Resoluções da Lista de 0 de MAT2455 – Cálculo III para Engenharia 2015 – POLI-USP
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1. $\int \frac{x^7+x^2+1}{x^2}dx=\int x^5 dx+\int dx+\int \frac{1}{x^2}dx=\frac{x^6}{6}+x-\frac{1}{x}+k$

2. $\int e^{2x}dx=\int \frac{e^u}{2}du=\frac{e^{2x}}{2}+k$ (com $u=2x\Rightarrow dx=\frac{du}{2}$)

3. $\int \cos(7x)dx=\int \frac{\cos(u)}{7}du$ $\left(com\ 7x=u\Rightarrow dx=\frac{du}{7}\right)=\frac{sen(7x)}{7}+k$

4.1 $\int tg^2 x\,dx=\int (sec^2 x-1)dx=\int sec^2 x\,dx-\int dx=tg\,x-x+k$

4.2 $\int tg^2 x\,dx=\int sen\,x(sec\,x)'\,dx=senx\,secx-\int cosx\,secx\,dx=tgx-x+k$

5. $x-2=u\Rightarrow dx=du$ $\int \frac{7}{x-2}dx=\int \frac{7}{u}du=7\ln|x-2|+k$

6.1 $tgx=u\Rightarrow dx=\frac{du}{sec^2 x}$ $\int tg^3 x\ sec^2 x\,dx=\int u^3 du=\frac{tg^4 x}{4}+k$

6.2 $\int tg^3 x\ sec^2 x\,dx=\int \frac{sen^3 x}{\cos^5 x}dx=\int \frac{senx(1-\cos^2 x)}{\cos^5 x}dx=\int \frac{u^2-1}{u^5}du$ $\left(com\ \cos x=u\Rightarrow dx=\frac{-du}{senx}\right)=$
$\frac{1}{4\cos^4 x}-\frac{1}{2\cos^2 x}+k$

7. $\int \frac{sen^3 x}{\sqrt{cosx}}dx=\int \frac{-(1-u^2)}{\sqrt{u}}du=\int (u^{3/2}-u^{-1/2})du=\frac{2\cos^{5/2}x}{5}-2\cos^{1/2}x+k=2\sqrt{cosx}\left(\frac{\cos^2 x}{5}-1\right)+k$

8. $u=cosx\Rightarrow du=-senx\,dx$: $\int tgx\,dx=\int \frac{senx}{cosx}dx=-\int \frac{1}{u}du=-\ln|cosx|+k$

9.1 $\int tg^3 x\,dx=\int (1+tg^2 x-1)tgx\,dx=\int (sec^2 x-1)tgx\,dx=\int tgx(tgx)'dx-\int tgx\,dx=\frac{tg^2 x}{2}+\ln|cosx|+k$

9.2 $\int tg^3 x\,dx=\int \frac{(1-\cos^2 x)\,senx}{\cos^3 x}dx=\int \frac{u^2-1}{u^3}du=\ln|cosx|+\frac{1}{2\cos^2 x}+k$

Não importa muito se a resposta deu diferente do gabarito, observe que:
$\ln|cosx|+\frac{1}{2\cos^2 x}-\frac{1}{2}=\ln|cosx|+\frac{sec^2 x-1}{2}=\frac{tg^2 x}{2}+\ln|cosx|$
Ou seja, as respostas só diferem por uma constante.

10. $u=1+x^2\Rightarrow du=2x\,dx$: $\int \frac{x}{1+x^2}dx=\frac{1}{2}\int \frac{1}{u}du=\frac{\ln|1+x^2|}{2}+k=\frac{\ln(1+x^2)}{2}+k$

11. $u=x^2 \Rightarrow du=2\,x\,dx$: $\quad \int \frac{x}{1+x^4}\,dx=\frac{1}{2}\int \frac{1}{1+u^2}\,du=\frac{arctg\left(x^2\right)}{2}+k$

12. $\int \frac{x^2}{1+x^2}\,dx=\int\left(1-\frac{1}{1+x^2}\right)dx=x-arctgx+k$

13. $u=1-x^2 \Rightarrow du=-2\,xdx \quad \int x\sqrt{1-x^2}\,dx=-\frac{1}{2}\int \sqrt{u}\,du=\frac{-\sqrt{\left(1-x^2\right)^3}}{3}+k$

14.1 $\int secx\,dx=\int \frac{cosx}{\cos^2 x}\,dx=\int \frac{cosx}{1-sen^2 x}\,dx=\int \frac{1}{1-u^2}\,du=\frac{1}{2}\int\left(\frac{1}{1-u}+\frac{1}{1+u}\right)du=\frac{1}{2}\ln\left|\frac{1+u}{1-u}\right|+k=$
$=\frac{1}{2}\ln\left(\frac{1+u}{1-u}\right)+k$ $\left(porquê\ ?\right)$ $=\frac{1}{2}\ln\left(\frac{1+senx}{1-senx}\right)+k$

