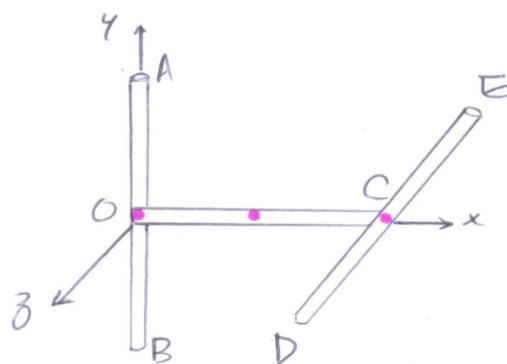


1.

$$AO = OB = CD = CE = \frac{a}{2}$$

$$OC = a$$



### BARRA DE

$$J_{DE} = \frac{ma^2}{12} + 0 = \frac{ma^2}{12}$$

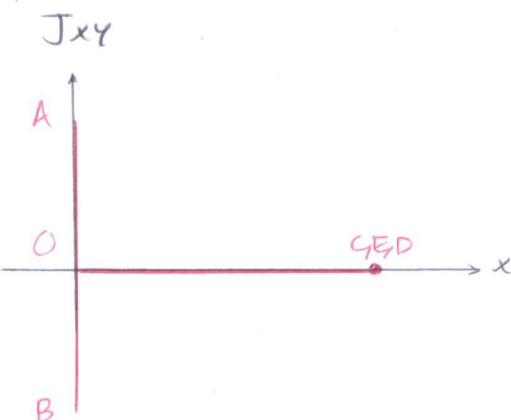
$$J_{YDE} = \frac{ma^2}{12} + m a^2 = \frac{13ma^2}{12}$$

$$J_{ZDE} = 0 + ma^2 = ma^2$$

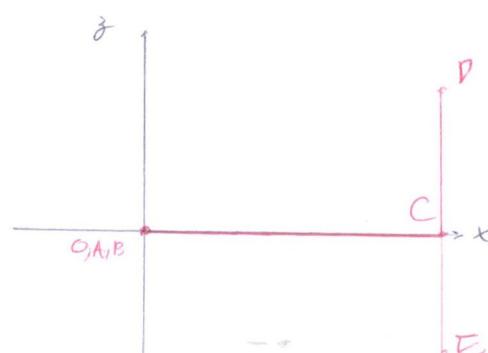
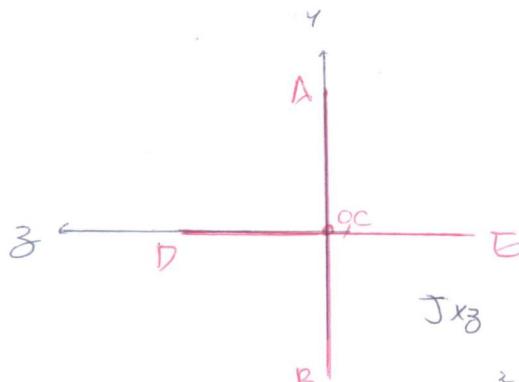
$$J_x = \frac{ma^2}{6}$$

$$J_y = \frac{17ma^2}{12}$$

$$J_z = \frac{17ma^2}{12}$$



$$J_{yz}$$



### CENTROS DE MASSA

Barra AB:  $(0, 0, 0)$

Barra OC:  $(\frac{a}{2}, 0, 0)$

Barra DE:  $(a, 0, 0)$

### Barra AB

$$J_{XAB} = \frac{ma^2}{12} + 0 = \frac{ma^2}{12}$$

$$J_{YAB} = 0 + 0 = 0$$

$$J_{ZAB} = \frac{ma^2}{72} + 0 = \frac{ma^2}{72}$$

### Barra OC

$$J_{XOC} = 0 + 0 = 0$$

$$J_{YOC} = \frac{ma^2}{12} + m\left(\frac{a}{2}\right)^2 = \frac{4ma^2}{12}$$

$$J_{ZOC} = \frac{ma^2}{12} + m\left(\frac{a}{2}\right)^2 = \frac{4ma^2}{12}$$

$$J_x = J_{XAB} + J_{XOC} + J_{XDE} = \frac{ma^2}{12} + 0 + \frac{ma^2}{12} = \frac{ma^2}{6}$$

$$J_y = J_{YAB} + J_{YOC} + J_{YDE} = 0 + \frac{4ma^2}{12} + \frac{13ma^2}{12} = \frac{17ma^2}{12}$$

$$J_z = J_{ZAB} + J_{ZOC} + J_{ZDE} = \frac{ma^2}{72} + \frac{4ma^2}{72} + ma^2 = \frac{17ma^2}{12}$$

