PEF – 3202 – Introdução à Mecânica dos Sólidos (20/05/2015)

Nome: ____ **nUSP:** ______________

Questão 1 (5,0)

Para a estrutura da figura 1:

(a) Encontre as reações de apoio e trace os diagramas de força normal, força cortante, momento fletor e momento torçor. Desenhe os diagramas na próxima página. (4,0)

(b) Dimensione a barra AB à torção. Determine o raio mínimo (considerando que AB é uma barra circular), com coeficiente de segurança 2, tensão de cisalhamento de ruptura $\tau_R = 200\ MPa$, ângulo máximo em B $\theta_{max}(B) = 0{,}05\ rad$ e módulo transversal de elasticidade $G = 70\ GPa$. (1,0).

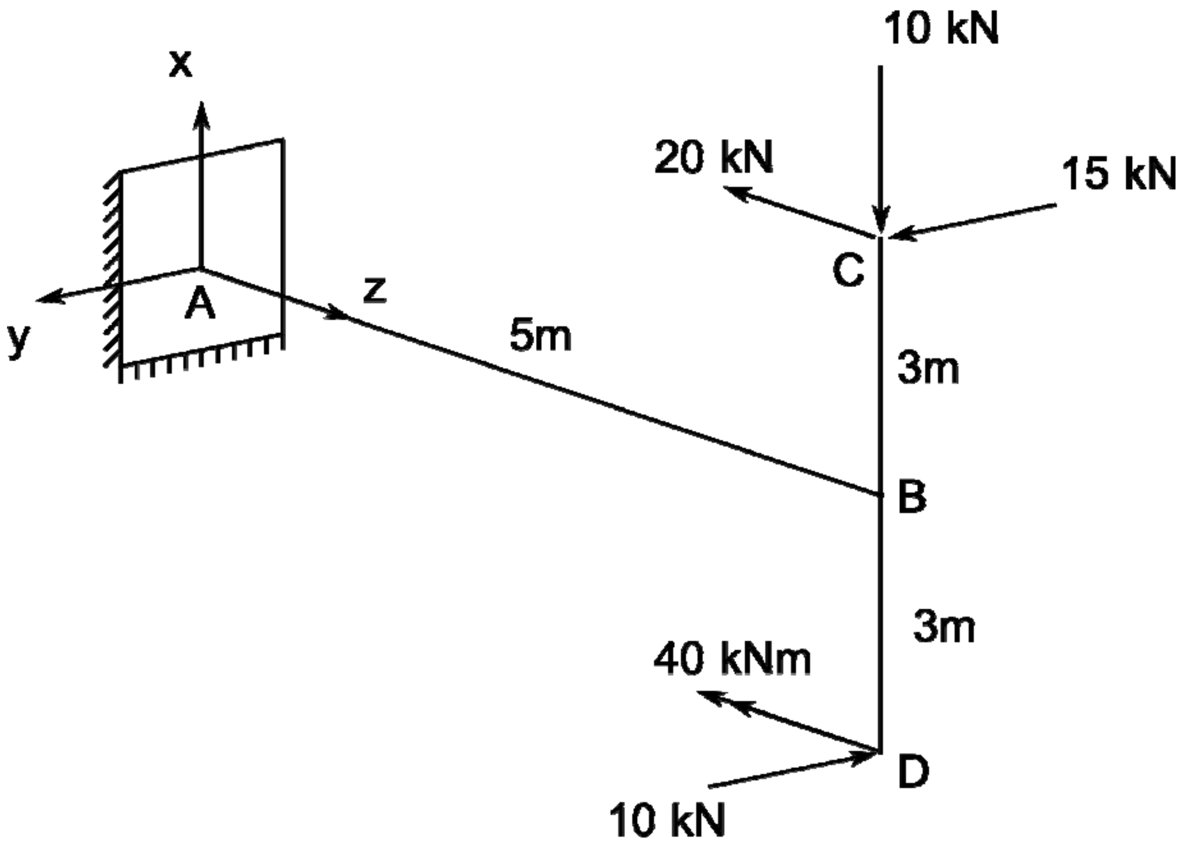

Figura 1

ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

Departamento de Engenharia de Estruturas e Geotécnica

PEF – 3202 – Introdução à Mecânica dos Sólidos (20/05/2015)