14.2 $\int secx\,dx=\int \frac{secx\left(secx+tgx\right)}{secx+tgx}\,dx=\int \frac{1}{u}\,du$ $\left(com\quad u=secx+tgx\right)=\ln\left|secx+tgx\right|+k$

Obs.: $\frac{1}{2}\ln\left(\frac{1+senx}{1-senx}\right)=\ln\left(\sqrt{\frac{\left(1+senx\right)\left(1+senx\right)}{\left(1-senx\right)\left(1+senx\right)}}\right)=\ln\left(\sqrt{\frac{\left(1+senx\right)^2}{\cos^2 x}}\right)=\ln\left|\frac{1+senx}{cosx}\right|=\ln\left|secx+tgx\right|.$

15. $\left(u=1+\ln x\right)\Rightarrow \int \frac{1}{x\sqrt{1+\ln x}}\,dx=\int \frac{1}{\sqrt{u}}\,du=2\sqrt{1+\ln x}+k$

16. $\left(u=x^3-1\right)\Rightarrow \int x^2\sqrt[5]{x^3+1}\,dx=\int \frac{\sqrt[5]{u}}{3}\,du=\frac{5\sqrt[5]{\left(x^3+1\right)^6}}{18}+k$

17. $\int \frac{4x+8}{2x^2+8x+20}\,dx=\int \frac{2x+4}{x^2+4x+10}\,dx=\int \frac{1}{u}\,du=\ln\left|x^2+4x+10\right|+k$ $\left(x^2+4x+10=\left(x+2\right)^2+6>0\right)=$
$=\ln\left(x^2+4x+10\right)+k$

18. $u=lnx \Rightarrow dx=x\,du \quad \int \frac{\sqrt{lnx}}{x}\,dx=\int \sqrt{u}\,du=2\,\frac{\sqrt{\left(lnx\right)^3}}{3}+k$

19. $\left(u=arcsenx\quad e\quad \left(arcsenx\right)'=\frac{1}{\sqrt{1-x^2}}\right)\Rightarrow \int \frac{dx}{\left(arcsenx\right)\sqrt{1-x^2}}=\int \frac{1}{u}\,du=\ln\left|arcsenx\right|+k$

20. $\left(1+e^x=u\Rightarrow du=e^x dx\right)$ $\int \frac{e^x}{1+e^x}\,dx=\int \frac{1}{u}\,du=\ln\left(1+e^x\right)+k$

21. $\left(1+\cos^2 x=u\Rightarrow du=-2\,senxcosxdx=-sen\,2\,x\,dx\right)$ $\int \frac{sen\,2x}{1+\cos^2 x}\,dx=-\int \frac{1}{u}\,du=-\ln\left(1+\cos^2 x\right)+k$

22. $\left(u=x^3\Rightarrow du=3\,x^2 dx\right)$ $\quad \int e^{x^3}\,x^2\,dx=\int \frac{e^u}{3}\,du=\frac{e^{x^3}}{3}+k$

23. $u=1+e^x \Rightarrow du=e^x\,dx \qquad \int e^x\sqrt[3]{1+e^x}\,dx=\int \sqrt[3]{u}\,du=\frac{3}{4}\sqrt[3]{(1+e^x)^4}+k$

24. $\sqrt{x}=u \Rightarrow dx=2\sqrt{x}\,du \qquad \int \frac{sen\sqrt{x}}{\sqrt{x}}dx=2\int sen\,u\,du=-2\cos\sqrt{x}+k$

25. $u=arctgx \Rightarrow du(1+x^2)=dx \qquad \int \frac{e^{arctgx}}{1+x^2}dx=\int e^u\,du=e^{arctgx}+k$

26. $\int 2x(x+1)^{2006}dx=2\int (x+1-1)(x+1)^{2006}dx=2\int (x+1)^{2007}dx-2\int (x+1)^{2006}dx=$
$=\frac{(x+1)^{2008}}{1004}-2\frac{(x+1)^{2007}}{2007}+k$

27. $\int xsenx\,dx=\int x(-cosx)'\,dx=-xcosx+\int cosx\,dx=senx-xcosx+k$

28. $\int e^x cosx\,dx=\int (e^x)'cosx\,dx=e^x cosx+\int e^x senx\,dx=e^x cosx+e^x senx-\int e^x cosx\,dx$
$2\int e^x cosx\,dx=e^x cosx+e^x senx \Leftrightarrow \int e^x cosx\,dx=\frac{e^x(cosx+senx)}{2}+k$