$$J_{XYAB} = 0 + 0 \cdot m = 0$$

$$J_{XYOC} = 0 + 0 \cdot \frac{a}{2} \cdot m = 0$$

$$J_{XYDE} = 0 + 0 \cdot \frac{a}{2} \cdot m = 0$$

$$\boxed{J_{XY} = 0}$$

$$J_{YZAB} = 0 + m \cdot 0 \cdot 0 = 0$$

$$J_{YZOC} = 0 + m \cdot 0 \cdot 0 = 0$$

$$J_{YZDE} = 0 + m \cdot 0 \cdot 0 = 0$$

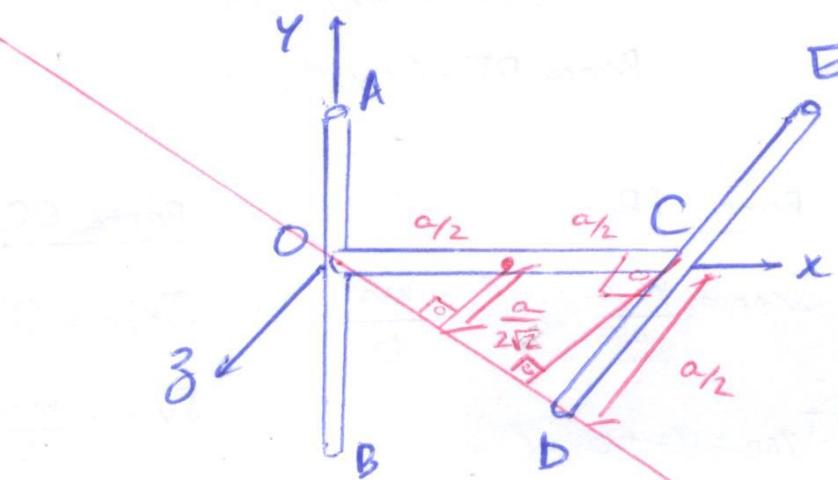
$$\boxed{J_{YZ} = 0}$$

$$J_{XZAB} = 0 + m \cdot 0 \cdot 0 = 0$$

$$J_{XZOC} = 0 + m \cdot 0 \cdot \frac{a}{2} = 0$$

$$J_{XZDE} = 0 + m \cdot 0 \cdot a = 0$$

$$\boxed{J_{XZ} = 0}$$



$$J_{op AB} = \frac{ma^2}{12} + 0 = \frac{ma^2}{12}$$

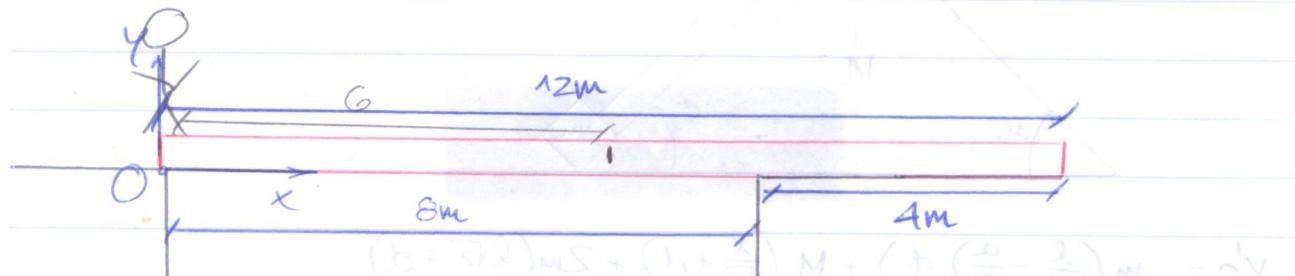
$$J_{op OC} = \frac{ma^2}{12} + m\left(\frac{a}{2\sqrt{2}}\right)^2 = \frac{ma^2}{12} + \frac{ma^2}{8}$$

$$J_{op DE} = \frac{ma^2}{12} + m\left(\frac{c}{\sqrt{2}}\right)^2 = \frac{ma^2}{12} + \frac{mc^2}{2}$$

$J_{op}$

3.

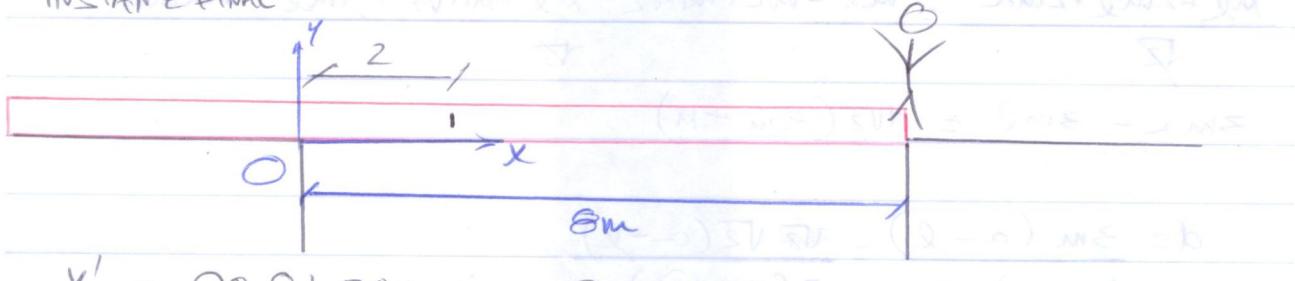
10 INSTANCE



$$X_G = \frac{80 \cdot 0 + 20n \cdot 6}{80 + 20n} = \frac{6n}{4+n}$$

$$80 + 20n = 80 + 20n$$

INSTANCE FINAL



$$X'_G = \frac{80 \cdot 8 + 20n \cdot 2}{80 + 20n} = \frac{32 + 2n}{4+n}$$

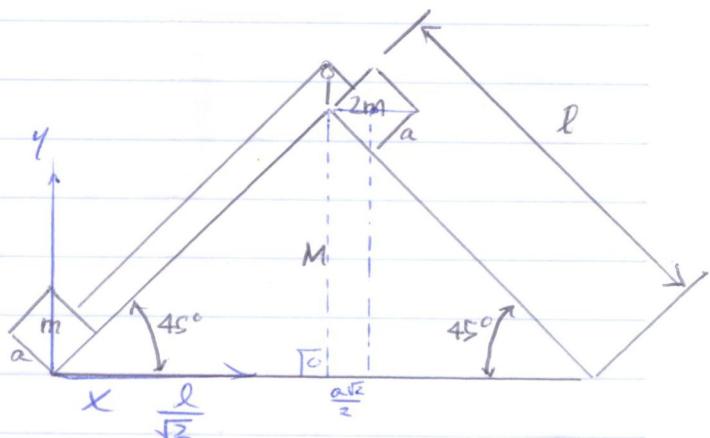
$$\text{Como } \overrightarrow{O_G} = \overrightarrow{O} \Rightarrow X_G = X'_G$$

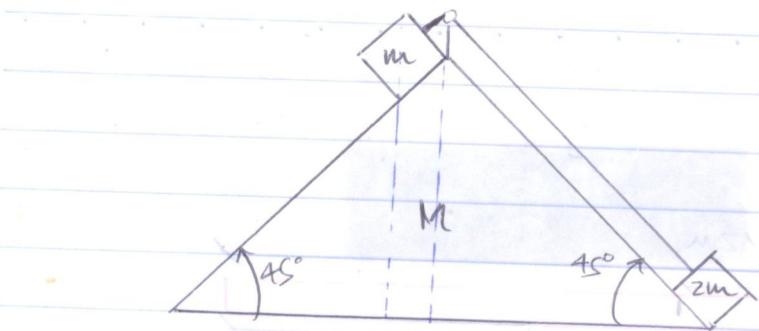
$$\frac{6n}{4+n} = \frac{32 + 2n}{n+4} \Rightarrow 4n = 32 \Rightarrow n=8$$

2.

$$X_G = m \cdot 0 + M \cdot \frac{l}{\sqrt{2}} + 2m \left( \frac{l}{\sqrt{2}} + \frac{a\sqrt{2}}{2} \right)$$

$$X_G = \frac{M \cdot l + 2ml + 2ma}{\sqrt{2}(m+M+2m)}$$





$$X'_G = \frac{m\left(\frac{l}{\sqrt{2}} - \frac{a}{\sqrt{2}} + d\right) + M\left(\frac{l}{\sqrt{2}} + d\right) + 2m(l\sqrt{2} + d)}{m + M + 2m}$$

NO SIST. SÓ HA FORÇAS NA DIREÇÃO Z.  $\Sigma F_z = 0 \Rightarrow a_G(z) = 0 \Rightarrow v_G = \text{constante}$ .  $v_{G0} = 0 \Rightarrow v_G = 0 \Rightarrow X_G = cte$

$$\frac{Ml + 2ml + 2ma}{\sqrt{2}} = ml - ma + md\sqrt{2} + Ml + Md\sqrt{2} + 4ml + md2\sqrt{2}$$

$$3ma - 3ml = d\sqrt{2}(3m + M)$$

$$d = \frac{3m(a-l)}{(3m+M)\cdot\sqrt{2}} = \frac{3m\sqrt{2}(a-l)}{2(3m+M)}$$

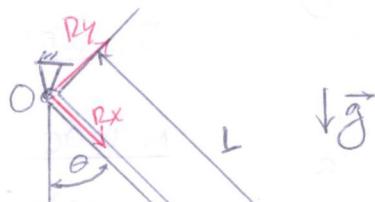
$$d = \frac{3m\sqrt{2}(a-l)}{2(3m+M)}$$



4.

