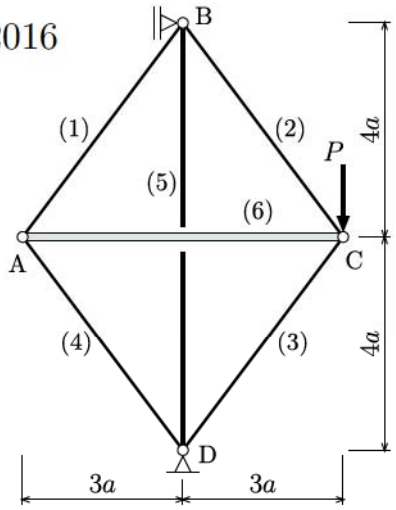
Nome: _____ Nº USP: _____

(Colocar nome em todas as folhas!)

2ª Prova— 1º semestre de 2016

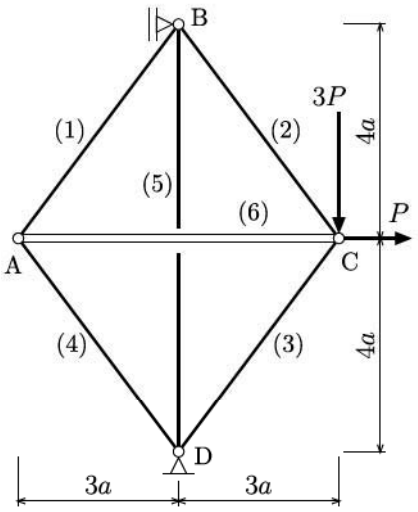
1ª **Questão** (3,5 pontos)

Determine as forças normais nas barras 5 e 6 da treliça ao lado sujeita à uma força vertical P aplicada no nó C. Admita a barra horizontal como rígida e considere um produto de rigidez axial EA constante para as demais barras.



2ª **Questão** (1,5 pontos)

Calcule o deslocamento vertical do ponto C da treliça ao lado sujeita ao carregamento indicado. Admita a barra horizontal como rígida e considere um produto de rigidez axial EA constante para as demais barras. São dadas as forças normais nas barras da treliça.



Barra	$\hat{N}(P)$
1	−0,891
2	1,817
3	−1,933
4	−0,891
5	−0,741
6	1,069

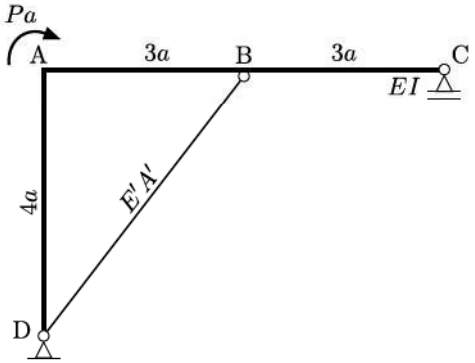
[2ª Prova, 1º semestre de 2016] Nome: _____ Nº USP: _____

3ª Questão (5,0 pontos)

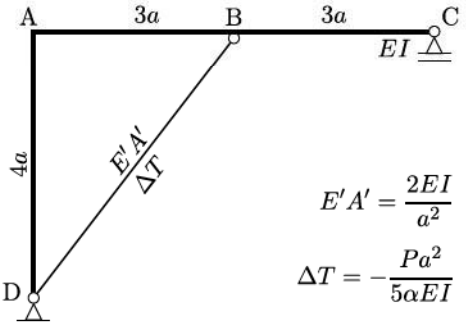
No pórtico esquematizado abaixo, as barras AD e ABC têm produto de rigidez à flexão EI e produto de rigidez axial $EA \approx \infty$, enquanto o tirante tem produto de rigidez axial $E'A' = 2EI/a^2$.

a) Determine o *diagrama de momentos fletores* no pórtico sujeito à ação do momento Pa aplicado no ponto A, e calcule o *deslocamento vertical* do ponto B.

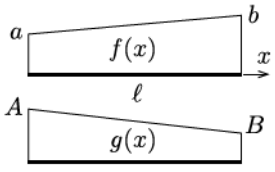
b) Se apenas o tirante DB sofrer uma redução de temperatura $\Delta T = -Pa^2/(5\alpha EI)$, qual seria a intensidade da força normal no tirante?



