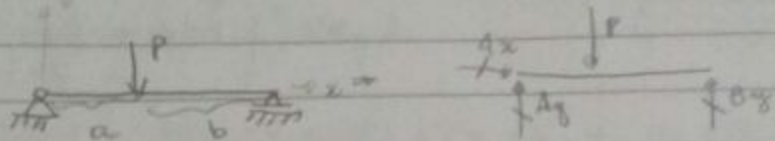


Lista 2 - PEF 3202

1) a)



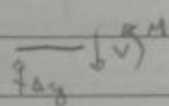
$$\begin{cases} A_x = 0 \\ A_y + B_y = P \end{cases}$$

$$M_A = (a+b) \cdot B_y - P \cdot a = 0$$

$$B_y = \frac{P \cdot a}{(a+b)}$$

$$A_y = \frac{P \cdot b}{(a+b)}$$

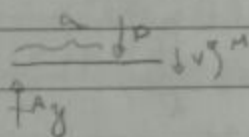
I) $0 < x < a$



$$V = A_y = \frac{P \cdot b}{a+b}$$

$$M = V \cdot x = \frac{P \cdot b \cdot x}{(a+b)}$$

II) $a < x < b$



$$A_y = P + V \rightarrow V = -\frac{P \cdot a}{a+b}$$

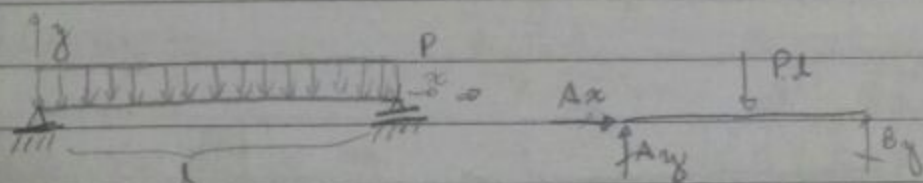
$$M = a \cdot P + x \cdot V$$

$$= \frac{a^2 P + a b P - x P a}{(a+b)}$$

$$V(x) = \begin{cases} \frac{P \cdot b}{a+b}, & 0 < x < a \\ -\frac{P \cdot a}{a+b}, & a < x < b \end{cases}$$

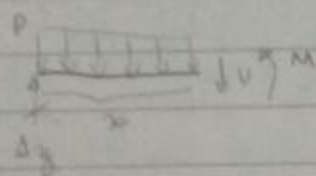
$$M(x) = \begin{cases} \frac{P \cdot b \cdot x}{a+b}, & 0 < x < a \\ \frac{P \cdot a^2 + P \cdot a \cdot b - P \cdot a \cdot x}{a+b}, & a < x < b \end{cases}$$

b)



$$\begin{cases} A_y + B_y = P l \\ B_y \cdot l = \frac{P l^2}{2} \Rightarrow B_y = \frac{P l}{2} = A_y \end{cases}$$

$0 < x < l$



$$A_y = V + \int_0^x p dx = \frac{pl}{2} - V + p x$$

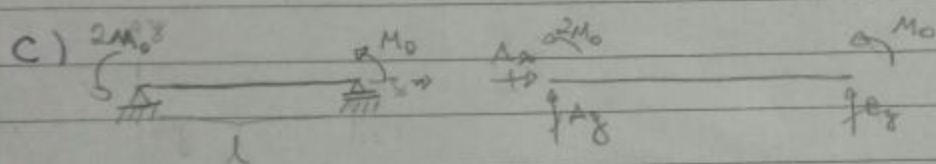
$$V = p \left(\frac{l}{2} - x \right)$$

$$M = V x + \int_0^x p x dx$$

$$= \frac{pl}{2} x - p x^2 + \frac{p x^2}{2} = \frac{plx}{2} - \frac{p x^2}{2} = \frac{px}{2} (l - x)$$