Barra homog. OA

massa m



Peso concentrado massa 2m

Centrada de reposo na posição horizontal



$$\Rightarrow X_G = \frac{m \cdot \frac{L}{2} + 2m \cdot L}{3m} = \frac{5L}{6} \quad \overrightarrow{OG} = \frac{5L}{6} \vec{i}$$

$$J_0 = \frac{m L^2}{12} + \frac{m L^2}{2^2} + 2m L^2 = \frac{7m L^2}{3}$$

$$J_0 = \frac{7m L^2}{3}$$

b)



$$E_{antes} = 3mgH$$

$$E_{depois} = 3m\left(H - \frac{\epsilon}{G} L \cos \theta\right) + \frac{3m V_G^2}{2} + \frac{J_G \omega^2}{2}$$

$$\overrightarrow{V_G} = \overrightarrow{V_0} + \omega \overrightarrow{k} \wedge (\overrightarrow{G} - \overrightarrow{O}) = \overrightarrow{0} + \omega \overrightarrow{k} \wedge (S_G L \vec{i}) = S_G \omega L \vec{j} \Rightarrow V_G^2 = \frac{2\epsilon}{3G} \omega^2 L^2$$

$$3mgH = 3m\left(H - \frac{\epsilon}{G} L \cos \theta\right) + \frac{3m}{2} \cdot \frac{2\epsilon}{3G} \omega^2 L^2 + \underbrace{\frac{m}{4} \frac{L^2}{2} \omega^2}_{J_G}$$

$$\omega^2 = \frac{15}{7} \frac{g}{L} \cos \theta$$

TQMA

$$\overrightarrow{a_0} = \overrightarrow{0}$$

$$\dot{\overrightarrow{a}_0} = \frac{d}{dt} [i \ j \ k] [j]_0 [\omega] = \overrightarrow{n_0} \omega^2$$

$$-\frac{7}{3} m L^2 \dot{\omega} \overrightarrow{k} = \frac{\epsilon}{2} mg L \sin \theta \overrightarrow{k}$$

$$\dot{\omega} = -\frac{15}{14} \frac{g}{L} \sin \theta$$

$$c) \overrightarrow{a_G} = \overrightarrow{a_0} + \dot{\omega} \overrightarrow{k} \wedge (\overrightarrow{G} - \overrightarrow{O}) + \omega \overrightarrow{k} \wedge [\omega \overrightarrow{k} \wedge (\overrightarrow{G} - \overrightarrow{O})]$$

$$\overrightarrow{a_G} = \overrightarrow{0} + \dot{\omega} \overrightarrow{k} \wedge (S_G \vec{i}) + \omega \overrightarrow{k} \wedge [\omega \overrightarrow{k} \wedge (S_G \vec{i})]$$

$$\overrightarrow{a_G} = -\frac{7\epsilon g}{84} \sin \theta \vec{j} - \frac{7\epsilon g}{72} \cos \theta \vec{i}$$

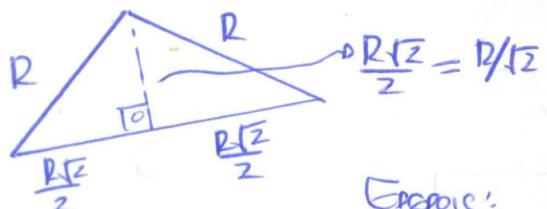
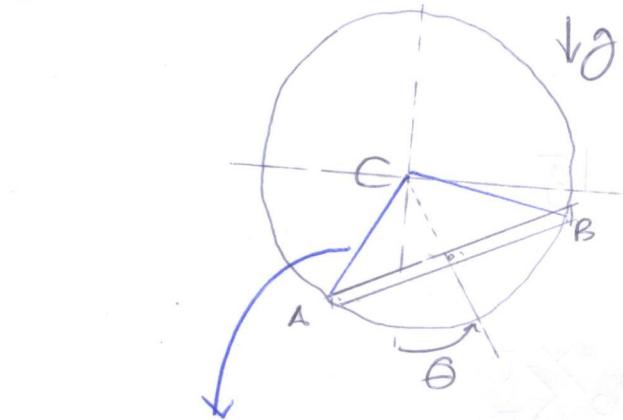
$$Dy - P \sin \theta = -\frac{7\epsilon g}{84} \sin \theta \cdot 3m$$

$$d) R_x + P \cos \theta = -\frac{7\epsilon}{42} \omega \sin \theta \cdot 3m$$

$$R_x = -\frac{117}{14} mg \cos \theta$$

$$Dy = \frac{9}{28} mg \sin \theta$$

10.

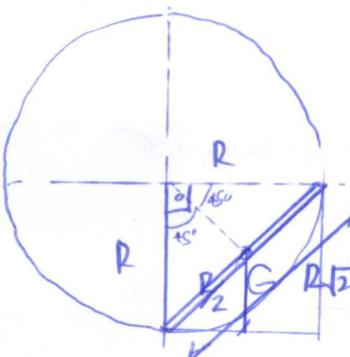


Energia:

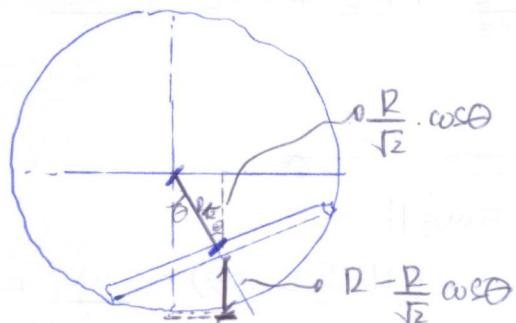
$$mg(R - \frac{R}{\sqrt{2}} \cos\theta) + \frac{mv^2}{2} + \frac{JG \cdot w^2}{2}$$

$$v_G = w \cdot \frac{R}{\sqrt{2}}$$

MOVIM.  
CIRCULAR  
AO RAIO DE  $\frac{R}{\sqrt{2}}$



$$\text{Energia: } \frac{mRg}{2}$$

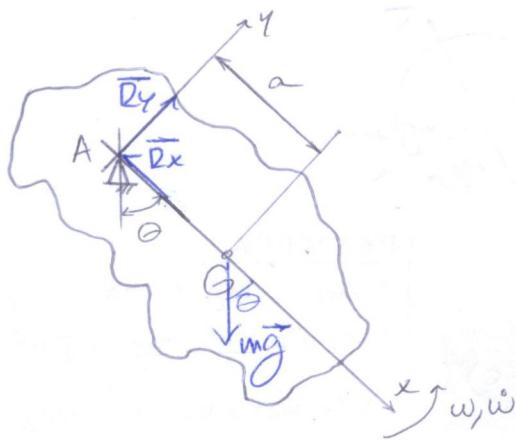


$$E_{total} = E_{pot} \Rightarrow mg \frac{R}{2} = mg(R - \frac{R}{\sqrt{2}} \cos\theta) + \frac{mw^2 R^2}{4} + \frac{mR^2}{G} \cdot \frac{w^2}{2}$$

$$mgR(\cos\theta \cdot \frac{\sqrt{2}}{2} - \frac{1}{2}) = \frac{mR^2 w^2}{3} \Rightarrow \boxed{w^2 = \frac{3\sqrt{2}g}{2R} (\cos\theta - \frac{1}{2})}$$

$$T = \frac{mR^2 w^2}{3} = \frac{mR^2}{3} \cdot \frac{3\sqrt{2}}{2} \frac{2}{R} (\cos\theta - \frac{1}{2}) \Rightarrow \boxed{T = mRg \frac{\sqrt{2}}{2} (\cos\theta - \frac{1}{2})}$$

5.

