MAT2456 - Cálculo Diferencial e Integral para Engenharia IV
Escola Politécnica - Prova Substitutiva - 09/12/2013

Turma A

Nome : GABARITO

NºUSP : ______

Professor(a) : ______ Turma : ______

Q	N
1	
2	
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Total	

JUSTIFIQUE TODAS AS SUAS AFIRMAÇÕES

A

1ª Questão: (2,5) Determine a solução geral da equação diferencial

$$4y'' - 4y' + y = x^2 + x + \sqrt{e^x}\ln x.$$

EQ. Homogênea.

$P(\lambda) = 4\lambda^2 - 4\lambda + 1 = 0$

$\lambda = \frac{4 \pm \sqrt{16-16}}{8} = \frac{1}{2}\ (m=2)$

$y_H = C_1 e^{x/2} + C_2 x e^{x/2}$

SOLUÇÕES PARTICULARES

a) $b(x) = x^2$

$y_p(x) = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$\Rightarrow 8A - 8Ax - 4B + Ax^2 + Bx + C = x^2$

$Ax^2 + (B - 8A)x + (8A - 4B + C) = x^2$

$A = 1 \quad B = 8 \quad 8 - 32 + C = 0$

$C = 24$

$y_{p_1} = x^2 + 8x + 24$

b) $b(x) = x$

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$\Rightarrow -4A + Ax + B = x$

$A = 1$

$B = 4$

$y_{p_2} = x + 4$

c) $b(x) = \sqrt{e^x}\ \ln x$

$$\begin{pmatrix} e^{x/2} & xe^{x/2} \\ \frac{e^{x/2}}{2} & e^{x/2} + \frac{xe^{x/2}}{2} \end{pmatrix} \begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\sqrt{e^x}\ \ln x}{4} \end{pmatrix}$$

$$W = \begin{vmatrix} e^{x/2} & x e^{x/2} \\ \frac{e^{x/2}}{2} & e^{x/2} + \frac{x e^{x/2}}{2} \end{vmatrix} = e^{x} + \frac{x e^{x}}{2} - \frac{x e^{x}}{2} = e^{x}$$

$$\Delta_1 = \begin{vmatrix} 0 & x e^{x/2} \\ \frac{\sqrt{e^x}\ln x}{4} & e^{x/2} + \frac{x e^{x/2}}{2} \end{vmatrix} = -\frac{x e^{x} \ln x}{4}$$

$$\Delta_2 = \begin{vmatrix} e^{x/2} & 0 \\ \frac{e^{x/2}}{2} & \frac{\sqrt{e^x}\ln x}{4} \end{vmatrix} = \frac{e^{x}\ln x}{4}$$

$$C_1' = \frac{\Delta 1}{W} = -\frac{x \ln x}{4} \Rightarrow C_1 = -\int \frac{x \ln x}{4}\, dx = -\frac{1}{4}\int \overbrace{x}^{dv} \underbrace{\ln x}_{u}\, dx$$

$$= -\frac{1}{4}\left[\frac{x^2}{2}\ln x - \int x\, dx\right]$$

$$= -\frac{1}{4}\left[\frac{x^2}{2}\ln x - \frac{x^2}{2}\right] = \frac{x^2}{8}(1 - \ln x)$$

$$C_2' = \frac{\Delta_2}{W} = \frac{\ln x}{4} \Rightarrow C_2 = \frac{1}{4}\int \underbrace{\ln x}_{u} \underbrace{dx}_{dv} = x\ln x - \int dx$$

$$= x(\ln x - 1)$$

$$y_{p_3} = \frac{x^2}{8}(1 - \ln x)\, e^{x/2} + x^2 e^{x/2}(\ln x - 1)$$

$$y_G = C_1 e^{x/2} + C_2 x e^{x/2} + x^2 + 8x + 24 + x + 4$$
$$+ \frac{x^2}{8}(1 - \ln x)e^{x/2} + x^2 e^{x/2}(\ln x - 1)$$

A

2ª Questão: Decida se cada série a seguir converge absolutamente, converge condicionalmente ou diverge. Justifique suas respostas

a) (1,0) $\sum_{n=1}^{\infty} \text{sen}\left(\frac{1}{n^3+n}\right)$ b) (1,5) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln n}}$

a) $\frac{1}{n^3+n} < \frac{1}{n^3} \rightarrow \sum \frac{1}{n^3+n}$ converge por comparação por $\sum \frac{1}{n^3}$ converge

comparação no limite:

$$\lim_{n\to\infty} \frac{\text{sen}\left(1/n^3+n\right)}{1/n^3+n} = 1 \Rightarrow \sum_{n=1}^{\infty} \text{sen}\left(\frac{1}{n^3+n}\right)$$ converge absolutamente pelo critério da Comparação no limite

b) $\lim_{n\to\infty} \frac{1}{n \ln n} = 0 \rightarrow$ a série converge pelo critério de Leibnitz

b) $\left|\sum a_n\right| = \sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