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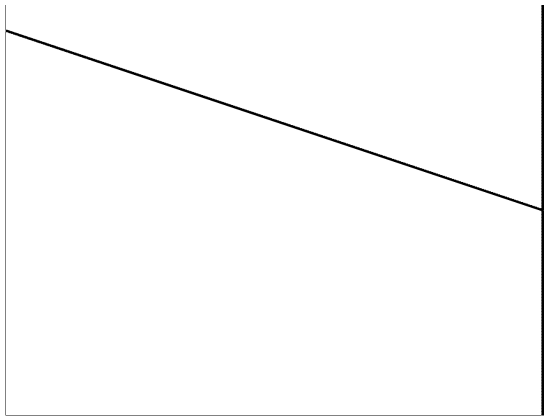
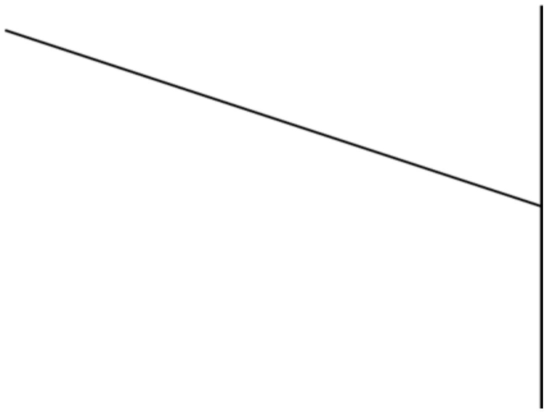
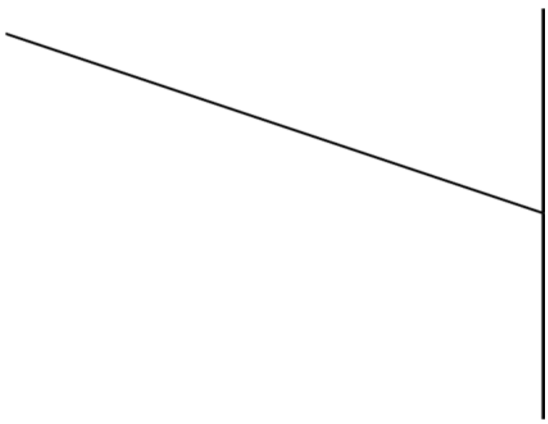


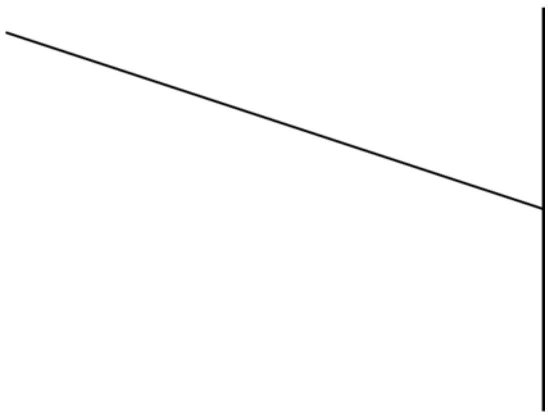
N[kN]



V[kN]



Mf[kN]



T[kN]

PEF – 3202 – Introdução à Mecânica dos Sólidos (20/05/2015)

Nome: ______________________________ **nUSP:** ______________

Questão 2 (2,0)

Calcule as forças nas treliças da figura 2. Calcule o raio mínimo, considerando que as barras são circulares, para que $\bar{\sigma} = 50\ MPa$. Considere que $\cos\alpha = 0{,}6$ e $\operatorname{sen}\alpha = 0{,}8$.

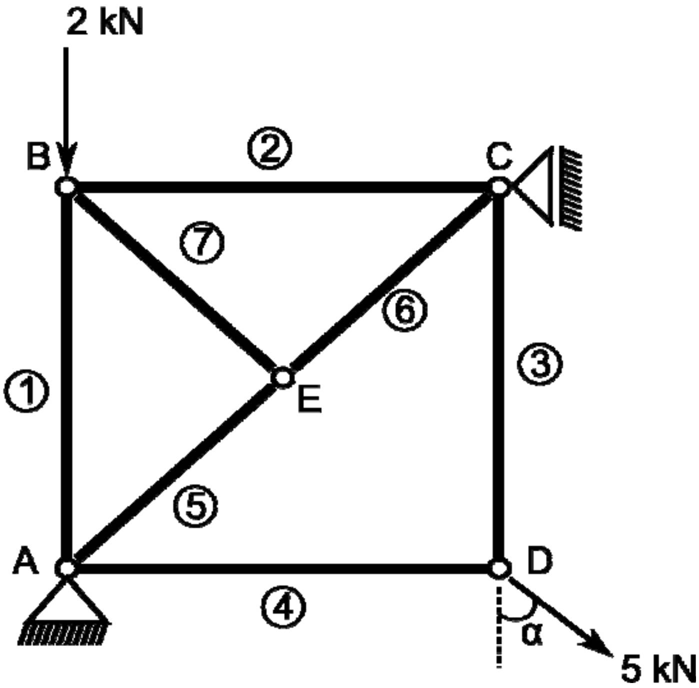


Figura 2

N_1	
N_2	
N_3	
N_4	
N_5	
N_6	
N_7	

PEF – 3202 – Introdução à Mecânica dos Sólidos (20/05/2015)