PÉNDULO COMPOSTO: massa  $m$ ,  $\omega$ ,  $\dot{\omega}$ .

$$\text{Então: } m(-\omega^2 a) = -Rx + mg \cos \theta$$

$$m \omega a = Ry - mg \sin \theta$$

$$\text{logo } Rx = m(g \cos \theta + \omega^2 a)$$

$$Ry = m(\omega a + g \sin \theta)$$

TMB

$$m a_{Tx} = -Rx + mg \cos \theta$$

$$m a_{Ty} = Ry - mg \sin \theta$$

$$\vec{a}_G = \vec{a}_A + \vec{\omega} \times (\vec{r}_G - \vec{r}_A) + \vec{\omega} \times [\vec{\omega} \times (\vec{r}_G - \vec{r}_A)]$$

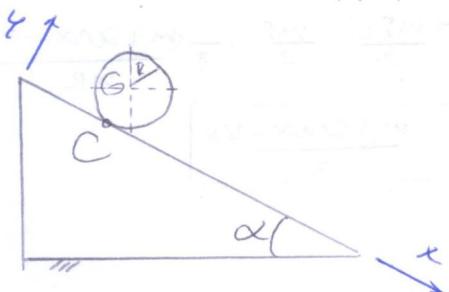
$$\vec{a}_G = \vec{0} + \vec{\omega} \times (\vec{a}_A) + \omega^2 \vec{r}_A \times [\vec{\omega} \times \vec{r}_A]$$

$$\vec{a}_G = \omega a_A \hat{j} - \omega^2 a_A \hat{i}$$

$$R_x = -m(g \cos \theta + \omega^2 a) \hat{i}$$

$$R_y = m(\omega a + g \sin \theta) \hat{j}$$

6.

Cilindro de massa  $m$ , raio  $R$ ,  $\mu$ ,  $J_{BG} = \frac{1}{2} mR^2$ 

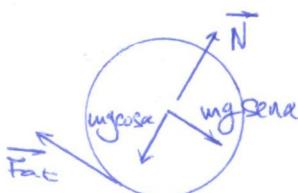
$$\vec{a}_C = \omega^2 R \hat{j}$$

$$\vec{a}_G = \vec{a}_C + (-\vec{\omega} \times (\vec{r}_G - \vec{r}_C)) + \vec{\omega} \times [\vec{\omega} \times (\vec{r}_G - \vec{r}_C)]$$

$$\vec{a}_G = \omega^2 R \hat{j} - \vec{\omega} \times (R \hat{j}) + \omega^2 \hat{k} \times [\hat{k} \times R \hat{j}]$$

$$\vec{a}_G = \omega^2 R \hat{j} + \vec{\omega} \hat{i} - \omega^2 R \hat{j}$$

$$\vec{a}_G = \vec{\omega} R \hat{i}$$



$$a_{Gy} = 0$$

$$m a_{Gy} = N - mg \cos \alpha \Rightarrow N = mg \cos \alpha$$

$$\begin{aligned} a) \quad m a_{Gx} &= mg \sin \alpha - F_f \\ m \cdot \omega R &= mg \sin \alpha - F_f \end{aligned}$$

TQMA

$$\frac{d}{dt} [I \vec{\omega} \vec{r}_G] = \vec{M}_{ext}$$

Casos planos, então vale

$$\frac{mR^2}{2} (-\dot{\omega}) \vec{r} = -F_f R \hat{i} \times R \hat{j}$$

$$F_f = \frac{mR}{2} \dot{\omega}$$

Substituindo, temos:

$$\frac{3m \omega R}{2} = mg \sin \alpha \Rightarrow \dot{\omega} = \frac{2}{3} \frac{g}{R} \sin \alpha$$

$$F_f = \frac{mR}{2} \cdot \frac{2}{3} \frac{g}{R} \sin \alpha = \frac{m}{3} g \sin \alpha = F_f$$

$$\vec{a}_G = \frac{2}{3} \frac{g}{R} \sin \alpha \cdot R \hat{i}$$

$$\vec{a}_G = \frac{2}{3} g \sin \alpha \hat{i}$$

$$b) \text{ Com escorregamento, vale } F_f = N \cdot \mu$$

$$F_f = mg \cos \alpha \cdot \mu$$

$$m a_{Gx} = mg \sin \alpha - mg \cos \alpha \mu$$

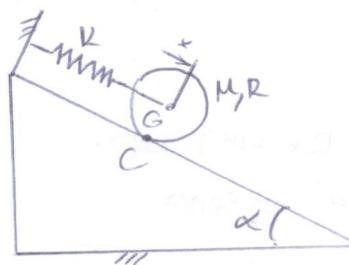
$$a_{Gx} = \vec{a}_G = g(\sin \alpha - \mu \cos \alpha) \hat{i}$$

$$\mu g \cos \alpha = \frac{mR}{2} \dot{\omega} \Rightarrow \dot{\omega} = \frac{2 \mu g \cos \alpha}{R}$$

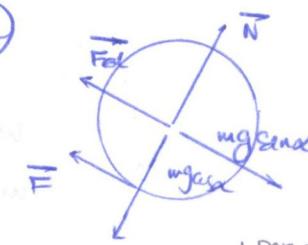
$$c) \frac{mg \sin \alpha}{3} = mg \cos \alpha \mu \Rightarrow \tan \alpha = 3\mu$$

Saída do repouso

7.



a)



$$\vec{a}_c = \omega^2 R \vec{j}$$

$$\vec{a}_G = \vec{a}_c + (-\vec{\omega} \vec{k}) \times (\vec{G} - \vec{c}) + (\vec{\omega} \vec{k}) \times [-\vec{\omega} \vec{k} \times (\vec{G} - \vec{c})]$$

$$\vec{a}_G = \omega^2 R \vec{j} + (-\vec{\omega} \vec{k}) \times R \vec{j} + \omega^2 \vec{k} \times [\vec{k} \times R \vec{j}]$$

$$\vec{a}_G = \omega^2 R \vec{j} + \vec{\omega} R \vec{i} - \vec{\omega} R \vec{j}$$

$$\vec{a}_G = \vec{\omega} R \vec{i}$$

TMB

$$m a_{Gy} = N - mg \cos \alpha = 0 \Rightarrow N = mg \cos \alpha$$

$$m a_{Gx} = mg \sin \alpha - kx - F$$

TQMA

$$\frac{d}{dt} [\vec{i} \vec{j} \vec{k}] \cdot \vec{F} \cdot \vec{I} G[\vec{\omega}] = \vec{r}_{ext}$$