Critério da Integral:

$$\int_0^{\infty} \frac{1}{x(\ln x)^{1/2}}\,dx = \lim_{M\to\infty} \int_{\ln 2}^{\ln M} u^{-1/2}\,du = \lim_{M\to\infty} 2u^{1/2}\Big|_{\ln 2}^{\ln M}$$

$u = \ln x$

$du = dx/x$

$= \lim_{M\to\infty} 2\left((\ln M)^{1/2} - (\ln 2)^{1/2}\right) = \text{"}\infty\text{"}$

a série diverge em modulo

$\rightarrow \sum a_n$ converge condicionalmente

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3ª Questão: (2,5) Determine o intervalo de convergência da série de potências

$$\sum_{n=1}^{\infty} \frac{x^n}{\sqrt[3]{2^n n^2}}.$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x|^{n+1}}{\sqrt[3]{2^{n+1}(n+1)^2}} \cdot \frac{\sqrt[3]{2^n n^2}}{|x|^n}$$

$$= \lim_{n\to\infty} |x| \cdot \sqrt[3]{\frac{1}{2}} \cdot \cancel{\sqrt[3]{\left(\frac{n}{n+1}\right)^2}} \rightarrow 1$$

$$= |x| \cdot \frac{1}{2^{1/3}} < 1$$

$$|x| < 2^{1/3} \longrightarrow -2^{1/3} < x < 2^{1/3}$$

Teste dos extremos:

$x = -2^{1/3}$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cancel{(2^{1/3})^n}}{\cancel{2^{n/3}} \cdot n^{2/3}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}} \Rightarrow \text{Converge por Leibniz}$$

$x = 2^{1/3}$

$$\sum_{n=1}^{\infty} \frac{1}{n^{2/3}} \rightarrow \text{Diverge} \cdot \sum \frac{1}{n^p}, \; p < 1$$

$\therefore$ a série converge em: $\left[-\sqrt[3]{2}, \sqrt[3]{2}\right[$ //

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4ª Questão:

a) (1,0) Obtenha a série de Fourier de cossenos da função $f(x) = \begin{cases} 1, & 0 \le x \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le x \le \pi \end{cases}$.

b) (1,0) Esboce o gráfico da soma da série $S(x)$ do item (a) para $-3\pi \le x \le 3\pi$. Justifique.

c) (0,5) Calcule $S(12)$ e $S(37\pi)$. Justifique a resposta.

a) $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi/2} dx = \frac{2}{\pi}\cdot\frac{\pi}{2} = 1$

$a_n = \frac{2}{\pi}\int_0^{\pi/2} \cos(nx)\,dx = -\frac{2}{\pi}\,\frac{\operatorname{sen}(nx)}{n}\Big|_0^{\pi/2}$

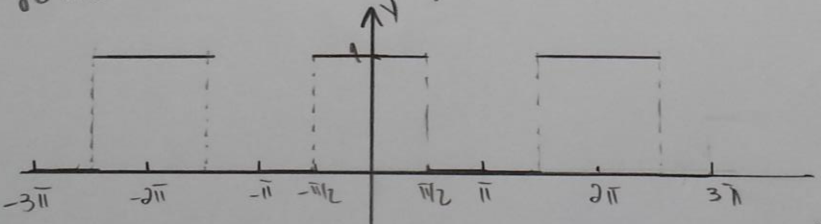
$= -\frac{2}{n\pi}\operatorname{sen}(n\pi/2)$

$n = 2m \rightarrow a_m = 0$

$n = 2m+1 \rightarrow a_n = \frac{-2\cdot(-1)^{m+1}}{(2m+1)\pi}$

$$\boxed{SF(x) = \frac{1}{2} + 2\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\pi} \cos[(2n+1)x]}$$

b) Tomando a extensão par de f, 2π-periódica:



c) $3\pi < 12 < 4\pi$

$\frac{3\pi}{2} < 6 < 2\pi$

$\frac{3\pi}{2} - 2\pi < 6 - 2\pi < 0$

$-\pi/2 < 6 - 2\pi < 0$

$S(6) = f(6 - 2\pi) = 1$

$S(37\pi) = f(\pi) = f(-\pi) = 0$