{i} + V\text{sen}\,\alpha\cos(\theta - \theta_0)\,\hat{j} + V\text{sen}\,\alpha\,\text{sen}(\theta - \theta_0)\,\hat{k}}$$

e) A aceleração tangencial faz aumentar a velocidade tangencial, que é $\vec{V_t} = V\,\hat{t} \rightarrow \vec{a_t} = \frac{d}{dt}(V)\,\hat{t} \rightarrow \boxed{\vec{a_t} = \dot{V}\,\hat{t}}$

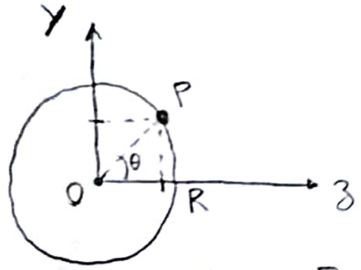
Já a aceleração normal só é afetada por V_r. Logo

$\vec{a_n} = \frac{|\vec{V_r}|^2}{R}\,\hat{n} \rightarrow \boxed{\vec{a_n} = \frac{(V\text{sen}\,\alpha)^2}{R}\,\hat{n}}$

a)



Pela vista traseira



Vê-se que: $\begin{cases} y = R\,\text{sen}(\theta - \theta_0) \\ z = R\cos(\theta - \theta_0) \end{cases}$

$$tg\alpha = \frac{R(\theta - \theta_0)}{x} \longrightarrow \boxed{x = \frac{R(\theta - \theta_0)}{tg\alpha}}$$ (também não entendi isso hahaha)

b) Nas coordenadas cilíndricas

$$(\hat{i}, \hat{j}, \hat{k}) \xrightarrow[\text{cilind.}]{\text{coord.}} (\hat{i}, \hat{r}, \hat{\theta})$$

$$\begin{cases} x = x \\ y = R\,\text{sen}(\theta - \theta_0) \\ z = R\cos(\theta - \theta_0) \end{cases} > \quad (P - O) = R \cdot \hat{r} + x\,\hat{i}$$

onde $\hat{r}$ é o versor radial ao tubo, e que acompanha o ponto P.

c) O raio de curvatura não se altera com a variação de θ. Como visto $a_n = \dfrac{v^2 \text{sen}^2\alpha}{R}$; mas, $a_n = \dfrac{v^2}{\rho}$, logo

$$\frac{v^2}{\rho} = \frac{v^2 \text{sen}^2\alpha}{R} \longrightarrow \boxed{\rho = \frac{R}{\text{sen}^2\alpha}}$$

C.P. 9)

a) i. $\vec{v} = \frac{d\vec{r}}{dt} \rightarrow \boxed{\vec{v} = (-\operatorname{sen} t)\,\hat{i} + \frac{\sqrt{2}}{2}\cos t\,\hat{j} - \frac{\sqrt{2}}{2}\cos t\,\hat{k}}$

$|\vec{v}| = \sqrt{\operatorname{sen}^2 t + \frac{1}{2}\cos^2 t + \frac{1}{2}\cos^2 t} = 1 \longrightarrow \boxed{|\vec{v}| = 1}$

$\boxed{\hat{t} = \frac{\vec{v}}{|\vec{v}|} = \vec{v}}$

ii. $\boxed{\vec{a} = \frac{d\vec{v}}{dt} = (-\cos t)\,\hat{i} - \left(\frac{\sqrt{2}}{2}\operatorname{sen} t\right)\hat{j} + \left(\frac{\sqrt{2}}{2}\operatorname{sen} t\right)\hat{k}}$

iii. $|\vec{a_t}| = \vec{a}\cdot\hat{t} = \operatorname{sen} t \cos t - \frac{1}{2}\operatorname{sen} t\cos t - \frac{1}{2}\operatorname{sen} t \cos t = 0$

$\boxed{\vec{a_t} = 0}$

iv. $\vec{a_n} = \vec{a} - \cancel{\vec{a_t}}^{\,0} \longrightarrow \boxed{\vec{a_n} = \vec{a}}$

$|\vec{a_n}| = \sqrt{\cos^2 t + \frac{1}{2}\operatorname{sen}^2 t + \frac{1}{2}\operatorname{sen}^2 t} = 1$

Como $|a_n| = \frac{|\vec{v}|^2}{\rho} \longrightarrow 1 = \frac{1^2}{\rho} \longrightarrow \boxed{\rho = 1}$