Caso particular, plano, vale:

$$\frac{mR^2}{2} \cdot (-\vec{\omega} \vec{i}) = -\vec{F} \vec{k} \times R \vec{j}$$

$$\frac{mR}{2} \dot{\omega} \vec{i} = \vec{F}$$

$$m \cdot \vec{\omega} R = mg \sin \alpha - kx - \frac{R}{2} \dot{\omega}$$

$$\dot{\omega} = \frac{2}{3} \frac{mg \sin \alpha - kx}{m \cdot R}$$

$$\text{Então } \vec{a}_G = \frac{2}{3} \frac{mg \sin \alpha - kx \vec{i}}{m}$$

c)  $F = \frac{mR \dot{\omega}}{2}$  é nula quando  $\dot{\omega}$  é nula

$$\dot{\omega} = 0 \Rightarrow mg \sin \alpha - kx = 0$$

$$x = \frac{mg \sin \alpha}{k}$$

POR ENERGIA

$$E_{antes} = mg \cdot x \operatorname{sen} \alpha \quad \frac{mR^2}{2}$$

$$E_{depois} = \frac{kx^2}{2} + \frac{mv^2}{2} + \frac{J\omega^2}{2}$$

Vale  $v_G = \omega R$  (SI DESCUAR)

$$mg \cdot x \operatorname{sen} \alpha = \frac{kx^2}{2} + \frac{mv^2}{2} + \frac{J\omega^2}{2}$$

$$mg \cdot x \operatorname{sen} \alpha - \frac{kx^2}{2} = \frac{3mv^2}{4}$$

DERRIVANDO, temos:

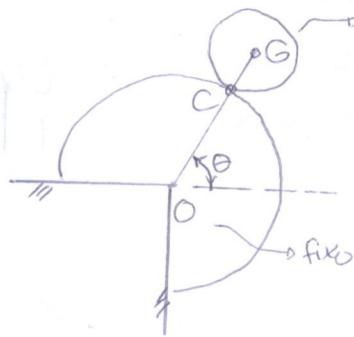
$$v \operatorname{sen} \alpha - kx v = \frac{3}{2} m \omega v$$

$$\omega = \frac{2}{3} \frac{(mg \operatorname{sen} \alpha - kx)}{m}$$

$$b) F = \frac{mR \dot{\omega}}{2} = \frac{mR}{2} \cdot \frac{2}{3} \frac{(mg \operatorname{sen} \alpha - kx)}{mR}$$

$$F = \frac{mg \operatorname{sen} \alpha - kx}{3}$$

8.



CILINDRO homog. de raio  $R$ , massa  $m$   
não se escorregar

POR CONSERVAÇÃO DE ENERGIA, TEMOS:

$$mg(R+r)\sin\theta_0 = E_{ANES}$$

$$mg(R+r)\sin\theta + \frac{mv_G^2}{2} + \frac{J\omega^2}{2} = E_{ENERG}$$

$$\text{Vale } v_G = \omega r$$

$$m(R+r)\sin\theta \cdot g = m(R+r)\sin\theta g + \frac{mr^2\omega^2}{2} + \frac{mr^2\omega^2}{4}$$

$$(R+r)g[\sin\theta_0 - \sin\theta] = \frac{3}{4}\omega^2r^2$$

$$\boxed{\omega^2 = \frac{4}{3}\frac{(R+r)}{r^2}g(\sin\theta_0 - \sin\theta)}$$



$$\frac{m \cdot v^2}{(R+r)} = mg \sin\theta - N$$

$$\frac{m \cdot w^2 \cdot r^2}{(R+r)} = mg \sin\theta - N$$

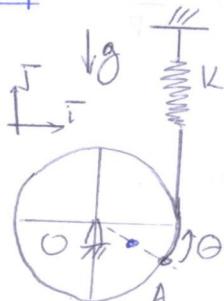
$$\frac{4}{3}\frac{g}{r^2}(R+r)\left(\frac{\sin\theta_0 - \sin\theta}{(R+r)}\right)r^2 = mg \sin\theta - N$$

$$\boxed{N = \frac{mg}{3}(7\sin\theta - 4\sin\theta_0)}$$

Para  $N=0$

$$\boxed{\sin\theta = \frac{4}{7}\sin\theta_0}$$

9.



DISCO de  $R, m$   
MASSA CONC. M EM A

PARTIR DO REPOSO  $\theta = 0 \Rightarrow F_{d0} = 0$

$$\theta \geq 0$$

$$(G-O) = -\frac{R}{2}\sin\theta \hat{j} + \frac{R}{2}\cos\theta \hat{i}$$

$$I_O = I_G + ML^2$$

$$I_G = \frac{mR^2}{2} + m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2$$

$$E_{ANES} = 2mg \frac{R}{2} \sin\theta$$

$$E_{ENERG} = \frac{Kx^2}{2} + \frac{2mv_G^2}{2} + \frac{J\omega^2}{2}$$

$$2mg \sin\theta = \frac{Kx^2}{2} + m \cdot \frac{w^2 R^2}{4} + \frac{mR^2\omega^2}{2}$$

$$I_G = mR^2$$

$$w^2 = \frac{4R \sin\theta \cdot mg - 2Kx^2}{3mR^2} \Rightarrow \theta = \theta R$$

$$\boxed{w^2 = -\frac{2}{3}\frac{K}{m}\theta^2 + \frac{4}{3}\frac{g}{R}\sin\theta}$$

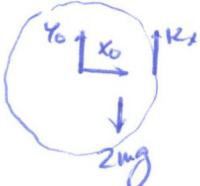
DARAVANDE,  $2\omega \dot{\theta} = -\frac{4}{3}\frac{K}{m}\theta \cdot \dot{\theta} + \frac{4}{3}\frac{g}{R} \cos\theta \cdot \omega$   
TEMOS:

$$\boxed{\dot{\omega} = -\frac{2}{3}\frac{K}{m}\theta + \frac{2}{3}\frac{g}{R}\cos\theta}$$

$$\overline{\alpha}_G = \overline{\alpha}_0 + (-\omega K) \times (G-O) + (-\omega K) \times [-\overline{wK} \times (G-O)]$$

$$\overline{\alpha}_G = \overline{\alpha} - \overline{wK} \times \left( \frac{R}{2}\cos\theta \hat{i} - \frac{R}{2}\sin\theta \hat{j} \right) + \frac{R}{2}\sin\theta \omega^2 \hat{j} - \frac{R}{2}\cos\theta \omega^2 \hat{i}$$

$$\boxed{\overline{\alpha}_G = \left( -\frac{\omega R}{2}\sin\theta - \frac{R}{2}\cos\theta \omega^2 \right) \hat{i} + \left( -\frac{\omega R}{2}\cos\theta + \frac{R}{2}\sin\theta \omega^2 \right) \hat{j}}$$

