PEF – 3202 – Introdução à Mecânica dos Sólidos (24/06/2015)

Nome: ____ **nUSP:** ______________

Questão 1 (6,0)

Para a estrutura da figura 1, com a seção transversal dada na figura 2, calcule:

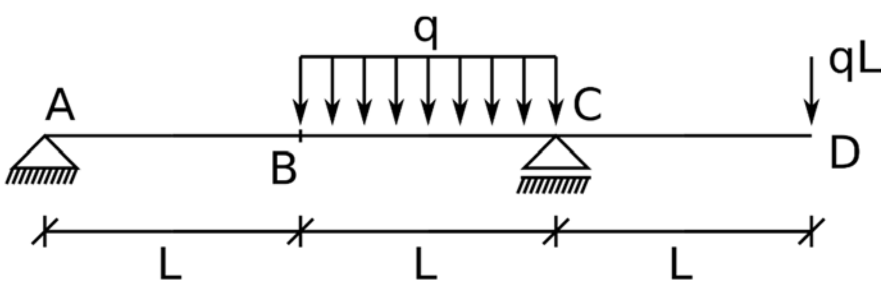

Figura 1

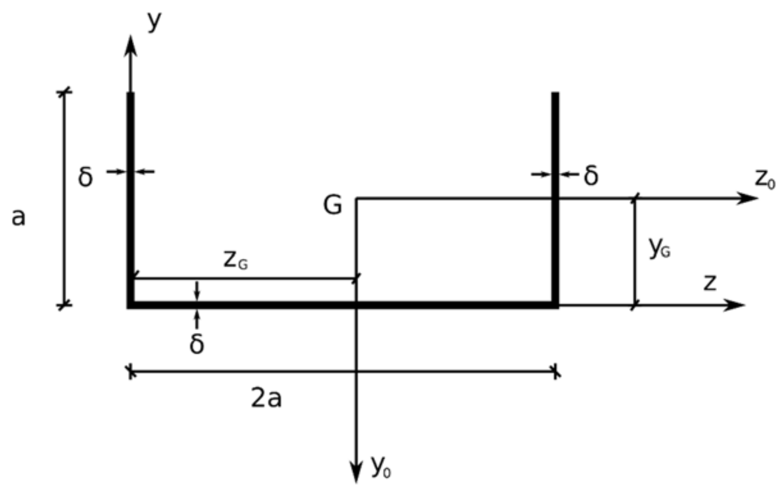

Figura 2

(a) Encontre as reações de apoio em A e B. Trace os diagramas de força cortante e momento. (1,0)

(b) Determine as propriedades da seção transversal (z_G, y_G e I_{Z0}). Considere que $\delta \ll a$. (1,0)

(c) Determine as tensões normais na seção de maior momento fletor (em módulo). (1,0)

(d) Determine as tensões de cisalhamento para a seção de maior força cortante (em módulo). (1,5)

(e) Determine o fluxo de cisalhamento para a mesma seção do item anterior. (1,5)

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Questão 2 (4,0)

Para a estrutura da figura 3, com a seção transversal dada na figura 4, calcule:

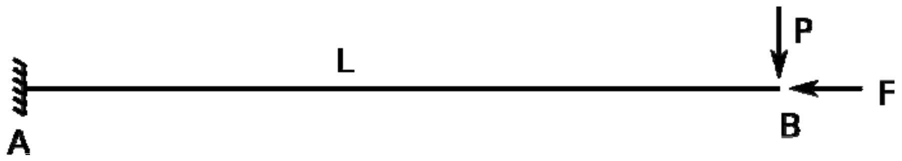

Figura 3

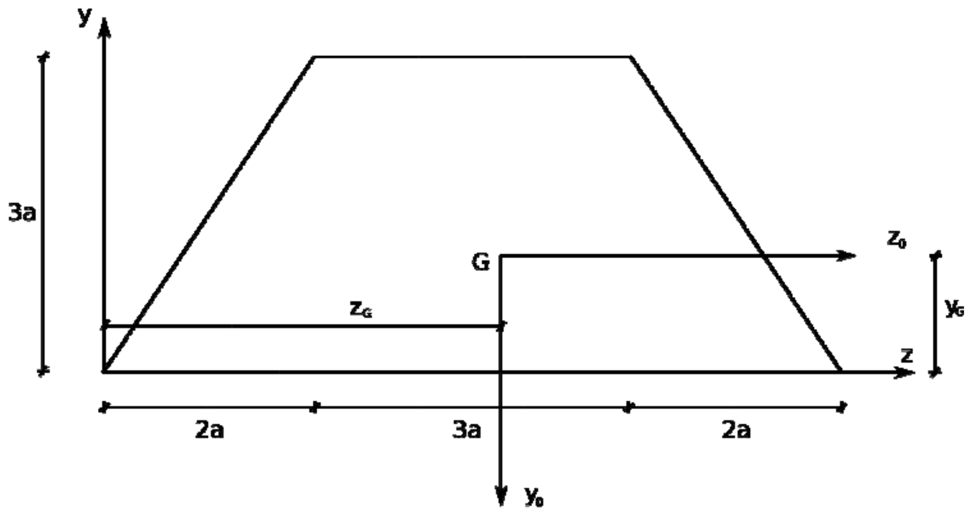

Figura 4

(a) Encontre as reações de apoio em A. Trace os diagramas de força normal, força cortante e momento. (1,0)

(b) Determine as propriedades da seção transversal (z_G, y_G e I_{Z0}). (1,0)

(c) Determine as tensões normais na seção de maior momento fletor (em módulo). (1,0)

(d) Determine a relação $\frac{P}{F}$, para que a tensão de tração máxima seja igual à tensão de compressão máxima em módulo. (0,5)

(e) Determine a relação $\frac{P}{F}$ considerando $L = 1{,}42m$ e $a = 8\ cm$. (0,5)

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Formulário

Propriedades de Figuras Planas

Momento estático: $M_{sz} = \int ydA, M_{sy} = \int zdA$

Momento de inércia: $I_z = \int y^2 dA, I_y = \int z^2 dA$

Retângulo: $I_{Z0} = \frac{bh^3}{12}$

Triângulo $I_{Z0} = \frac{bh^3}{36}$

Steiner $I_z = I_{Z0} + A \cdot d^2$

Tensões

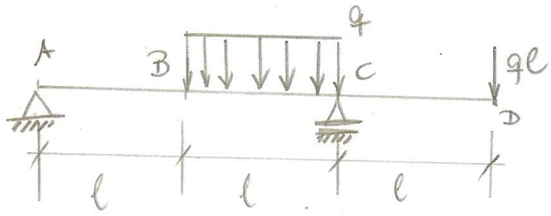
Tensão normal: $\sigma = \frac{N}{A} + \frac{M}{I_{Z0}} y$

Tensão de cisalhamento: $\tau = \frac{V \cdot M_S^*}{b^* \cdot I_{Z0}}$

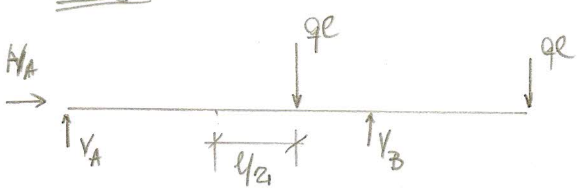
Dica

Quando for calcular o momento estático com 2 áreas com CG em posições opostas em relação ao eixo (uma no positivo, outra no negativo), use a seguinte fórmula:

$M_S = M_{S1} - M_{S2}$



Reações



$\boxed{H_A = 0}$

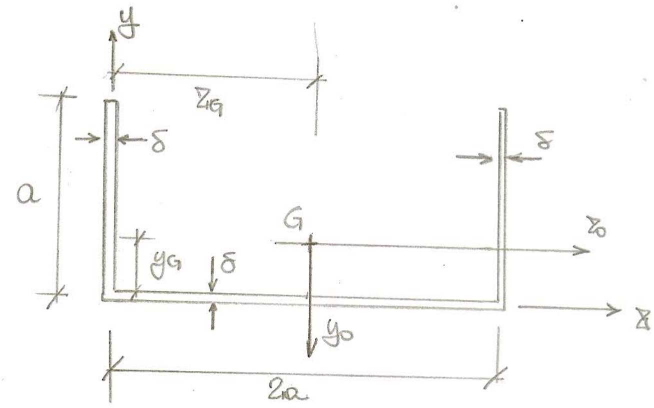
$V_A + V_B = 2q\ell$

$-q\ell\cdot(\tfrac{3}{2}\ell) + V_B\cdot 2\ell - q\ell\cdot 3\ell = 0$

$-\frac{3}{2}q\ell + 2V_B - 3q\ell = 0 \Rightarrow -\frac{9}{2}q\ell + 2V_B = 0$

$\boxed{V_B = \frac{9}{4}q\ell}$; $V_A = 2q\ell - V_B$

$V_A = 2q\ell - \frac{9}{4}q\ell \Rightarrow \boxed{V_A = -\frac{q\ell}{4}}$



$\delta \ll a$

Seção Transversal:

$A = (\delta a)\cdot 2 + (\delta\cdot 2a) \Rightarrow \boxed{A = 4\delta a}$

$\boxed{z_G = a}$ (simetria)

$$y_G = \frac{(\delta a)\cdot a/2 + (2\delta a)\cdot 0 + (\delta a)\cdot a/2}{4\delta a} \Rightarrow \boxed{y_G = \frac{a}{4}}$$

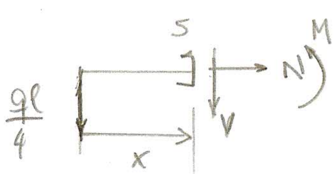
$$I_{z0} = 2\left[\frac{\delta a^3}{12} + (\delta a)\cdot\left(\frac{a}{4} - \frac{a}{2}\right)^2\right] + \left[\cancel{\frac{2a\delta^3}{12}} + (2\delta a)\cdot\left(0 - \frac{a}{4}\right)^2\right]$$

$$I_{z0} = \frac{2\delta a^3}{12} + 2\delta a\cdot\frac{a^2}{16} + 2\delta a\cdot\frac{a^2}{16} = \frac{\delta a^3}{6} + \frac{\delta a^3}{4}$$

$$I_{z0} = \left(\frac{2+3}{12}\right)\delta a^3 \Rightarrow \boxed{I_{z0} = \frac{5}{12}\delta a^3}$$

Diagramas

Trecho AB $(0 < x < \ell)$

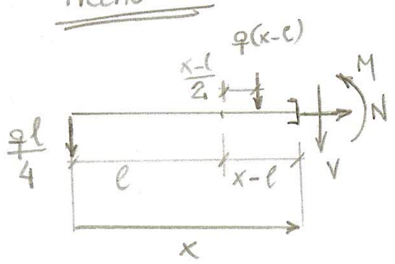


$\boxed{N = 0}$

$-\frac{q\ell}{4} - V = 0 \Rightarrow \boxed{V = -\frac{q\ell}{4}}$

$M + \frac{q\ell}{4}\cdot x = 0 \Rightarrow \boxed{M = -\frac{q\ell}{4}x}$

Trecho BC $(\ell < x < 2\ell)$



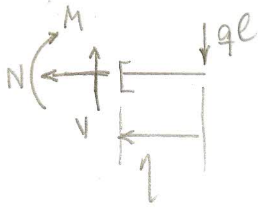
$\boxed{N = 0}$

$-\frac{q\ell}{4} - q(x-\ell) - V = 0 \Rightarrow V = -\frac{q\ell}{4} + q\ell - qx \Rightarrow \boxed{V = \frac{3}{4}q\ell - qx}$

$M + \frac{q\ell}{4}x + \frac{q}{2}(x-\ell)^2 = 0 \Rightarrow M = -\frac{q\ell}{4}x - \frac{q}{2}(x^2 - 2\ell x + \ell^2) = -\frac{q\ell^2}{2} + \frac{3}{4}q\ell x - \frac{q}{2}x^2$