Atenção nesse exercício (r é um parâmetro):

29. $Se \quad r=-1: \int \frac{lnx}{x}dx=\int (lnx)'lnx\,dx=(lnx)^2-\int lnx(lnx)'\,dx \Rightarrow \int \frac{lnx}{x}dx=\frac{(lnx)^2}{2}+k$
$Se \quad r\neq -1: \int x^r lnx\,dx=\int (\frac{x^{r+1}}{r+1})'lnx\,dx=\frac{x^{r+1}}{r+1}lnx-\frac{1}{r+1}\int x^r dx=\frac{x^{r+1}}{r+1}lnx-\frac{x^{r+1}}{(r+1)^2}+k$

30. $\int (lnx)^2dx=\int (x)'(lnx)^2dx=x(lnx)^2-2\int lnx\,dx=x(lnx)^2-2\int (x)'lnx\,dx=$
$=x(lnx)^2-2(xlnx-\int dx)=x(lnx)^2-2(xlnx-x)+k$

31. $\int x\,e^{-x}dx=-xe^{-x}+\int e^{-x}dx=-x\,e^{-x}-e^{-x}+k$

32. $Como \quad \int \frac{x^2}{1+x^2}dx=\int (1-\frac{1}{1+x^2})dx=x-arctgx+k, \int xarctgx\,dx=\frac{x^2}{2}arctgx-\frac{1}{2}\int \frac{x^2}{1+x^2}dx=$
$=\frac{x^2}{2}arctgx+\frac{arctgx}{2}-\frac{x}{2}+k$

33. $1-x^2=u \Rightarrow du=-2\,xdx \qquad \int arcsenx\,dx=xarcsenx-\int \frac{x}{\sqrt{1-x^2}}dx=xarcsenx+\frac{1}{2}\int \frac{1}{\sqrt{u}}du=$
$=xarcsenx+\sqrt{1-x^2}+k$

34. $\int sec^3x\,dx=\int sec^2x\,secx\,dx=\int (tgx)'secx\,dx=secx\,tgx-\int secx\,tg^2x\,dx=$
$=secx\,tgx-\int secx(sec^2x-1)\,dx=secx\,tgx-\int sec^3x\,dx+\int secx\,dx=$
$\int sec^3x\,dx=secx\,tgx+\ln|secx+tgx|+k-\int sec^3x\,dx \Rightarrow \int sec^3x\,dx=\frac{secx\,tgx+\ln|secx+tgx|}{2}+k$

Por identidades trigonométricas:

35.1 $\int \cos^2 x\,dx = \frac{1}{2}\int (2\cos^2 x - 1 + 1)\,dx = \frac{1}{2}\int (\cos 2x + 1)\,dx = \frac{sen\,2x}{4} + \frac{x}{2} + k = \frac{1}{2}(x + senxcosx) + k$

Por partes:

35.2 $\begin{cases} \int \cos^2 x\,dx = senxcosx + \int sen^2 x\,dx \\ \int \cos^2 x\,dx = x - \int sen^2 x\,dx \end{cases} \quad + \Rightarrow \quad \int \cos^2 x\,dx = \frac{1}{2}(x + senxcosx) + k$

36. $senx = u \Rightarrow du = cosx\,dx \qquad \int sen^2 x \cos^3 x\,dx = \int sen^2 x(1 - sen^2 x)\,cosx\,dx = \int u^2(1-u^2)\,du =$
$= \frac{u^3}{3} - \frac{u^5}{5} + k = \frac{sen^3 x}{3} - \frac{sen^5 x}{5} + k$

37. $\int sen^2 x \cos^2 x\,dx = \frac{1}{4}\int (sen\,2x)^2\,dx = -\frac{1}{8}\int (1 - 2\,sen^2 2x - 1)\,dx == -\frac{1}{8}\int (\cos 4x - 1)\,dx =$
$= -\frac{sen\,4x}{32} + \frac{x}{8} + k$

38.1 $\int \frac{1 - senx}{cosx}\,dx = \int secx\,dx - \int tgx\,dx = \ln|secx + tgx| + \ln|cosx| + k = \ln|1 + senx| + k = \ln(1 + senx) + k$

38.2 $\int \frac{1 - senx}{cosx}\,dx = \int \frac{1 - sen^2 x}{cosx(1 + senx)}\,dx = \int \frac{cosx}{1 + senx}\,dx = \int \frac{1}{u}\,du = \ln(1 + senx) + k$