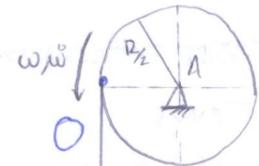


$$2m\left(-\frac{\omega R}{2}\sin\theta - \frac{R}{2}\cos\theta \omega^2\right) = Kx \Rightarrow \boxed{x_0 = -\omega R \sin\theta \cdot m - R \cos\theta \omega^2 m}$$

$$2m\left(-\frac{\omega R}{2}\cos\theta + \frac{R}{2}\sin\theta \omega^2\right) = Y_0 + Kx - 2mg$$

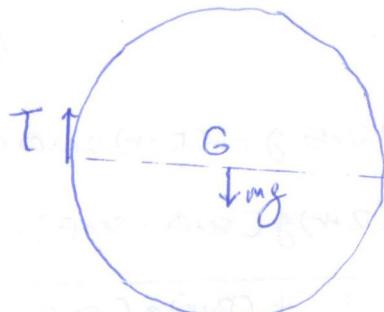
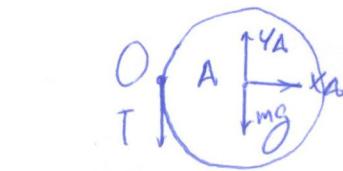
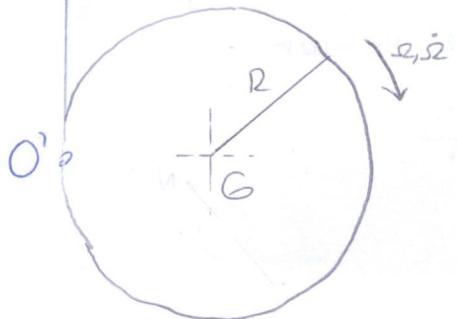
$$\boxed{Y_0 = 2mg - KR\theta - \omega^2 R \cos\theta + \omega^2 R \sin\theta}$$

19.



DISCO A: massa m

DISCO G: massa M



$$\overline{\alpha}_A = \overline{0}$$

TMA

$$\overline{m\alpha}_G = -mg\hat{j} + T\hat{j}$$

$$4A = T + mg$$

$$x_A = 0$$

$$\overline{m\alpha}_G = (T - mg)\hat{j}$$

TMA NO DISCO A $\overline{\alpha}_A = \overline{0}$ , problema planar, vale:

$$\dot{\overline{H}}_A = \overline{M}_A$$

$$\frac{m}{2} \left(\frac{R}{2}\right)^2 \cdot \dot{\omega} \hat{k} = (-T\hat{j}) \wedge \left(\frac{R}{2}\hat{i}\right)$$

$$\dot{\omega} = \frac{4T}{mR}$$

$$\text{ASSUM: } \dot{\omega} = 2\dot{r}$$

TMA NO DISCO G

$$\dot{\overline{H}}_G = \overline{M}_G$$

$$\frac{mR^2}{2} \cdot (-\dot{r}\hat{k}) = T\hat{j} \wedge (R\hat{i})$$

$$\dot{r} = \frac{2T}{mR}$$

$$\overline{\alpha}_O = \overline{\alpha}_A + (\dot{\omega} \hat{k}) \wedge (O-A) + \omega \hat{k} \wedge [\omega \hat{k} \wedge (O-A)]$$

$$\overline{\alpha}_O = \overline{0} + \dot{\omega} \hat{k} \wedge \left(-\frac{R}{2}\hat{i}\right) + \omega^2 \hat{k} \wedge [\hat{k} \wedge \left(-\frac{R}{2}\hat{i}\right)]$$

$$\overline{\alpha}_O = \boxed{-\frac{\dot{\omega}R}{2}\hat{j} + \frac{\omega^2 R}{2}\hat{i}}$$

Vai no fio

$$\overline{\alpha}_G = \overline{\alpha}_{O\text{fio}} + (-\dot{r}\hat{k}) \wedge (G-O) + (-\dot{r}\hat{k}) \wedge [-\dot{r}\hat{k} \wedge (G-O)]$$

$$\overline{\alpha}_G = -\frac{\dot{\omega}R}{2}\hat{j} + (-\dot{r}\hat{k}) \wedge (R\hat{i}) + \dot{r}^2 \hat{k} \wedge [\hat{k} \wedge R\hat{i}]$$

$$\overline{\alpha}_G = -\frac{\dot{\omega}R}{2}\hat{j} - \dot{r}\hat{i}\hat{j} - \dot{r}^2 \hat{R}\hat{i}$$

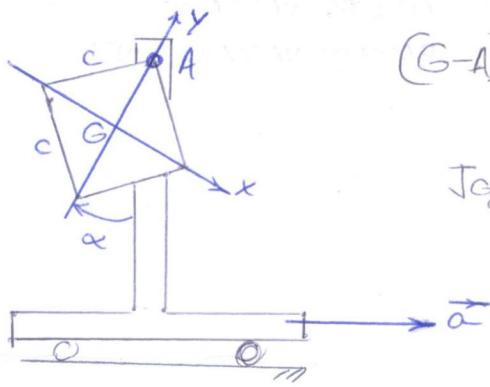
$$\left(-\frac{\dot{\omega}R}{2} - \dot{r}\hat{i}\right) \cdot m = T - mg \Rightarrow \frac{4TR}{mR \cdot 2} + \frac{2\dot{r}R}{mR} = \frac{mg - T}{m} \Rightarrow \boxed{T = \frac{mg}{5}}$$

$$\dot{\omega} = \frac{4T}{mR} = 4 \cdot \frac{mg}{5} \cdot \frac{1}{mR} = \frac{4g}{5R} \Rightarrow \boxed{\dot{\omega} = \frac{4g}{5R}} \Rightarrow \boxed{\dot{r} = \frac{2g}{5R}}$$

$$\overline{\alpha}_G = -\frac{4g}{5R} \cdot \frac{R}{2}\hat{j} - \frac{2g}{5R} \cdot R\hat{i} = -\frac{4g}{5R}\hat{j}$$

$$\boxed{\alpha_G = \frac{4}{5}g}$$

12.



$$(G-A) = -\frac{c}{\sqrt{2}} \vec{j}$$

$$J_{Gz} = \frac{m}{6} c^2$$

$$J_{A\bar{x}} = \frac{2}{3} m c^2$$

$$mg = mg \sin \alpha \vec{i} - mg \cos \alpha \vec{j}$$

$$\vec{a} = a \cos \alpha \vec{i} + a \sin \alpha \vec{j}$$

$$\vec{N_A} = -\frac{c}{\sqrt{2}} \vec{j} \times (mg \sin \alpha \vec{i} - mg \cos \alpha \vec{j})$$

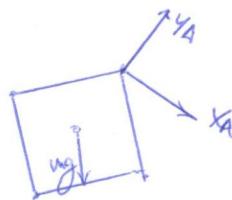
TQMA da placa em A

$$\frac{d}{dt} [I_{Gz} \dot{\alpha}] + m(G-A) \cdot \vec{a}_A = \vec{N_A}$$

$$J_{Gz} \cdot (-\ddot{\alpha} \vec{k}) + m \left( -\frac{c}{\sqrt{2}} \vec{j} \right) \cdot (a \cos \alpha \vec{i} + a \sin \alpha \vec{j}) = \left( -\frac{c}{\sqrt{2}} \vec{j} \right) \cdot (mg \sin \alpha \vec{i} - mg \cos \alpha \vec{j})$$

$$(J_{Gz} \dot{\alpha}) \vec{k} + m \frac{c \cdot a \cos \alpha}{\sqrt{2}} \vec{k} = \frac{mg c \sin \alpha}{\sqrt{2}} \vec{k}$$

$$\ddot{\alpha} = \frac{mc\sqrt{2}}{2J_{Gz}} (a \cos \alpha - g \sin \alpha) \vec{k}$$