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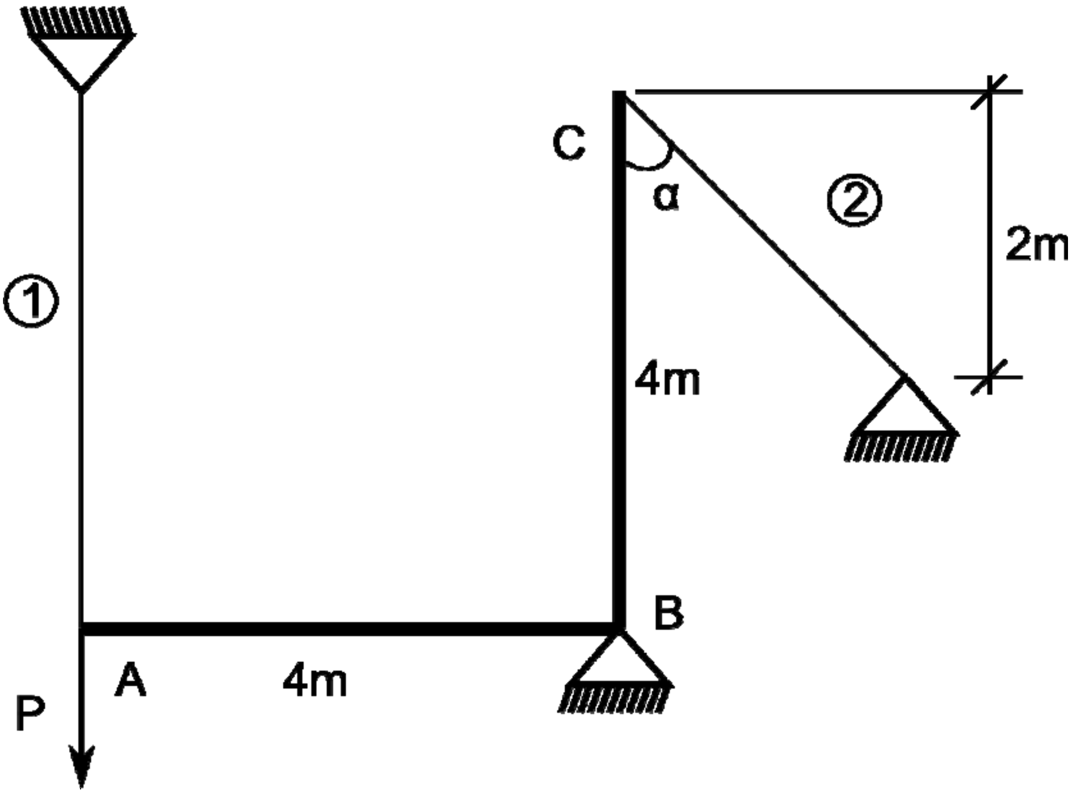
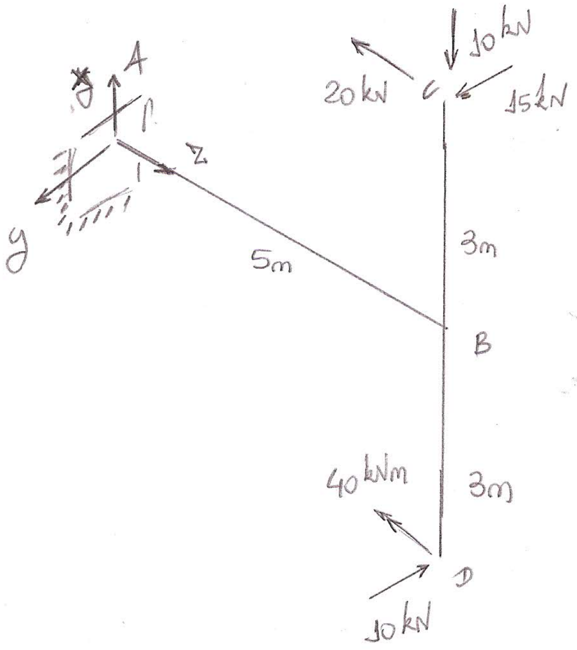
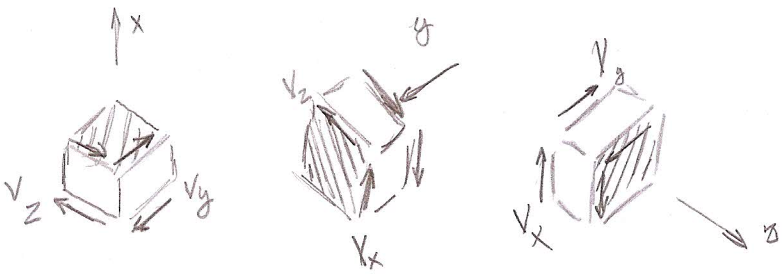
Questão 3 (3,0)

Dimensione as barras 1 e 2 da figura 3 considerando que a barra ABC é rígida, $\bar{\sigma} = 150\ MPa$, $E = 210\ GPa$ e $P = 4\ kN$. Considere que $\alpha = 45^o$.