(a)



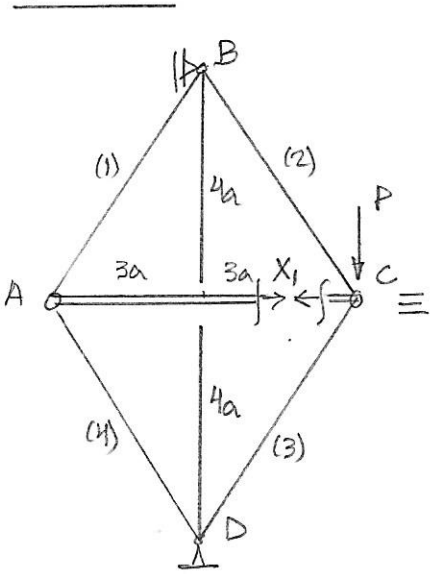
(b)



$$\int_0^\ell f(x)g(x)\,\mathrm{d}x = \frac{\ell}{6}\big(2aA + 2bB + aB + bA\big)$$

2016 may 31

16 P2 Q1



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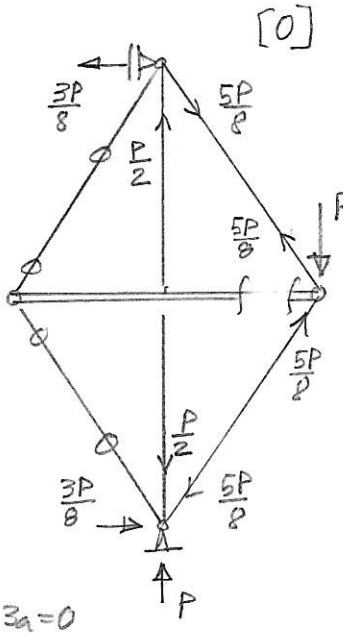
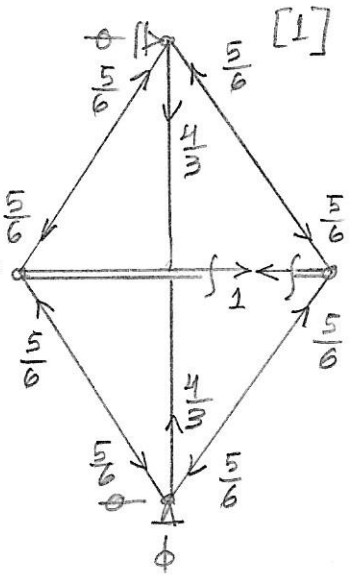


+ X_1 ·



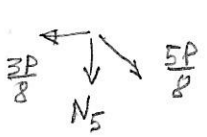
$$\curvearrowleft_D \; -R_B \times 8a + P \times 3a = 0$$

$$R_B = \frac{3P}{8}$$

Ⓒ

$$N_4 = -N_3 = \frac{5P}{8}$$

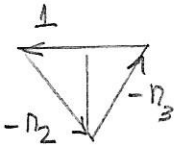
Ⓑ



$$\uparrow \; \frac{5P}{8} \times \frac{4}{5} = -N_5$$

$$N_5 = -\frac{P}{2}$$

Ⓒ

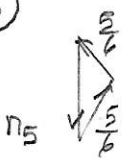


$$n_2 = n_3 = -\frac{5}{6}$$

simetria

$$n_4 = n_1 = -\frac{5}{6}$$

Ⓑ



$$n_5 = 2\left(\frac{8}{6} \times \frac{4}{5}\right) = \frac{4}{3}$$

Equação de Fortviolant

$$F_{11} = \sum_{b=1}^{nb} \frac{n_1 n_1 l_b}{EA} = \frac{1}{EA}\left\{4\left[\left(-\frac{5}{6}\right)^2 \times 5a\right] + \left(\frac{4}{3}\right)^2 \times 8a\right\} = \frac{a}{EA}\left\{\frac{500}{36} + \frac{128}{9}\right\} = \frac{1012}{36}\frac{a}{EA} = \frac{253}{9}\frac{a}{EA}$$

$$d_{10} = \sum_{b=1}^{nb} \frac{n_1 N_0 l_b}{EA} = \frac{1}{EA}\left[\frac{5P}{8}\left(-\frac{5}{6}\right) \times 5a + \left(-\frac{5P}{8}\right)\left(-\frac{5}{6}\right) \times 5a + \left(-\frac{P}{2}\right)\left(\frac{4}{3}\right) \times 8a\right] = -\frac{16}{3}\frac{Pa}{EA}$$

$$F_{11} X_1 = -d_{01} \Rightarrow \frac{253}{9}\frac{a}{EA} X_1 = \frac{16}{3}\frac{Pa}{EA} \Rightarrow X_1 = \frac{48}{253}P = 0{,}1897P \quad (\hat{N}_6)$$

$$\hat{N}_5 = -\frac{P}{2} + \frac{48P}{253}\left(\frac{4}{3}\right) = -\frac{253 + 128}{506} = -\frac{125}{506}P = -0{,}2470P$$