$V(x) = p \left(\frac{l}{2} - x \right)$

$M(x) = \frac{px}{2} (l - x)$

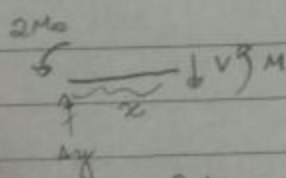


$$\begin{cases} A_y + B_y = 0 \\ A_x = 0 \end{cases}$$

$$M_A = 3M_0 + l B_y = 0$$

$$B_y = -\frac{3M_0}{l}$$

$$A_y = \frac{3M_0}{l}$$



$$V = \frac{3M_0}{l}$$

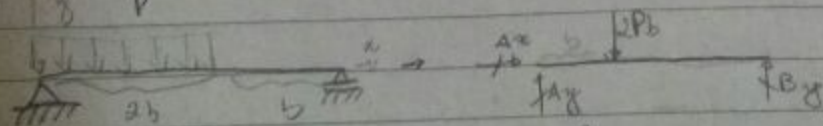
$$V = \frac{3M_0}{l}$$

$$M + 2M_0 = V \cdot x$$

$$M = M_0 \left(\frac{3x}{l} - 2 \right)$$

$$M = \frac{3M_0 x}{l} - 2M_0$$

d) p

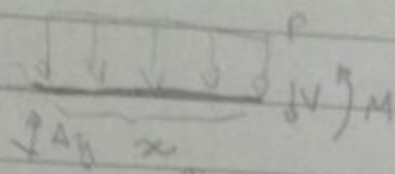


$$\begin{cases} A_x = 0 \\ A_y + B_y = 2Pb \end{cases}$$

$$M_A = 3b B_y - 2Pb^2 = 0$$

$$B_y = \frac{2Pb}{3} \rightarrow A_y = \frac{4Pb}{3}$$

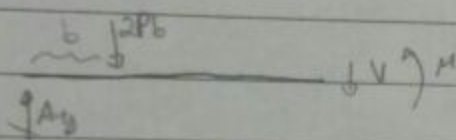
$$I) 0 < x < 2b$$



$$\Delta y = \int_0^x p dx + V \Rightarrow \frac{4pb}{3} = px + V \Rightarrow V = p\left(\frac{4b}{3} - x\right)$$

$$M = Vx + \int_0^x p dx = \frac{4pbx}{3} - px^2 + \frac{px^2}{2} = px\left(\frac{4b}{3} - \frac{x}{2}\right)$$

$$II) 2b < x < 3b$$



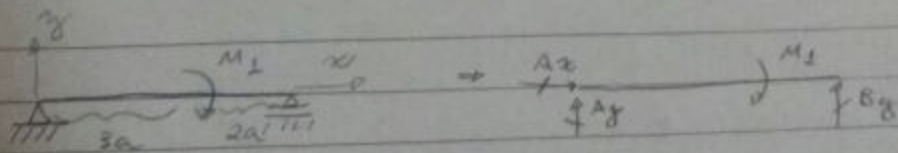
$$\Delta y = 2pb + V \quad V = -\frac{2pb}{3}$$

$$M = 2pb^2 + Vx = 2pb^2 - 2pbx = 2pb\left(b - \frac{x}{3}\right)$$

$$V(x) = \begin{cases} p\left(\frac{4b}{3} - x\right), & 0 < x < 2b \\ p\left(\frac{4b}{3} - \frac{x}{2}\right), & 2b < x < 3b \end{cases}$$

$$M(x) = \begin{cases} px\left(\frac{4b}{3} - \frac{x}{2}\right), & 0 < x < 2b \\ 2pb\left(b - \frac{x}{3}\right), & 2b < x < 3b \end{cases}$$

e)