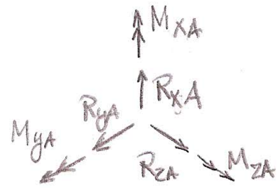


Q1 (5,0)





Reações do apoio em A



$\sum F_x = 0 \Rightarrow R_{XA} - 10 = 0 \Rightarrow \boxed{R_{XA} = 10\,kN}$

$\sum F_y = 0 \Rightarrow R_{YA} + 15 - 10 = 0 \Rightarrow \boxed{R_{YA} = -5\,kN}$

$\sum F_z = 0 \Rightarrow R_{ZA} - 20 = 0 \Rightarrow \boxed{R_{ZA} = 20\,kN}$

$\sum M_{A,x} = 0 \Rightarrow M_{XA} + 10.5 - 15.5 = 0 \Rightarrow M_{XA} + 50 - 75 = 0 \Rightarrow \boxed{M_{XA} = 25\,kNm}$

$\sum M_{A,y} = 0 \Rightarrow M_{YA} - 10.5 + 20.3 = 0 \Rightarrow M_{YA} - 50 + 60 = 0 \Rightarrow \boxed{M_{YA} = -10\,kNm}$

$\sum M_{A,z} = 0 \Rightarrow M_{ZA} - 40 + 15.3 + 10.3 = 0 \Rightarrow M_{ZA} - 40 + 45 + 30 = 0 \Rightarrow \boxed{M_{ZA} = -35\,kNm}$

método alternativo (momentos):

$\vec{M} = \sum \vec{r}_i \wedge \vec{F}_i + \sum \vec{M}_i$

$\vec{F}_1 = -20\vec{k}, \quad \vec{r}_1 = 3\vec{i} + 5\vec{k}$

$\vec{F}_2 = -10.\vec{i}, \quad \vec{r}_2 = \vec{r}_1$

$\vec{F}_3 = 15\vec{j}, \quad \vec{r}_3 = \vec{r}_1$

$\vec{F}_4 = -10\vec{j}, \quad \vec{r}_4 = -3\vec{i} + 5\vec{k}$

$\vec{M}_1 = -40\vec{k}$

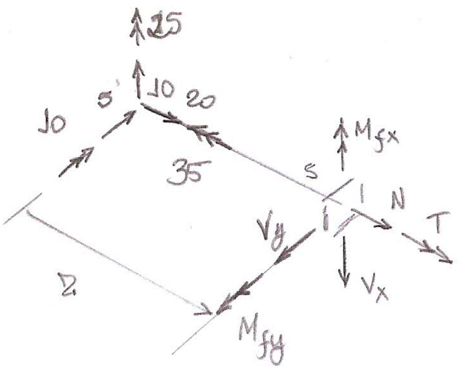
$\vec{M} = -40\vec{k} + (3\vec{i} + 5\vec{k}) \wedge (-10\vec{i} + 15\vec{j} - 20\vec{k}) +$
$+ (-3\vec{i} + 5\vec{k}) \wedge (-10\vec{j})$

$\vec{M} = -40\vec{k} + (45\vec{k} + 60\vec{j} - 50\vec{j} - 75\vec{i})$
$+ (30\vec{k} + 50\vec{i})$

$\boxed{\vec{M} = -25\vec{i} + 10\vec{j} + 35\vec{k}}$

$$\begin{cases} M_{XA} - 15 = 0 \\ M_{YA} + 10 = 0 \\ M_{ZA} + 35 = 0 \end{cases}$$

Barra AB:



$\sum F_x = 0: \; 10 - V_x = 0 \Rightarrow \boxed{V_x = 10\,kN}$

$\sum F_y = 0: \; -5 + V_y = 0 \Rightarrow \boxed{V_y = 5\,kN}$

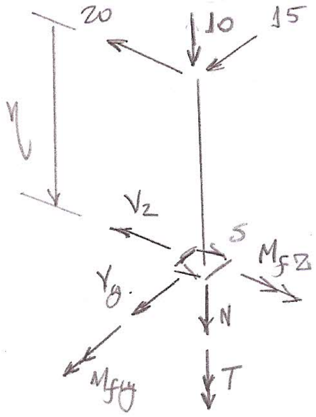
$\sum F_z = 0: \; 20 + N = 0 \Rightarrow \boxed{N = -20\,kN}$

$\sum M_{S,x} = 0: \; M_{fx} - 5z + 25 = 0 \Rightarrow \boxed{M_{fx} = 5z - 25}$ (⊕ na frente)

$\sum M_{S,y} = 0: \; M_{fy} - 10 - 10z = 0 \Rightarrow \boxed{M_{fy} = 10 + 10z}$ (⊕ embaixo)

$\sum M_{S,z} = 0: \; -35 + T = 0 \Rightarrow \boxed{T = 35\,kNm}$

Barra BC:



$\sum F_x = 0: \; -10 - N = 0 \Rightarrow \boxed{N = -10\,kN}$

$\sum F_y = 0: \; 15 + V_y = 0 \Rightarrow \boxed{V_y = -15\,kN}$

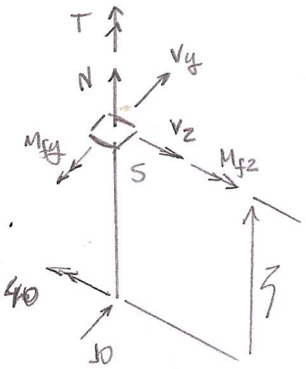
$\sum F_z = 0: \; -V_z - 20 = 0 \Rightarrow \boxed{V_z = -20\,kN}$