$\boxed{M = -\frac{q}{2}\ell^2 + \frac{3}{4}q\ell x - \frac{q}{2}x^2}$

Trecho CD

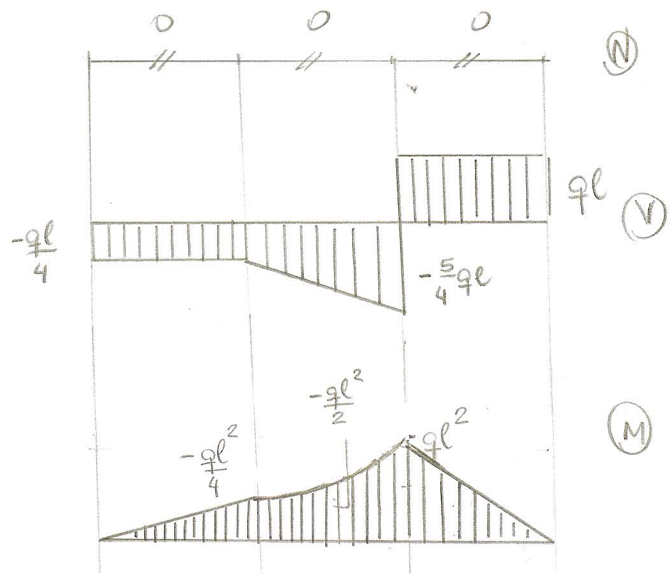


$$\boxed{N=0}$$

$$V-q\ell=0 \Rightarrow \boxed{V=q\ell}$$

$$-M-q\ell\eta=0 \Rightarrow \boxed{M=-q\ell\eta}$$

Diagramas

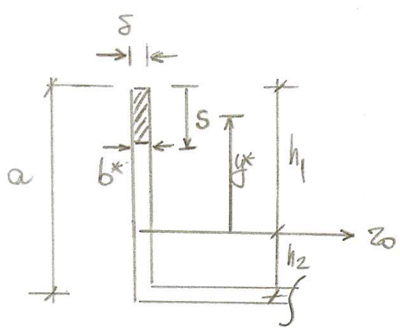


Máxima força cortante (em módulo): $V=-\frac{5}{4}q\ell$

Máximo momento fletor (em módulo) $M=-q\ell^2$

* Ambos em C.

Tensões de cisalhamento e fluxo de cisalhamento



$$S^*=b^*.s$$

$$b^*=\delta \Rightarrow S^*=\delta s$$

$$y^*=h_1-s/2$$

$$h_1=a-y_G=a-a/4=3/4\,a$$

$$M_s^*=S^*y^*=\delta s\left(\frac{3}{4}a-\frac{s}{2}\right)=\frac{\delta s}{4}(3a-2s)$$

$$\tau=\frac{VM_s^*}{b^*I_{z_0}}=\frac{\cancel{5}}{4}q\ell\left[\frac{\cancel{\delta}s}{4}(3a-2s)\right]\cdot\frac{1}{\cancel{\delta}}\cdot\frac{\cancel{12}^{3}}{\cancel{5}\delta a^3}=\frac{3}{4}\frac{q\ell}{\delta a^3}(3as-2s^2)$$

$$\tau(s=0)=0$$

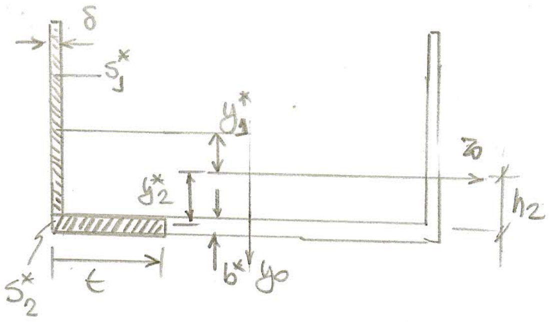
$$\tau(s=a)=\frac{3}{4}\frac{q\ell}{\delta a^3}.a^2=\frac{3}{4}\frac{q\ell}{\delta a}$$

$$\tau_{máx} \Rightarrow \frac{\partial\tau}{\partial s}=0 \Rightarrow 3a-4s=0 \Rightarrow s=3a/4$$

$$\tau(s=3a/4)=\tau_{máx}=\frac{3}{4}\frac{q\ell}{\delta a^3}\left(\frac{9a^2}{4}-\frac{2.9a^2}{16}\right)=\frac{3}{4}\frac{q\ell}{\delta a}\underbrace{\left(\frac{9}{4}-\frac{9}{8}\right)}_{9/8}$$

$$\boxed{\tau_{máx}=\frac{27}{32}\frac{q\ell}{\delta a}}$$

* As tensões no outro lado são idênticas. (simetria)