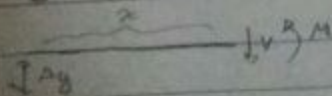


$$\begin{cases} \Delta x = 0 \\ \Delta y + B_y = 0 \end{cases}$$

$$M_1 = 3a B_y - M_2 = 0$$

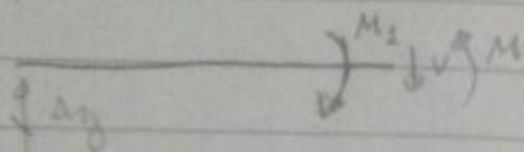
$$B_y = \frac{M_1}{3a} \Rightarrow \Delta y = -\frac{M_1}{3a}$$

$$I) 0 < x < 3a$$



$$V = \Delta y = -\frac{M_1}{3a} \quad M = Vx = -\frac{M_1}{3a}x$$

$$\text{II) } 3a \leq x < 5a$$

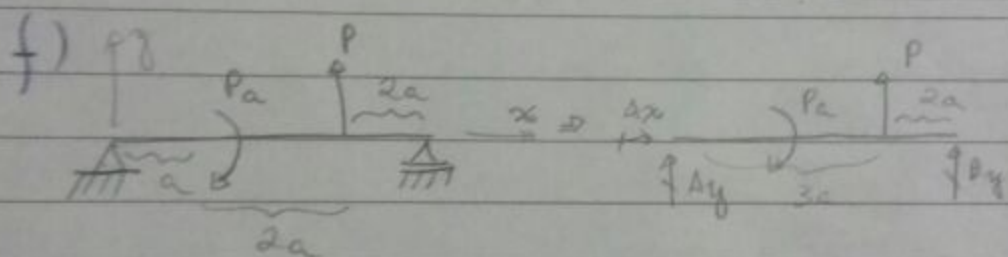


$$V = \Delta y = -\frac{M_1}{5a}$$

$$M = Vx + M_1 = -\frac{M_1 x}{5a} + M_1 = M_1 \left(1 - \frac{x}{5a}\right)$$

$$V = -\frac{M_1}{5a}$$

$$M(x) = \begin{cases} -\frac{M_1 x}{5a}, & 0 < x < 3a \\ M_1 \left(1 - \frac{x}{5a}\right), & 3a \leq x < 5a \end{cases}$$



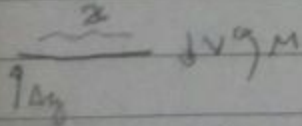
$$\begin{cases} \Delta x = 0 \\ A_y + B_y + P = 0 \end{cases}$$

$$M_A = 3Pa + 5a B_y - Pa = 0$$

$$B_y = -\frac{2P}{5}$$

$$A_y = -\frac{3P}{5}$$

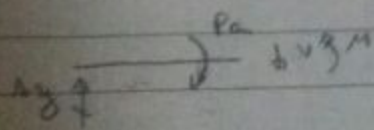
$$\text{I) } 0 < x < a$$



$$V = A_y = -\frac{3P}{5}$$

$$M = Vx = -\frac{3P}{5}x$$

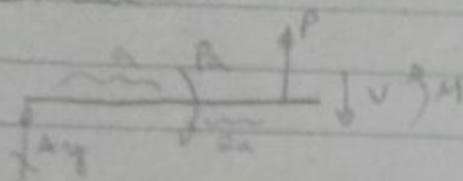
$$\text{II) } a \leq x < 3a$$



$$M = Pa + Vx = P \left(a - \frac{3x}{5}\right)$$

$$V = A_y = -\frac{3P}{5}$$

III) $3a \leq x < 5a$



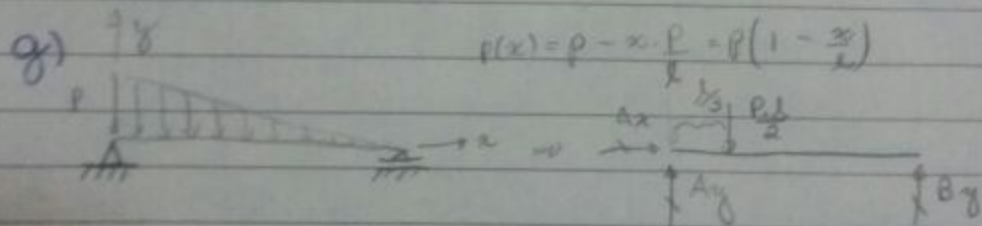
$$A_y + P = V \rightarrow V = \frac{2P}{5}$$