$\sum M_{S,x} = 0: \; \boxed{T = 0}$

$\sum M_{S,y} = 0: \; M_{fy} + 20\eta = 0 \Rightarrow \boxed{M_{fy} = -20\eta}$ (⊕ esquerda)

$\sum M_{S,z} = 0: \; M_{fz} + 15\eta = 0 \Rightarrow \boxed{M_{fz} = -15\eta}$ (⊕ na frente)

Barra BD:



$\sum F_x = 0: \; \boxed{N = 0}$

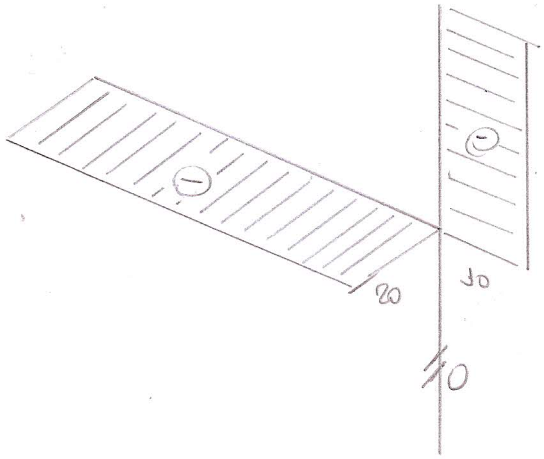
$\sum F_y = 0: \; -V_y - 10 = 0 \Rightarrow \boxed{V_y = -10\,kN}$

$\sum F_z = 0: \; \boxed{V_z = 0}$

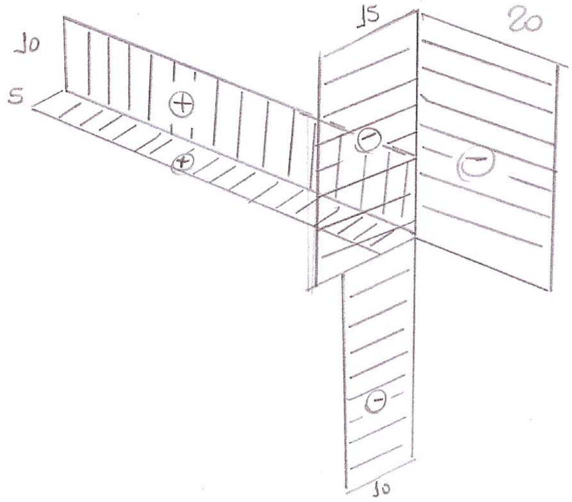
$\sum M_{S,x} = 0: \; \boxed{T = 0}$

$\sum M_{S,y} = 0: \; \boxed{M_{fy} = 0}$

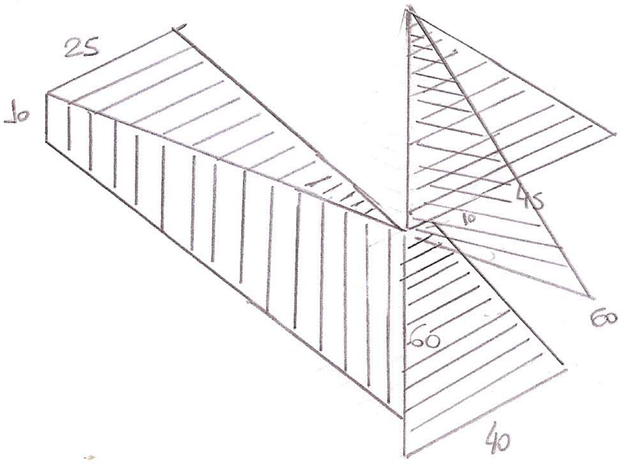
$\sum M_{S,z} = 0: \; M_{fz} - 40 + 10\zeta = 0 \Rightarrow \boxed{M_{fz} = 40 - 10\zeta}$ (⊕ atrás)



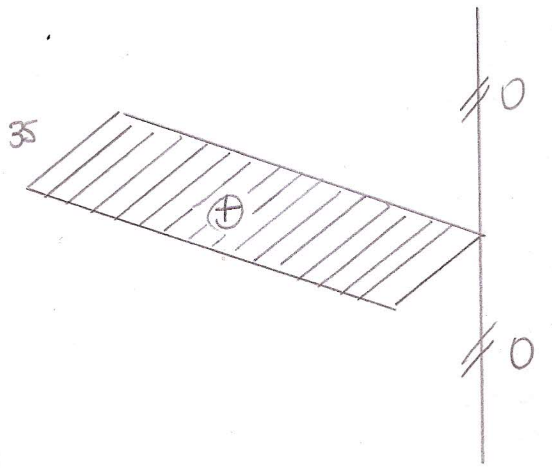
N[kN]