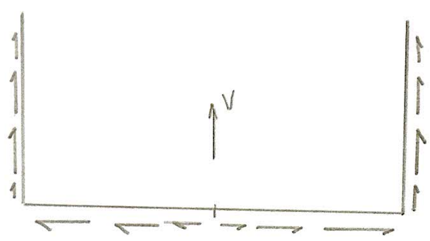
$b^* = \delta$ $\qquad S_2^* = b^* . t = \delta t$

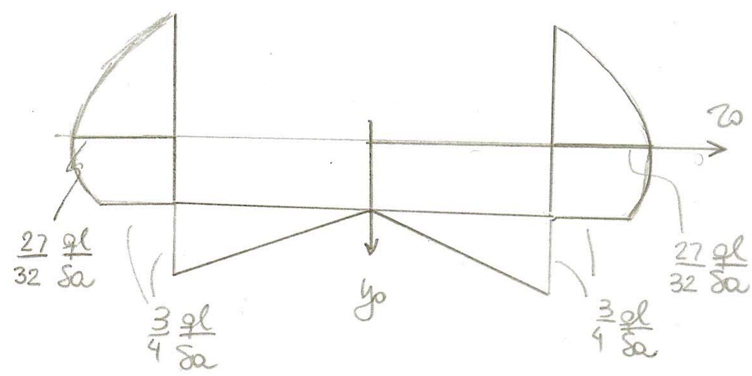
$S_1^* = \delta a$ $\qquad y_2^* = h_2 = -a/4 \ (y_G)$

$y_1^* = a/2 - a/4 = a/4$ $\qquad M_{S2}^* = S_2^* y_2^* = -\dfrac{\delta a t}{4}$

$M_{S1}^* = \dfrac{\delta a^2}{4}$ $\qquad M_S^* = M_{S1}^* + M_{S2}^* = \dfrac{\delta a^2}{4} + \dfrac{\delta a t}{4} = \dfrac{\delta a}{4}(a-t)$

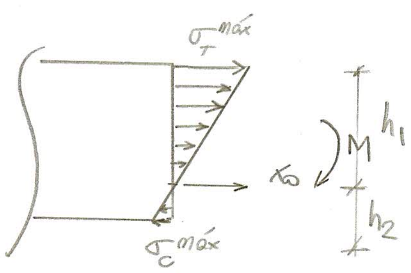
$$\tau = \frac{V . M_S^*}{b^* I_{z_0}} = \frac{\cancel{3}}{4} q l \cdot \left[\frac{\cancel{\delta} a (a-t)}{4}\right] \cdot \frac{1}{\cancel{\delta}} \cdot \frac{\cancel{12}^{3}}{\cancel{5}\delta a^3} = \frac{3}{4}\frac{ql}{\delta a^2}(a-t)$$





<u>Tensões Normais</u>

$$\sigma = \frac{My}{I_{z_0}}$$

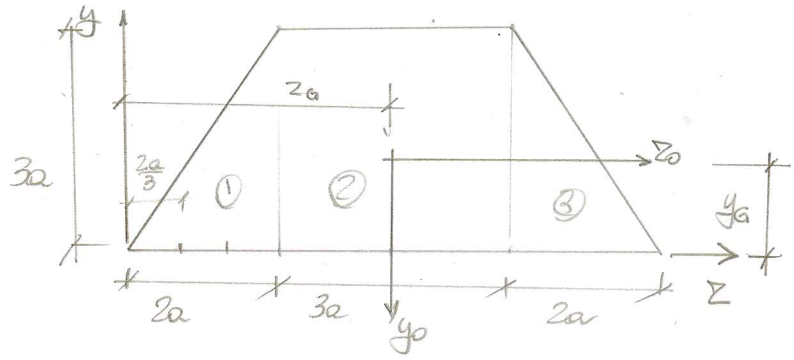


$$\sigma_T^{máx} = \frac{M.(-h_1)}{I_{z_0}} = \frac{-ql^2.(-3/4 a)}{5/12 \delta a^3} = \frac{3}{\cancel{4}} q l^2 a \cdot \frac{\cancel{12}^{3}}{5\delta a^3}$$