39. Se você não lembra muito bem sobre frações parciais não tem problema, apenas leia os exemplos do Guidorizzi (vol.1).

$$\frac{3x^2 + 4x + 5}{(x-1)(x-2)(x-3)} \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3} \equiv \frac{6}{x-1} + \frac{22}{x-3} - \frac{25}{x-2}$$

pois:

$$3x^2 + 4x + 5 \equiv A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) = f(x)$$

$$\begin{cases} x = 1 \Rightarrow 12 = 2A \Leftrightarrow A = 6 \\ x = 2 \Rightarrow B = -25 \\ x = 3 \Rightarrow C = 22 \end{cases}$$

e não há problema nenhum em ter testado esses valores. f(x) possui os pontos que faltam na função original para a torná-la contínua.

$$\int \frac{3x^2 + 4x + 5}{(x-1)(x-2)(x-3)}\,dx = 6\ln|x-1| - 25\ln|x-2| + 22\ln|x-3| + k$$

40. $\int \frac{1}{2x^2 + 8x + 20}\,dx = \frac{1}{2}\int \frac{1}{x^2 + 4x + 4 + 6}\,dx = \frac{1}{12}\int \frac{1}{\left(\frac{x+2}{\sqrt{6}}\right)^2 + 1}\,dx = \frac{\sqrt{6}}{12}\int \frac{1}{1 + u^2}\,du = \frac{\sqrt{6}}{12}\,arctg\left(\frac{x+2}{\sqrt{6}}\right) + k$

41. Como:

$$\frac{3x^2+4x+5}{(x-1)^2(x-2)}\equiv\frac{25}{x-2}-\frac{22}{x-1}-\frac{12}{(x-1)^2}$$ Então:

$$\int\frac{3x^2+4x+5}{(x-1)^2(x-2)}dx=25\ln|x-2|-22\ln|x-1|+\frac{12}{x-1}+k$$

42. Como:

$$\frac{x^5+x+1}{x^3-8}\equiv x^2+\frac{8x^2+x+1}{(x-2)(x^2+2x+4)}\equiv x^2+\frac{1}{12}\left(\frac{35}{x-2}+\frac{61x+64}{x^2+2x+4}\right)$$ Então:

$$\int\frac{x^5+x+1}{x^3-8}dx=\int\left(x^2+\frac{1}{12}\left(\frac{35}{x-2}+\frac{61x+64}{x^2+2x+4}\right)\right)dx=\frac{x^3}{3}+\frac{35}{12}\ln|x-2|+\frac{1}{12}\int\frac{61x+64}{x^2+2x+4}dx$$

Como:

$$\frac{61x+64}{x^2+2x+4}\equiv\frac{61(x+1)}{(x+1)^2+3}+\frac{3}{(x+1)^2+3}$$ e fazendo x+1=u:

$$\int\frac{61x+64}{x^2+2x+4}dx=\int\left(61\frac{u}{u^2+3}+\frac{3}{u^2+3}\right)du=\frac{61}{2}\ln(u^2+3)+\sqrt{3}\,arctg\left(\frac{u}{\sqrt{3}}\right)+k=$$

$$=\frac{61}{2}\ln(x^2+2x+4)+\sqrt{3}\,arctg\left(\frac{x+1}{\sqrt{3}}\right)+k$$ e finalmente:

$$\int\frac{x^5+x+1}{x^3-8}dx==\frac{x^3}{3}+\frac{35}{12}\ln|x-2|+\frac{61}{24}\ln(x^2+2x+4)+\frac{\sqrt{3}}{12}arctg\left(\frac{x+1}{\sqrt{3}}\right)+k$$

43. $x=senu,\ -\frac{\pi}{2}<u<\frac{\pi}{2}$ $\quad\int\frac{x^2}{\sqrt{1-x^2}}dx=\int sen^2u\,du=\frac{1}{2}\int(1-\cos 2u)du=\frac{u-senu\,cosu}{2}+k$

$$=\frac{arcsenx-x\sqrt{1-x^2}}{2}+k$$

44. $x=senu,\ -\frac{\pi}{2}<u<\frac{\pi}{2}$ $\quad\int x^2\sqrt{1-x^2}dx=\int sen^2u\cos^2u\,du=\frac{1}{4}\int sen^2 2u\,du=\frac{1}{8}\int(1-\cos 4u)du=$

$$=\frac{u}{8}-\frac{sen\,4u}{32}+k=\frac{u-senu\,cosu(1-2sen^2u)}{8}+k=\frac{1}{8}\left(arcsenx+(2x^2-1)\sqrt{1-x^2}\right)+k$$