$$M + P \cdot 3a = P a + V x$$

$$M = -2Pa + \frac{2Px}{5} = 2P\left(\frac{x}{5} - a\right)$$

$$V(x) = \begin{cases} -\frac{3P}{5}, & 0 < x < 3a \\ \frac{2P}{5}, & 3a \leq x < 5a \end{cases}$$

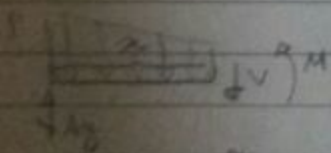
$$M(x) = \begin{cases} -\frac{3P}{5}x, & 0 < x < a \\ P\left(a - \frac{3}{5}x\right), & a \leq x < 3a \\ 2P\left(\frac{x}{5} - a\right), & 3a \leq x < 5a \end{cases}$$



$$\begin{cases} A_x = 0 \\ A_y + B_y = \frac{Pl}{2} \end{cases}$$

$$B_y \cdot l = \frac{Pl^2}{6} \rightarrow B_y = \frac{Pl}{6}$$

$$A_y = \frac{1Pl}{3}$$

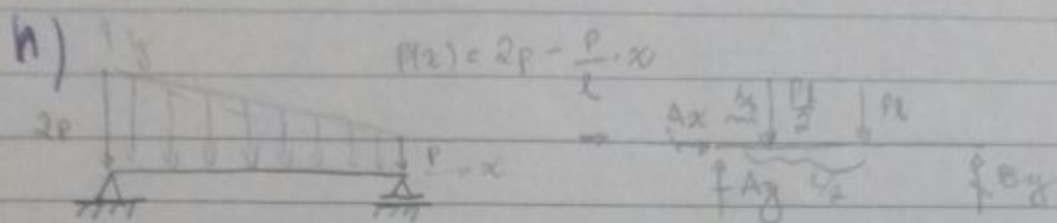


$$A_y = V + \int_0^x \left(p - \frac{xp}{l}\right) dx = V + Px - \frac{x^2 p}{2l} = \frac{Pl}{3} \rightarrow V = p\left(\frac{l}{3} + \frac{x^2}{2l} - x\right)$$

$$M = Vx + \int_0^x \left(p x - \frac{x^2 p}{l}\right) dx = \frac{1Plx}{3} + \frac{Px^3}{2l} - Px^2 + \frac{Px^2}{2} - \frac{Px^3}{3l} = \frac{1Plx}{6} + \frac{Px^3}{3l} - \frac{Px^2}{2}$$

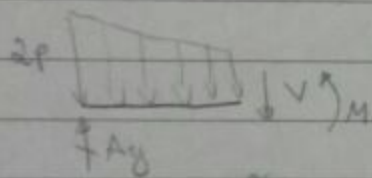
$$V(x) = P \left(\frac{x^2}{2l} - x + \frac{l}{3} \right)$$

$$M(x) = P \left(\frac{x^3}{6l} - \frac{x^2}{2} + \frac{l}{3} x \right)$$



$$\begin{cases} A_y + B_y = \frac{3Pl}{2} \\ \Delta x = 0 \end{cases} \quad M_a = B_y \cdot l = \frac{Pl^2}{6} + \frac{Pl^2}{2} = \frac{2Pl^2}{3}$$