Y[kN]

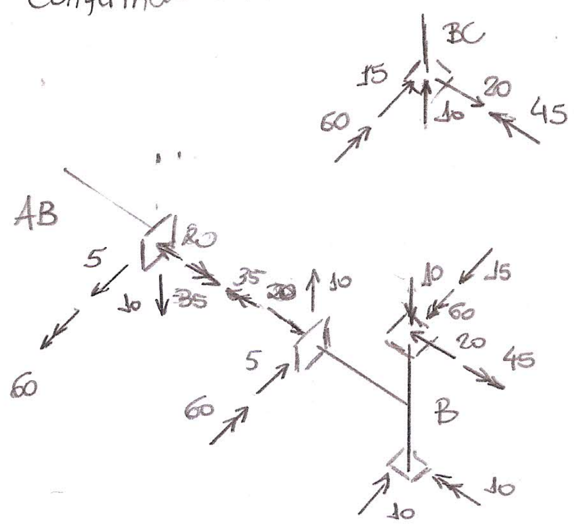


M[kNm]

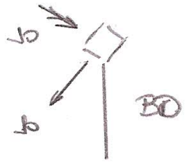


T[kNm]

Conferindo em B os esforços:



OK!



Dimensionar a barra AB à torção. (qual o raio mínimo para $s=2$; $\tau_R = 200\,MPa$; $\theta_{máx} = 0{,}05\,rad$; considerar $G = 70\,GPa$).

- tensão máxima de cisalhamento (admissível)

$$\bar{\tau} = \frac{\tau_R}{s} = 100\,MPa$$

$\tau = \frac{T}{J} r$. Logo $\tau_{máx} \leq \bar{\tau}$ $\quad \tau_{máx} = \frac{T}{J} \cdot R_{min}$, $J = \frac{\pi R_{min}^4}{2}$

$$\tau_{máx} = \frac{2T}{\pi R_{min}^3} \leq \bar{\tau} \Rightarrow \frac{\pi R_{min}^3}{2T} \geqslant \frac{1}{\bar{\tau}} \Rightarrow R_{min} \geqslant \sqrt[3]{\frac{2T}{\pi \bar{\tau}}} = \sqrt[3]{\frac{2 \cdot 35 \cdot 10^3}{\pi \cdot 100 \cdot 10^6}}$$

$$\boxed{R_{min} \geqslant 0{,}061\,m}$$

Ângulo máximo em B:

$\theta_{máx}(B) = 0{,}05$ rad

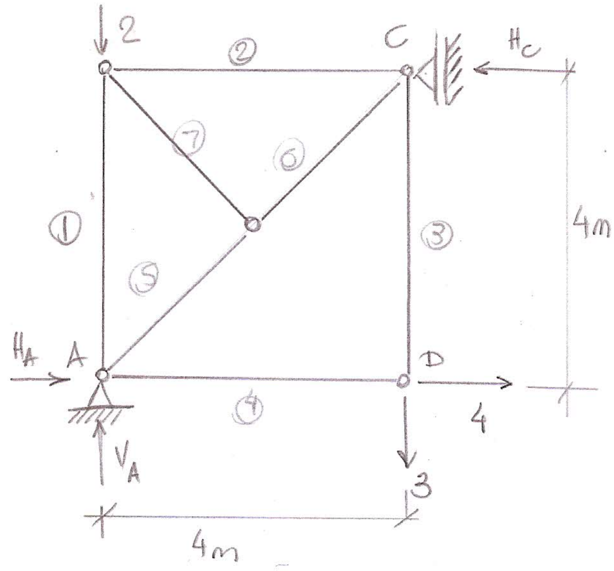
$$\theta \leq \theta_{máx} \qquad \theta = \frac{Tl}{GJ} \leq \theta_{máx} \quad \frac{2Tl}{G\pi R_{min}^4} < \theta_{máx}$$

$$\frac{R_{min}^4 \pi G}{2Tl} \geqslant \frac{1}{\theta_{máx}} \Rightarrow R_{min}^4 \geqslant \frac{2Tl}{\pi G \theta_{máx}} \Rightarrow R_{min} \geqslant \sqrt[4]{\frac{2Tl}{\pi G \theta_{máx}}}$$

$$R_{min} \geqslant \sqrt[4]{\frac{2 \cdot 35 \cdot 10^3 \cdot 5}{\pi \, 70 \cdot 10^9 \, 0{,}05}} \Rightarrow \boxed{R_{min} \geqslant 0{,}075m}$$

Logo $\boxed{R_{min} = 0{,}075m}$

Q2 (2,0)



Equilíbrio:

$\Sigma F_x = 0: \ H_A - H_C + 4 = 0$

$\Sigma F_V = 0: \ V_A - 2 - 3 = 0 \Rightarrow \boxed{V_A = 5 kN}$

$\Sigma M_A = 0: \ -3.4 + H_C . 4 = 0$

$\boxed{H_C = 3 kN}$

Logo $H_A = H_C - 4 \Rightarrow \boxed{H_A = -1 kN}$

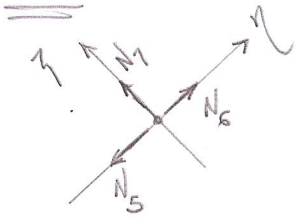
H_A	-1
V_A	5
H_C	3
N_1	-2
N_2	0
N_3	3
N_4	4
N_5	$-3\sqrt{2}$
N_6	$-3\sqrt{2}$
N_7	0

$b = 7, \ n = 5$

$2n = b + 3 \rightarrow$ treliça isostática!