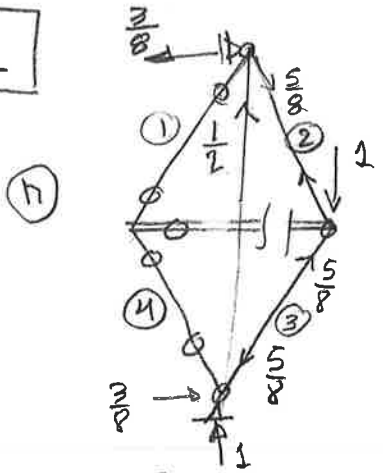
Demais forças normais:

$$\hat{N}_4 = \hat{N}_1 = 0 + \frac{48}{253}P\left(-\frac{5}{6}\right) = -\frac{40}{253}P = -0{,}1581P$$

$$\hat{N}_2 = \frac{5P}{8} + \frac{48P}{253}\left(-\frac{5}{6}\right) = \frac{5P}{8} - \frac{40}{253}P = \frac{945}{2024}P = 0{,}4669P$$

$$\hat{N}_3 = -\frac{5P}{8} + \frac{48P}{253}\left(-\frac{5}{6}\right) = -\frac{5P}{8} - \frac{40}{253}P = -\frac{1585}{2024}P = -0{,}7831P$$

EIF$_1$



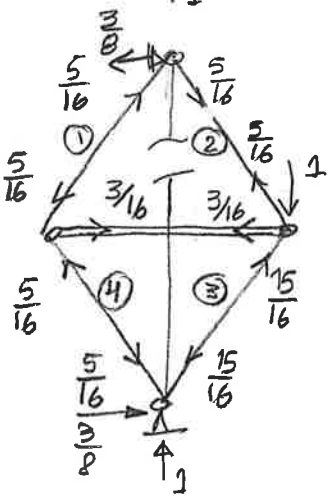
(h)

$$v_C = \sum_{b=1}^{nb} \frac{\hat{N}_h}{EA} \cdot \beta_b$$

$$= 1{,}817 P_x \times \frac{5}{8} \times \frac{5a}{EA} - 1{,}933 P_x \left(-\frac{5}{8}\right) \times \frac{5a}{EA}$$

$$-0{,}741 P_x \left(-\frac{1}{2}\right) \frac{8a}{EA} = 14{,}68 \frac{Pa}{EA}$$

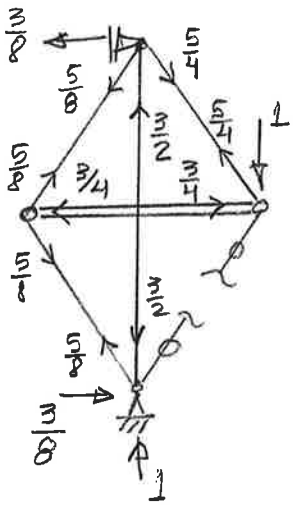
EIF$_2$



$$v_C = \left[-0{,}891 P_x \left(-\frac{5}{16}\right) + 1{,}817 P_x \left(\frac{5}{16}\right) - 1{,}933 \times \left(-\frac{15}{16}\right)\right.$$

$$\left. - 0{,}891 P_x \left(-\frac{5}{16}\right)\right] \frac{5a}{EA} = 14{,}68 \frac{Pa}{EA} \ (\downarrow)$$

EIF$_3$



$$v_C = \left[\, 2\left(-0{,}891 P_x \left(\frac{5}{8}\right) + 1{,}817 P_x \left(\frac{5}{4}\right)\right] \right. \frac{5a}{EA}$$