DCL da placa

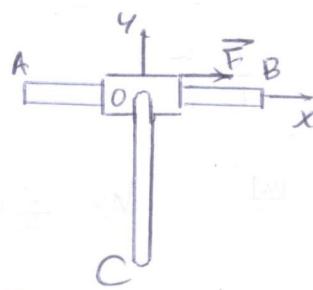
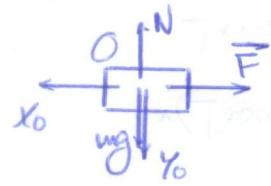
$$\vec{a}_G = \vec{a}_A + (-\dot{\alpha} \vec{k}) \times (G-A) + (-\dot{\alpha} \vec{k}) \times [-\dot{\alpha} \vec{k} \times (G-A)]$$

$$\vec{a}_G = a \cos \alpha \vec{i} + a \sin \alpha \vec{j} - \frac{\dot{\alpha}^2 c}{\sqrt{2}} \vec{i} + \frac{w^2 c}{\sqrt{2}} \vec{j}$$

$$m \cdot a_{Gx} = x_A + mg \sin \alpha \Rightarrow \left( a \cos \alpha - \frac{\dot{\alpha}^2 c}{\sqrt{2}} \right) = x_A + mg \sin \alpha \Rightarrow x_A = m \left( a \cos \alpha - \frac{\dot{\alpha}^2 c}{\sqrt{2}} - g \sin \alpha \right)$$

$$m a_{Gy} = y_A - mg \cos \alpha \Rightarrow m \left( a \sin \alpha + \frac{\dot{\alpha}^2 c}{\sqrt{2}} \right) = y_A - mg \cos \alpha \Rightarrow y_A = m \left( a \sin \alpha + \frac{\dot{\alpha}^2 c}{\sqrt{2}} + g \cos \alpha \right)$$

13.

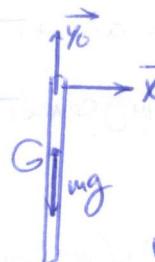
ANEL DE MASSA m  
BARRA DE MASSA m COMPL.DCL

$$ma_{0x} = F - x_0$$

$$ma_{0y} = N - mg - y_0 = 0$$

$$N = mg + y_0$$

$$\boxed{N = 2mg}$$



$$ma_{0x} = x_0$$

$$ma_{0y} = y_0 - mg = 0$$

$$\boxed{y_0 = mg}$$

$$\vec{a}_G = \vec{a}_O + \vec{\alpha}^\wedge (G-O) + \vec{\omega}^\wedge [\vec{\omega}^\wedge (G-O)]$$

$$\vec{a}_G = a_{0x} \vec{i} + \dot{\omega} R \vec{j} (-\frac{L}{2} \vec{j}) = a_{0x} \vec{i} + \dot{\omega} \frac{L}{2} \vec{i}$$

$$a_{0x} = a_{0x} + \frac{\dot{\omega} L}{2}$$

$$ma_{0x} = F - x_0$$

TQMA de BARRA

$$\frac{d}{dt} [I_0 \vec{\omega} \times \vec{w}] + m(G-O) \times \vec{a}_0 = \vec{F}_0$$

PADA O=G

$$\frac{mL^2}{12} \cdot \ddot{\omega} = -x_0 \frac{L}{2} \Rightarrow x_0 = -\frac{mL\ddot{\omega}}{6}$$

$$a_{0x} = \frac{F}{m} + \frac{mL\ddot{\omega}}{6}$$

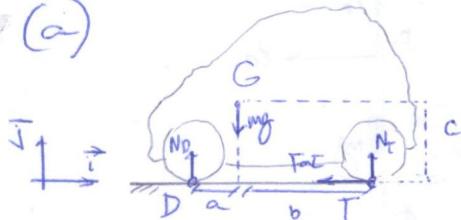
$$\frac{x_0}{m} = a_{0x} \Rightarrow \frac{x_0}{m} = \frac{F + m\ddot{\omega}L}{mG} + \frac{\ddot{\omega}L}{2} \Rightarrow -\frac{L\ddot{\omega}}{G} = \frac{F}{m} + \frac{L\ddot{\omega}}{G} + \frac{\ddot{\omega}L}{2} \Rightarrow \boxed{\ddot{\omega} = -\frac{GF}{5Lm}}$$

$$a_{0x} = \frac{F}{m} + \frac{L}{6} \cdot \left( -\frac{6F}{5Lm} \right) = \frac{4F}{5m} \Rightarrow \boxed{\ddot{a}_0 = \frac{4L}{5m} \vec{i}}$$

$$\vec{r}_0 = mg \vec{j}$$

$$\boxed{\vec{x}_0 = \frac{F}{5} \vec{i}}$$

14. (a)



Só a roda traseira sob tração.

Acelerarão max p/ não derrapar: Então vale  $F_{at} = N_T \cdot \mu$

Não está girando, Faz para esquerda.

Vamos calcular  $\vec{M}_G = \vec{0}$  (C.M.)

$$N_D \vec{j} \wedge (D - G) + (N_T \vec{j} \cdot N_T \mu \vec{i}) \wedge (T - G) = \vec{0}$$

$$N_D \vec{j} \wedge (-a\vec{i} - c\vec{j}) + (N_T \vec{j} - N_T \mu \vec{i}) \wedge (b\vec{i} - c\vec{j}) = \vec{0}$$

$$N_D a \vec{k} - N_T b \vec{k} + N_T \mu c \vec{k} = \vec{0}$$

$$N_D a = N_T (b - \mu c)$$

$$a(mg - N_T) = N_T (b - \mu c)$$

$$mga = N_T (a + b - \mu c)$$

$$\boxed{N_T = \frac{mga}{(a+b-\mu c)}}$$

$$(-1) m_{ax} = \frac{mga}{(a+b-\mu c)} \cdot \mu \Rightarrow$$

$$\frac{N_D + m_{ax}}{(a+b-\mu c)} = mg$$

$$\boxed{N_D = \frac{mg(b-\mu c)}{(a+b-\mu c)}}$$

$$\boxed{a_x = \frac{ag\mu}{(a+b-\mu c)}}$$

(b)



$$\vec{M}_G = \vec{0}$$

$$(N_D \vec{j} + F_{ad}) \wedge (D - G) + N_T \vec{j} \wedge (T - G) = \vec{0}$$

$$(N_D \vec{j} - N_D \mu \vec{i}) \wedge (-a\vec{i} - c\vec{j}) + N_T \vec{j} \wedge (b\vec{i} - c\vec{j}) = \vec{0}$$

$$N_D a \vec{k} + N_D \mu c \vec{k} - N_T b \vec{k} = \vec{0}$$

$$\left| \begin{array}{l} N_D(a + \mu c) = N_T b \\ N_D + N_T = mg \end{array} \right. \Rightarrow \left[ \begin{array}{l} [mg - N_T](a + \mu c) = N_T b \\ \frac{(a + \mu c)mg}{(a + b + \mu c)} = N_T(a + b + \mu c) \end{array} \right]$$