Equilíbrio nos nós:

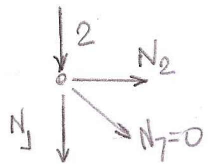
Nó E



$\Sigma F_\eta = 0 \Rightarrow \boxed{N_5 = N_6}$

$\Sigma F_\xi = 0 \Rightarrow \boxed{N_7 = 0}$

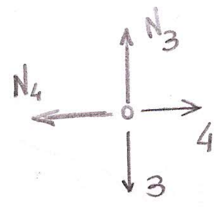
Nó B



$\Sigma F_H = 0 \Rightarrow \boxed{N_2 = 0}$

$\Sigma F_V = 0 \Rightarrow \boxed{N_1 = -2 kN}$

Nó D

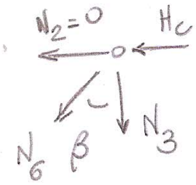


$\Sigma F_H = 0 \Rightarrow \boxed{N_4 = 4 kN}$

$\Sigma F_V = 0 \Rightarrow \boxed{N_3 = 3 kN}$

Nó C

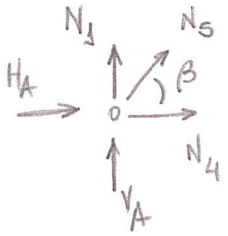
$\beta = 45°$



$\Sigma F_H = 0: -N_6 \cdot \sqrt{2}/2 - H_C = 0 \Rightarrow N_6 = -H_C\sqrt{2} \Rightarrow \boxed{N_6 = -3\sqrt{2}\ kN}$

$\Sigma F_V = 0: -N_3 - N_6\sqrt{2}/2 = 0 \Rightarrow -3 - (-3\sqrt{2})\sqrt{2}/2 = 0$ <u>ok</u>

Nó A



$\Sigma F_H = 0: \ H_A + N_4 + N_5\sqrt{2}/2 = 0 \Rightarrow -1 + 4 + (-3\sqrt{2})\sqrt{2}/2 = 0$ <u>ok</u>

$\Sigma F_V = 0: \ V_A + N_1 + N_5\sqrt{2}/2 = 0 \Rightarrow 5 - 2 + (-3\sqrt{2})\sqrt{2}/2 = 0$ <u>ok</u>

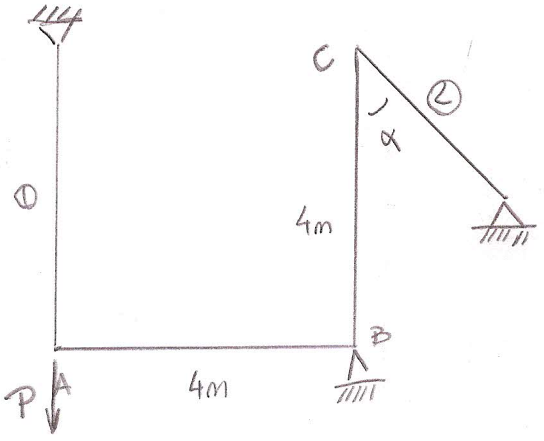
Dimensionar a treliça para que $\bar{\sigma} = 50$ MPa.

$\sigma_{máx} \leq \bar{\sigma}$ $\qquad \sigma_{máx} = |\sigma_5|, |\sigma_6| = \dfrac{|N_5|}{A} = \dfrac{|N_6|}{\pi R_{min}^2}$

$$\frac{|N_5|}{\pi R_{min}^2} \leq \bar{\sigma} \Rightarrow \frac{\pi R_{min}^2}{|N_5|} \geq \frac{1}{\bar{\sigma}} \Rightarrow R_{min} \geq \sqrt{\frac{|N_5|}{\pi \bar{\sigma}}}$$

$$R_{min} \geq \sqrt{\frac{3\sqrt{2} \cdot 10^3}{50 \cdot 10^6 \cdot \pi}} \Rightarrow R_{min} \geq 0{,}0052\, m \Rightarrow \boxed{R_{min} \geq 5{,}2\, mm}$$

Q3 (3,0)



$\alpha = 45°$

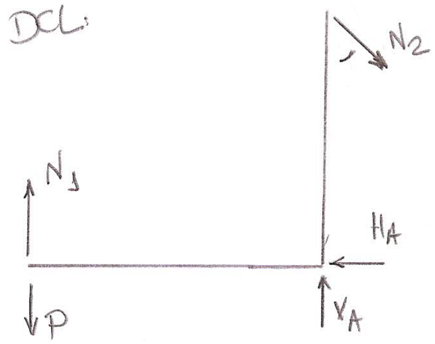
Barra ABC rígida

Dimensionar as barras ① e ②

para $\bar{\sigma} = 150 MPa$, $E = 210 GPa$.