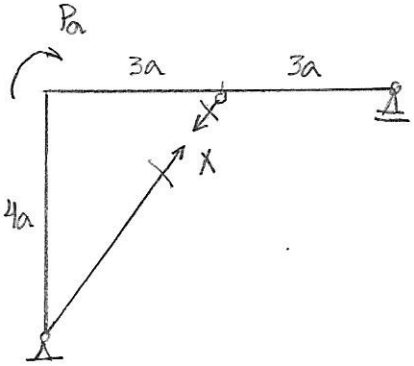
$$- 0{,}741 P_x \left(-\frac{3}{2}\right) \times \frac{8a}{EA}$$

$$= 14{,}68 \frac{Pa}{EA} \ (\downarrow)$$

2016 may 31

16P2Q3

a) EIF



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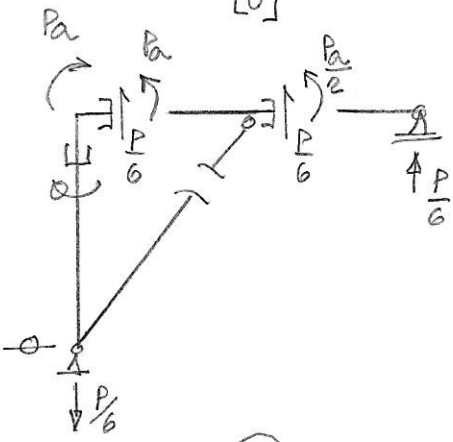

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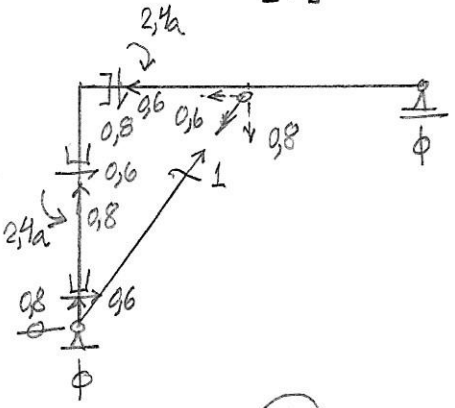
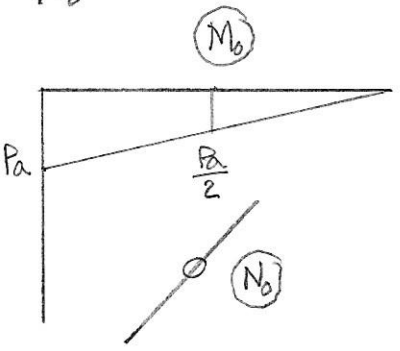
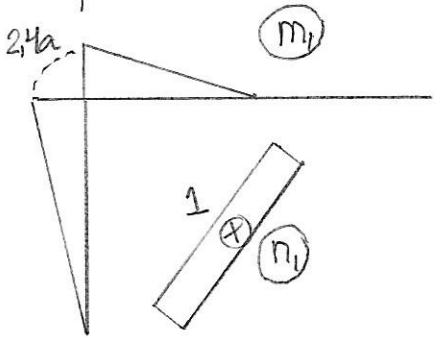