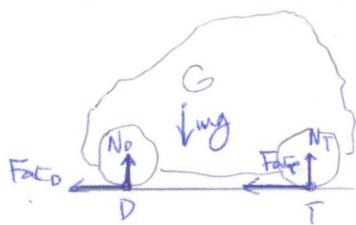
$$\boxed{\frac{mg(a + \mu c)}{(a + b + \mu c)} = N_T}$$

$$(-1) m_{ax} = + N_D \mu$$

$$m_{ax} = \frac{mg b}{(a+b+\mu c)} \cdot \mu (+1) \Rightarrow$$

$$\boxed{a_x = \frac{\mu g b}{(a+b+\mu c)}}$$

(c)



TMB CARRO

$$\vec{m}_{ax} = 2 N_D \vec{j} + 2 N_T \vec{j} + 2 F_{ad} \vec{i} + 2 F_{at} \vec{i} + mg \vec{j}$$

$$m_{ay} = 2 N_D + 2 N_T - mg = 0 \Rightarrow 2(N_D + N_T) = mg$$

$$m_{ax} = -2 F_{ad} - 2 F_{at} = -2 N_D \mu - 2 N_T \mu = (-2\mu)(N_D + N_T)$$

$$\vec{M}_G = 2(N_D \vec{j} + F_{ad} \vec{i}) \wedge (D - G) + 2(N_T \vec{j} + F_{at} \vec{i}) \wedge (T - G) = \vec{0}$$

$$\vec{M}_G = 2(N_D \vec{j} - N_D \mu \vec{i}) \wedge (-a\vec{i} - c\vec{j}) + 2(N_T \vec{j} - N_T \mu \vec{i}) \wedge (b\vec{i} - c\vec{j}) = \vec{0}$$

$$2N_D a \vec{k} + 2N_D \mu c \vec{k} - 2N_T b \vec{k} + 2N_T \mu c \vec{k} = \vec{0}$$

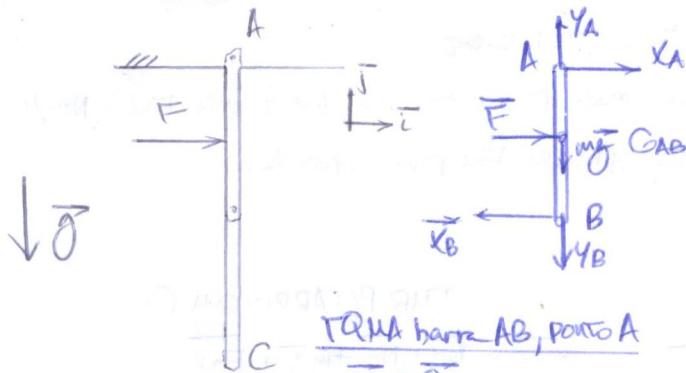
$$N_D(a + \mu c) = N_T(b - \mu c)$$

$$2(N_D + N_T) = mg$$

$$(-1) m_{ax} = -2 N_D \mu - 2 N_T \mu \Rightarrow \boxed{a_x = \frac{Mg}{2(a+b)}}$$

$$\boxed{N_D = \frac{(b-\mu c) \cdot mg}{2(a+b)}}$$

15.



TQMA barra AB, ponto A

$$\overrightarrow{\alpha_A} = \vec{0}$$

$$\frac{d}{dt} [I_A \vec{\omega}(w)] + m(G-A) \times \vec{\alpha_A} = \vec{F_A}$$

$$\left[ \frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2 \right] \alpha_{AB} \vec{k} = (-\frac{L}{2} \vec{j}) \times \vec{F} + (-L \vec{j}) \times (-x_B \vec{i})$$

$$\left[ \frac{mL^2}{3} \alpha_{AB} \vec{k} = \frac{FL}{2} \vec{k} - x_B L \vec{k} \right]$$

$$\overrightarrow{\alpha_B} = \overrightarrow{\alpha_A} + \alpha_{AB} \vec{k} \times (B-A) + w_{AB} \vec{k} \times [w_{AB} \vec{k} \times (B-A)]$$

$$\overrightarrow{\alpha_B} = \vec{0} + \alpha_{AB} \vec{k} \times (-L \vec{j})$$

$$\overrightarrow{\alpha_B} = \alpha_{AB} \cdot L \vec{i}$$

algemeen coörd. onv.  $\vec{j}$

$$\overrightarrow{\alpha_{GBC}} = \overrightarrow{\alpha_B} + (\alpha_{BC}) \vec{k} \times (G_{BC}-B) + w_{BC} \vec{k} \times [w_{BC} \vec{k} \times (G_{BC}-B)]$$

$$\overrightarrow{\alpha_{GBC}} = \alpha_{ABL} \vec{i} - \alpha_{BCL} \vec{i}$$

algemeen coörd. onv.  $\vec{j}$

$$\text{Entso}: m \left( \frac{3F}{2mL} \cdot L + \left( -\frac{3x_B}{mL} \right) \cdot L - \frac{Gx_B}{mL} \cdot \frac{L}{2} \right) = x_B$$

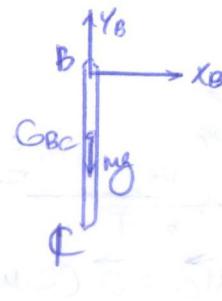
$$\frac{3F}{2} = 7x_B \Rightarrow x_B = \frac{3}{14} F$$

$$x_B = -\frac{3}{14} F$$

$$x_{AB} = \frac{3F}{2mL} - \frac{3}{mL} \cdot \frac{3}{14} F = \frac{21-9}{14} \frac{F}{mL} = \frac{6}{7} \frac{F}{mL}$$

$$\alpha = \frac{G}{AB} \frac{F}{mL}$$

$$\alpha_{AB} = \frac{G}{7} \frac{F}{mL} \vec{k}$$



TQMA barra BC, ponto G

$$\frac{d}{dt} [I_G \vec{\omega}(w)] + m(G-B) \times \vec{\alpha_G} = \vec{F_G}$$

$$\frac{mL^2}{12} \cdot \dot{w}_{BC} \vec{k} = (G-B) \times x_B \vec{i}$$

$$\left[ \frac{mL^2}{12} \cdot \alpha_{BC} \vec{k} = x_B \frac{L}{2} \vec{k} \right]$$

TMB barra BC

$$m \alpha_{GK} = x_B$$

$$m (\alpha_{ABL} + \alpha_{BCL} \frac{L}{2}) = x_B$$

$$\alpha_{ABL} = \frac{3F}{2mL} - \frac{3x_B}{mL}$$

$$\alpha_{BCL} = \frac{Gx_B}{mL}$$