$$\begin{cases} B_y = \frac{2}{3} Pl \\ A_y = \frac{5}{6} Pl \end{cases}$$



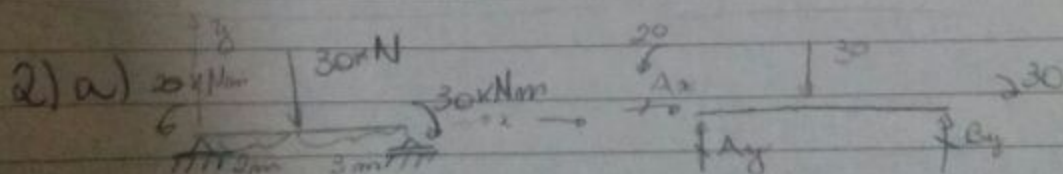
$$A_y = V + \int_0^x \left(2p - \frac{px}{l} \right) dx \Rightarrow \frac{5}{6} Pl = V + 2px - \frac{px^2}{2l} \Rightarrow$$

$$\Rightarrow V = \frac{px^2}{2l} - 2px + \frac{5}{6} Pl$$

$$M = Vx + \int_0^x \left(2px - \frac{px^2}{l} \right) dx \Rightarrow M = \frac{px^3}{2l} - 2px^2 + \frac{5}{6} Plx + \frac{px^2}{2} - \frac{px^3}{3l}$$

$$V(x) = \frac{px^2}{2l} - 2px + \frac{5}{6} Pl$$

$$M(x) = \frac{px^3}{6l} - px^2 + \frac{5}{6} Plx$$

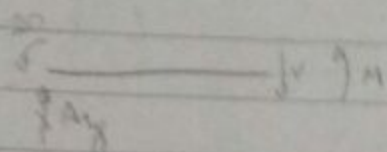


$$\begin{cases} \Delta x = 0 \\ A_y + B_y = 30 \end{cases}$$

$$20 + 8B_y = 30 + 60$$

$$B_y = 14 \text{ kN} \quad A_y = 16 \text{ kN}$$

I) $0 < x < 2 \text{ m}$

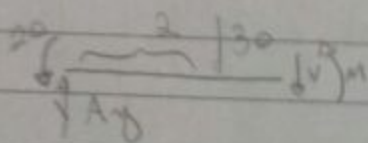


$$V = A_y = 16 \text{ kN}$$

$$20 + M = V \cdot x$$

$$M = 16x - 20$$

II) $2 \leq x < 5 \text{ m}$



$$16 = 30 + V$$

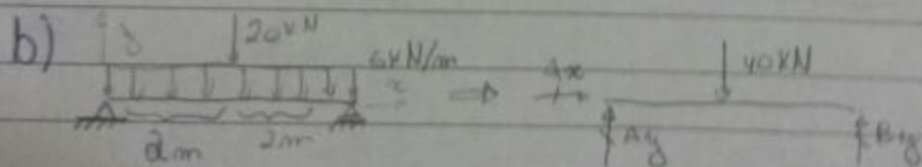
$$20 + M = Vx + 60$$

$$V = -14 \text{ kN}$$

$$M = 40 - 14x$$

$$V(x) = \begin{cases} 16 \text{ kN}, & 0 < x < 2 \\ -14 \text{ kN}, & 2 \leq x < 5 \end{cases}$$

$$M(x) = \begin{cases} 16x - 20, & 0 < x < 2 \\ 40 - 14x, & 2 \leq x < 5 \end{cases}$$



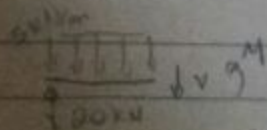
$$\begin{cases} A_x = 0 \\ A_y + B_y = 40 \end{cases}$$

$$M_{(A)} = 4 \cdot B_y - 80 = 0$$

$$B_y = 20 \text{ kN}$$

$$A_y = 20 \text{ kN}$$

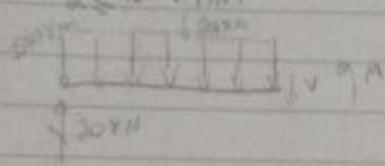
I) $0 < x < 2 \text{ m}$



$$20 = V + \int_0^x 5 dx + V = 20 - 5x$$

$$M = Vx + \int_0^x 5x dx = M = 20x - 5x^2 + \frac{5x^2}{2} = 20x - \frac{5x^2}{2}$$