Diagramas



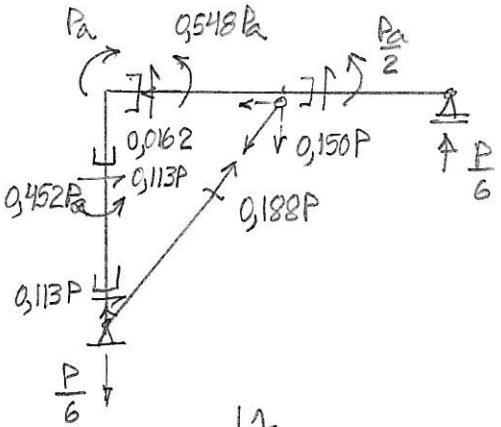


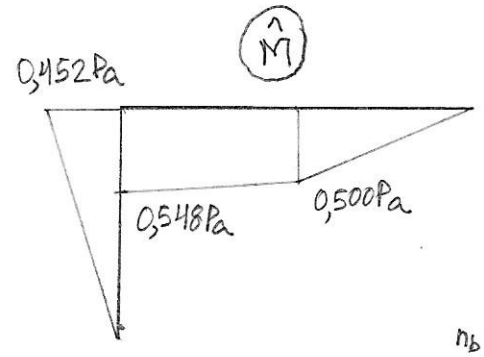
Eqs. de Continuidad

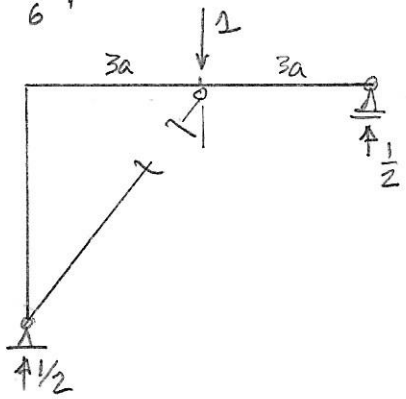
$$F_{11} = \sum_{b=1}^{n_b} \int_0^{\ell_b} \left(\frac{m_1 m_1}{EI} + \frac{n_1 n_1}{EA} \right) dx = \frac{4a}{3EI}(2{,}4a)^2 + \frac{3a}{3EI}(2{,}4a)^2 + \frac{1^2}{\frac{2EI}{a^2}} \times 5a = 13{,}44\frac{a^3}{EI} + \frac{5a^3}{2EI} = 15{,}94\frac{a^3}{EI}$$

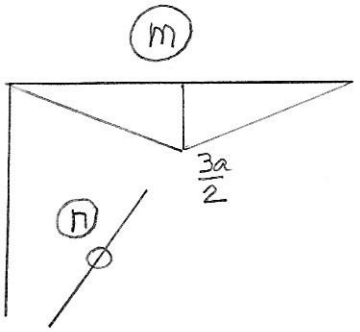
$$d_{10} = \sum_{b=1}^{n_b} \int_0^{\ell_b} \left(\frac{m_1 M_0}{EI} + \frac{n_1 N_0}{EA} \right) dx = \frac{1}{2}\frac{3a}{6EI}\left(-2Pa \times 2{,}4a - \frac{Pa}{2} \times 2{,}4a \right) = -3{,}0\frac{Pa^3}{EI}$$

$$F_{11} \cdot X_1 = -d_{01} \Rightarrow 15{,}94 \frac{X_1 a^3}{EI} = +3{,}0\frac{Pa^3}{EI} \Rightarrow \underline{X = 0{,}188P}$$









$$\sqrt{B} = \sum_{b=1}^{n_b} \int_0^{\ell_b} \left(\frac{\hat{M} m}{EI} + \frac{\bar{N} n}{EA} \right) dx$$

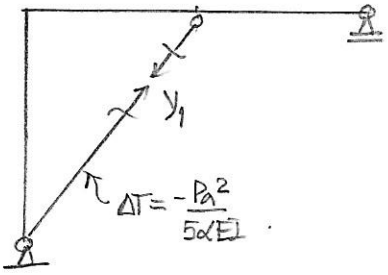
$$= \frac{1}{2}\frac{3a}{6EI}\left[\frac{3a}{2}(0{,}548 + 2 \times 0{,}5)\, Pa \right] + \frac{3a}{3EI}\left[\frac{3a}{2} \times 0{,}5\, Pa \right]$$

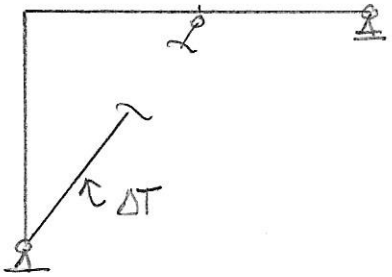
$$= (1{,}161 + 0{,}75)\frac{Pa^3}{EI}$$

$$= \underline{1{,}911\frac{Pa^3}{EI}\ (\downarrow)}$$

b)

[0]





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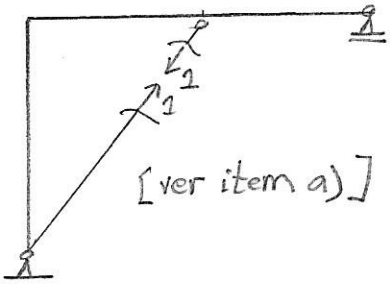

$$d_{10} = \sum_{b=1}^{n_b} \alpha \Delta T \int_0^{l_b} n_1 \, dx = \alpha \left(-\frac{Pa^2}{5\alpha EI}\right) \times 1 \times 5a = -\frac{Pa^3}{EI}$$

Equação de Fortviolant

$$F_{11} y_1 = -d_{01} \Rightarrow 15{,}94 \frac{a^3}{EI} y_1 = \frac{Pa^3}{EI} \Rightarrow y_1 = 0{,}0627P$$

Força no tirante.