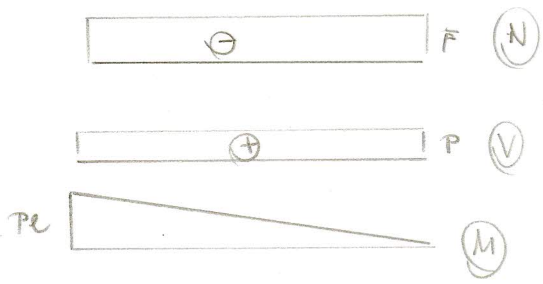
$$\boxed{\sigma_T^{máx} = \frac{9}{5}\frac{ql^2}{\delta a^2}}$$

$$\sigma_C^{máx} = \frac{M.h_2}{I_{z_0}} = \frac{-ql^2 . a/4}{5/12\,\delta a^3} = -\frac{1}{\cancel{4}_1} q l^2 a \cdot \frac{\cancel{12}^{3}}{5\delta a^3}$$

$$\boxed{\sigma_C^{máx} = -\frac{3}{5}\frac{ql^2}{\delta a^2}}$$



Reações de Apoio em A e diagramas V,M,N



$H_A = +F$

$V_A = P$

$M_A = Pl$

Propriedades da seção:

$$y_G = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$z_G = \frac{A_1 z_1 + A_2 z_2 + A_3 z_3}{A_1 + A_2 + A_3}$$

$A_1 = 3a^2$ $\quad$ $A_2 = 9a^2$ $\quad$ $A_3 = 3a^2$

$y_1 = a$ $\quad$ $y_2 = 3/2\, a$ $\quad$ $y_3 = a$

$z_1 = 4/3\, a$ $\quad$ $z_2 = 2a + 3/2\, a = 7/2\, a$ $\quad$ $z_3 = 5a + 2/3\, a = 17/3\, a$

$$y_G = \frac{3a^2 \cdot a + 9a^2 \cdot 3/2\, a + 3a^2 \cdot a}{3a^2 + 9a^2 + 3a^2} = \frac{3a^3 + 27/2\, a^3 + 3a^3}{15a^2} = \frac{1}{30a^2}\left(6a^3 + 27a^3 + 6a^3\right) = \frac{39}{30}a = \underline{\frac{13}{10}a}$$

$$z_G = \frac{3a^2 \cdot 4/3\, a + 9a^2 \cdot 7/2\, a + 3a^2 \cdot 17/3\, a}{3a^2 + 9a^2 + 3a^2} = \frac{1}{15a^2}\left(4a^3 + \frac{63}{2}a^3 + 17a^3\right) = \frac{1}{30a^2}\left(8a^3 + 63a^3 + 34a^3\right) = \frac{105}{30}a = \underline{\frac{7}{2}a}$$

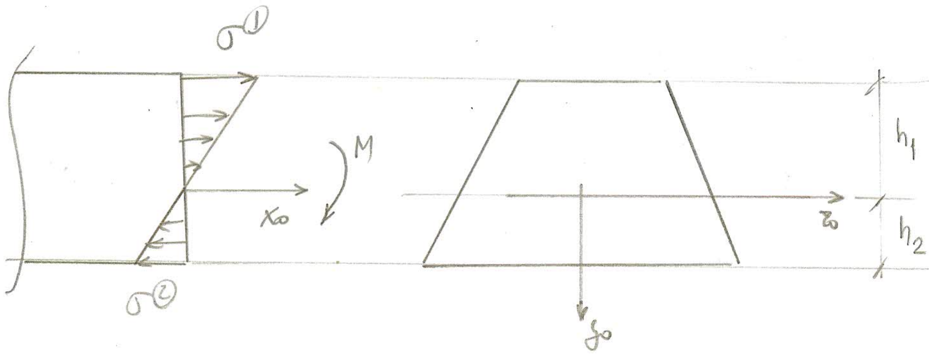
(obtido também por simetria)