II) $2 < x < 4m$

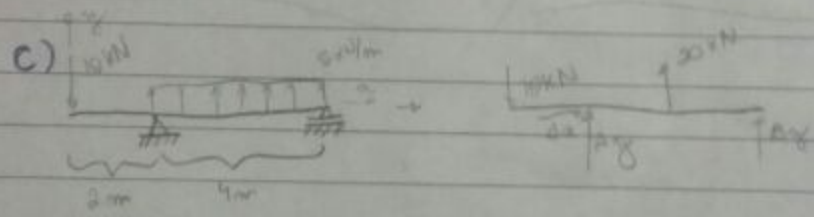


$$20 = 20 + V + \int_0^x 5 dx \Rightarrow V = -5x$$

$$M = Vx + 40 + \int_0^x 5x dx \Rightarrow M = -5x^2 + 40 + \frac{5x^2}{2} = 40 - \frac{5x^2}{2}$$

$$V(x) = \begin{cases} 20 - 5x, & 0 < x < 2m \\ -5x, & 2 < x < 4m \end{cases}$$

$$M(x) = \begin{cases} 20x - \frac{5x^2}{2}, & 0 < x < 2m \\ 40 - \frac{5x^2}{2}, & 2 < x < 4m \end{cases}$$



$$\begin{cases} \sum \Delta x = 0 \\ \sum A_y + B_y = -10 \end{cases} \quad \begin{cases} M_x = 20 + 40 + 4B_y \\ B_y = -15 \text{ kN} \\ A_y = 5 \text{ kN} \end{cases}$$

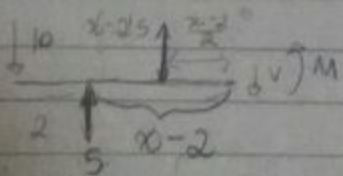
I) $0 < x < 2m$



$$V = -10 \text{ kN}$$

$$M = Vx = +10x$$

II) $2 < x < 6$



$$10 + V = 5 + 5(x-2) \Rightarrow V = 5x - 15$$

$$M = \frac{5}{2}x^2 - 10x + 10 - 10(x-2) + 5(x-2) + 0$$

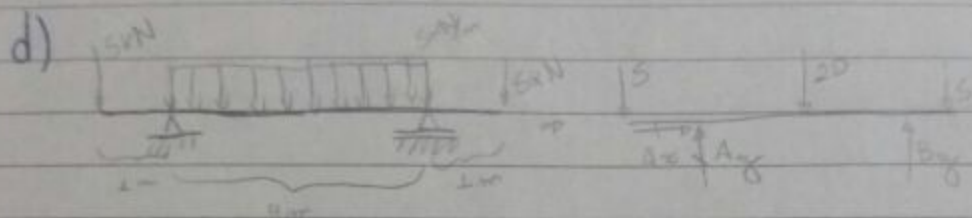
$$M = 10x - \frac{(x-2)^2}{2} \cdot 5 + (x-2) \cdot 5 + 0$$

$$\frac{5}{2}(x^2 - 4x + 4)$$

$$M = \frac{5}{2}x^2 - 15x$$

$$V(x) = \begin{cases} -10 & 0 \leq x < 2 \\ 5x - 15 & 2 \leq x < 6 \text{ m} \end{cases}$$

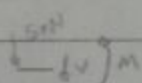
$$M(x) = \begin{cases} 10x & 0 \leq x < 2 \\ \frac{5}{2}x^2 - 15x & 2 \leq x < 6 \text{ m} \end{cases}$$



$$\begin{cases} A_x = 0 \\ A_y + B_y = 30 \end{cases}$$

$$\begin{aligned} M_{(6)} &= 5 + 4B_y = 40 + 25 \\ B_y &= 15 \text{ kN} \\ A_y &= 10 \text{ kN} \end{aligned}$$