$P = 4kN$.

DCL:



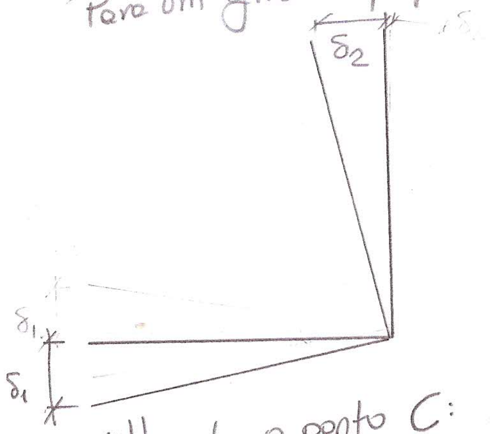
$\Sigma F_H = 0: \quad H_A = N_2 \frac{\sqrt{2}}{2}$

$\Sigma F_V = 0: \quad N_1 + V_A = N_2 \frac{\sqrt{2}}{2} + P$

$\Sigma M_A = 0: \quad -\cancel{4} . N_1 - \cancel{4} . \frac{\sqrt{2}}{2} N_2 + 4P = 0$

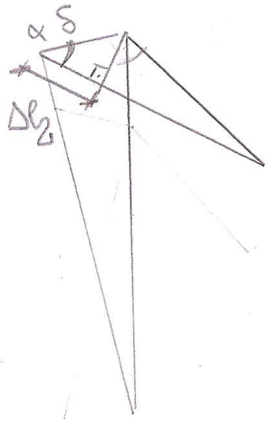
$$\boxed{N_1 = -\frac{\sqrt{2}}{2} N_2 + P}$$

Para um giro θ pequeno em A:



$\delta_1 = \delta_2 = \delta$

Olhando o ponto C:



$\frac{\Delta \ell_2}{\delta} = \cos\alpha \Rightarrow \Delta \ell_2 = \delta \cos\alpha = \underline{\frac{\sqrt{2}}{2}\delta}$

$\underline{\Delta \ell_1 = \delta}$

As deformações são:

$$\varepsilon_1 = \frac{\Delta \ell_1}{\ell_1} = \frac{\delta}{4}$$

$$\varepsilon_2 = \frac{\Delta \ell_2}{\ell_2} = \frac{\sqrt{2}/2 . \delta}{2\sqrt{2}} = \frac{\sqrt{2}\,\delta}{4\sqrt{2}} = \frac{\delta}{4}$$

Lembrando que $N = EA\varepsilon$:

$$\left.\begin{array}{l} N_1 = EA\varepsilon_1 \Rightarrow N_1 = \dfrac{EA\delta}{4} \\ N_2 = EA\varepsilon_2 = N_2 = \dfrac{EA\delta}{4} \end{array}\right\} \Rightarrow \boxed{N_1 = N_2}$$

Assim:

$$N_1 + \frac{\sqrt{2}}{2} N_2 = P \Rightarrow \left(\frac{2+\sqrt{2}}{2}\right) N_1 = P \Rightarrow$$

$$N_1 = N_2 = \frac{2P}{2+\sqrt{2}} = \boxed{2{,}34\ kN}$$

$$\boxed{H_A = 3{,}31\ kN} \qquad V_A = N_2\frac{\sqrt{2}}{2} - N_1 = \left(\frac{\sqrt{2}-2}{\cancel{2}}\right) \cdot \frac{\cancel{2}P}{(\sqrt{2}+2)} = \left(\frac{\sqrt{2}-2}{\sqrt{2}+2}\right)P$$

$$\boxed{V_A = -0{,}69\ kN} \qquad V_A = \frac{(\sqrt{2}-2)^2}{2-4}P = -\frac{P}{2}(2+4-4\sqrt{2}) = -P(3-2\sqrt{2})$$

Como $N_1 = N_2$, então:

$$\sigma_{m\acute{a}x} = \sigma_1 = \sigma_2 < \bar{\sigma} \qquad \frac{N_1}{A} \leq \bar{\sigma}$$

$$\frac{A}{N_1} \geq \frac{1}{\bar{\sigma}} \qquad \frac{\pi R_{min}^2}{N_1} \geq \frac{1}{\bar{\sigma}} \Rightarrow R_{min} \geq \sqrt{\frac{N_1}{\pi\bar{\sigma}}} = \sqrt{\frac{2{,}34 . 10^3}{\pi . 150 . 10^6}}$$

$$\boxed{R_{min} \geq 0{,}0022m} \quad \text{ou} \quad \boxed{R_{min} \geq 2{,}2mm}$$