Momento de inércia:

$$I_{z0} = I_{z1} + I_{z2} + I_{z3} = \left[\frac{2a(3a)^3}{36} + 3a^2\left(a - \frac{13}{10}a\right)^2\right] + \left[\frac{3a(3a)^3}{12} + 9a^2\left(\frac{3a}{2} - \frac{13}{10}a\right)^2\right] +$$

$$\left[\frac{2a(3a)^3}{36} + 3a^2\left(a - \frac{13}{10}a\right)^2\right] = 2\cdot\left[\frac{54}{36}a^4 + 3a^2 \cdot \frac{9a^2}{100}\right] + \left[\frac{81}{12}a^4 + 9a^2 \cdot \frac{4a^2}{100}\right] =$$

$$= \frac{2(5400 + 27\cdot 36) + 81\cdot 300 + 36\cdot 36}{3600}a^4 = \frac{12744 + 24300 + 1296}{3600}a^4 = \frac{38340}{3600}a^4 = \underline{\frac{213}{20}a^4}$$



$$h_1 = 3a - \frac{13}{10}a = \frac{17}{10}a$$

$$h_2 = \frac{13}{10}a$$

$$M = -P\ell \qquad \sigma = \frac{M}{I_{z_0}}y$$

$$\sigma^N = N/A$$

Nesse caso $\sigma^N = -F/A$.

$$A = 15a^2$$

$$\sigma^{①} = \frac{(-P\ell)\cdot(-h_1)}{I_{z_0}} = P\ell \cdot \frac{17}{\cancel{10}_{1}}a \cdot \frac{\cancel{20}^{2}}{213a^4} = \frac{34}{213}\frac{P\ell}{a^3}$$

$$\sigma^{②} = \frac{(-P\ell)\cdot h_2}{I_{z_0}} = -P\ell \cdot \frac{13}{\cancel{10}_{1}}a \cdot \frac{\cancel{20}^{2}}{213a^4} = -\frac{26}{213}\frac{P\ell}{a^3}$$

Logo, as tensões máximas são:

$$\sigma_T^{m\acute{a}x} = \frac{-F}{15a^2} + \frac{34}{213}\frac{P\ell}{a^3}$$

$$\sigma_C^{m\acute{a}x} = -\frac{F}{15a^2} - \frac{26}{213}\frac{P\ell}{a^3}$$

Relação P/F para $\sigma_T^{m\acute{a}x} = |\sigma_C^{m\acute{a}x}|$:

$$\frac{-F}{15a^2} + \frac{34}{213}\frac{P\ell}{a^3} = \frac{F}{15a^2} + \frac{26}{213}\frac{P\ell}{a^3}$$

$$\left(\frac{34-26}{213}\right)\frac{P\ell}{a^{\cancel{3}}} = \frac{2}{15}\frac{F}{\cancel{a^2}}$$

$$\frac{8}{213}\frac{P\ell}{a} = \frac{2}{15}F \Rightarrow \frac{P}{F} = \frac{\cancel{213}^{71}}{\cancel{8}_{4}} \cdot \frac{\cancel{2}^{1}}{\cancel{15}_{5}}\frac{a}{\ell} \Rightarrow \boxed{\frac{P}{F} = \frac{71}{20}\frac{a}{\ell}}$$

Supondo: $\ell = 1{,}42\,m = 142\,cm$ e $a = 8\,cm$:

$$\frac{P}{F} = \frac{\cancel{71}^{1}}{\cancel{20}_{5}} \cdot \frac{\cancel{8}^{2}}{\cancel{142}_{2}} = \frac{1}{5} \Rightarrow \boxed{P = 5F}$$

$$\begin{cases} A_1 = 192\,cm^2 & y_1 = 8\,cm & z_1 = 32/3\,cm \\ A_2 = 576\,cm^2 & y_2 = 12\,cm & z_2 = 28\,cm \\ A_3 = 192\,cm^2 & y_3 = 8\,cm & z_3 = 136/3\,cm \end{cases} \qquad \begin{cases} y_G = 10{,}4\,cm \\ z_G = 28\,cm \end{cases} \qquad \{ I_{z_0} = 43622{,}4\,cm^4$$