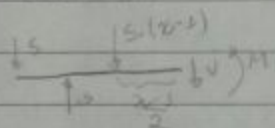
I) $0 \leq x < 2 \text{ m}$



$$V = -5x \text{ kN}$$

$$M = Vx = -5x^2$$

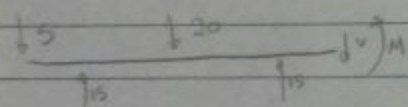
II) $2 \leq x < 6 \text{ m}$



$$5x + V = 15 \rightarrow V = 15 - 5x$$

III) $5 \leq x < 6 \text{ m}$

$$5x + M + \frac{5(x-5)^2}{2} = 15(x-5)$$



$$V + 25 = 30$$

$$V = 5 \text{ kN}$$

$$M = 10x - 15 - \frac{(5x^2 - 10x + 5)}{2}$$

$$M = -\frac{5}{2}x^2 + 15x - \frac{35}{2}$$

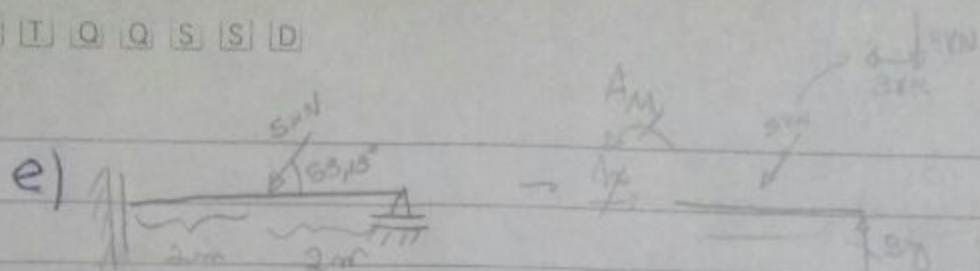
$$M + 15 + 75 = 60 + Vx$$

$$M = Vx - 30$$

$$= 5x - 30$$

$$V(x) = \begin{cases} -5x & 0 \leq x < 2 \\ 15 - 5x & 2 \leq x < 5 \\ 5 & 5 \leq x < 6 \end{cases}$$

$$M(x) = \begin{cases} -5x^2 & 0 \leq x < 2 \\ -\frac{5}{2}x^2 + 15x - \frac{35}{2} & 2 \leq x < 5 \\ 5x - 30 & 5 \leq x < 6 \end{cases}$$



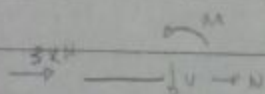
$$\sin(53,13^\circ) = 0,8 \quad , \quad \cos(53,13^\circ) = 0,6$$

$$\begin{cases} A_x = 3 \text{ kN} \\ B_y = 4 \text{ kN} \end{cases}$$

$$M_{(A)} = A_y + 4B_x = 8$$

$$A_y = -8 \text{ kNm}$$

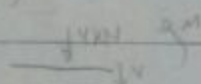
I) $0 < x < 2 \text{ m}$



$$V = 0 \quad M = 0$$

$$N = -3 \text{ kN}$$

II) $2,5 \text{ m} < x < 4 \text{ m}$



$$N = 0 \quad M = 8 + Vx = 8 - 4x$$

$$V = -4 \text{ kN}$$

$$N(x) = \begin{cases} -3 \text{ kN}, & 0 < x < 2 \text{ m} \\ 0, & 2 < x < 4 \text{ m} \end{cases}$$

$$M(x) = \begin{cases} 0, & 0 < x < 2 \\ 8 - 4x, & 2,5 < x < 4 \text{ m} \end{cases}$$

$$V(x) = \begin{cases} 0, & 0 < x < 2 \text{ m} \\ -4 \text{ kN}, & 2,5 < x < 4 \text{ m} \end{cases}$$