45. $\sqrt{x}=u,\ du=\frac{dx}{2\sqrt{x}}$ $\quad\int e^{\sqrt{x}}dx=2\int ue^u du=2ue^u-2\int e^u du=2e^2(u-1)+k=2e^{\sqrt{x}}(\sqrt{x}-1)+k$

46. $x=tgu,\ dx=dusec^2u$ $\quad\int\ln\left(x+\sqrt{1+x^2}\right)dx=\int sec^2u\ln(secu+tgu)du=$

$$=tgu\ln(secu+tgu)-\int secu\,tgu\,du=tgu\ln(secu+tgu)-secu+k=x\ln\left(x+\sqrt{1+x^2}\right)-\sqrt{1+x^2}+k$$

47. $\int \frac{dx}{\sqrt{5-2x+x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{1+\left(\frac{x-1}{2}\right)^2}} = \int \frac{1}{\sqrt{1+u^2}}du = \int \frac{sec^2 v}{\sqrt{1+tg^2 v}}dv = \int secv\, dv = \ln|secv+tgv|+k$

$= \ln\left|\frac{x-1}{2}+\sqrt{1+\left(\frac{x-1}{2}\right)^2}\right|+k = \ln\left(\frac{x-1}{2}+\sqrt{1+\left(\frac{x-1}{2}\right)^2}\right)+k = \ln\left(\sqrt{5-2x+x^2}+x-1\right)+k$

48. $\int \sqrt{x}\, lnx\, dx = \int \left(\frac{2x\sqrt{x}}{3}\right)' lnx\, dx = \frac{2x\sqrt{x}}{3} lnx - \int \frac{2\sqrt{x}}{3}dx = \frac{2}{3}x\sqrt{x}\left(lnx - \frac{2}{3}\right)+k$

49. $\int sen(lnx)dx = x\, sen(lnx) - \int \frac{\cancel{x}\cos(lnx)}{\cancel{x}}dx = x\, sen(lnx) - x\cos(lnx) - \int sen(lnx)dx$

$\int sen(lnx)dx = \frac{x}{2}(sen(lnx) - \cos(lnx))+k$

50. $x^2-4=u \;\Rightarrow\; du = 2x\,dx \qquad \int \frac{x}{x^2-4}dx = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|x^2-4|+k$

51. $\frac{3x^2+5x+4}{x^3+x^2+x-3} \equiv \frac{3x^2+5x+4}{x^3+2x^2-x^2+3x-2x-3} \equiv \frac{3x^2+5x+4}{(x-1)(x^2+2x+3)} \equiv \frac{2x^2+2*2x+2*3+x^2-x+2x-2}{(x-1)(x^2+2x+3)} \equiv$

$\equiv \frac{2(x^2+2x+3)+(x+2)(x-1)}{(x-1)(x^2+2x+3)} \equiv \frac{2}{x-1}+\frac{x+2}{x^2+2x+3} \equiv \frac{2}{x-1}+\frac{1}{2}\frac{2x+2}{x^2+2x+3}+\frac{1}{2}\frac{1}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1}$ Logo:

$\int \left(\frac{3x^2+5x+4}{x^3+x^2+x-3}\right)dx = \int \left(\frac{2}{x-1}+\frac{1}{2}\frac{2x+2}{x^2+2x+3}+\frac{1}{2}\frac{1}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1}\right)dx =$

$= 2\ln|x-1|+\frac{1}{2}\ln(x^2+2x+3)+\frac{\sqrt{2}}{2}arctg\left(\frac{x+1}{\sqrt{2}}\right)+k$

52. O próximo exercício é uma equação paramétrica. Analisemos caso a caso:

Se $a=0$:

$$\int \sqrt{a^2+b^2x^2}\,dx = \int \sqrt{b^2x^2}\,dx = \int |b||x|\,dx = \begin{cases} \frac{1}{2}|b|x^2+k & se \quad x \geq 0; \\ -\frac{1}{2}|b|x^2+k & se \quad x<0. \end{cases}$$

Se $a \neq 0 \quad com \quad b=0$:

$\int \sqrt{a^2+b^2x^2}\,dx = \int \sqrt{a^2}\,dx = |a|\int dx = |a|x+k$

Se $a \neq 0 \;\; com \;\; b \neq 0$:

$I = \int \sqrt{a^2+b^2x^2}\,dx = |a|\int \sqrt{1+\left(\frac{bx}{a}\right)^2}dx \quad Fazendo\, \frac{bx}{a} = tgu,\; -\frac{\pi}{2}<u<\frac{\pi}{2},\; dx = \frac{asec^2u\,du}{b}$:

$I = \frac{a|a|}{b}\int sec^3u\,du = \frac{a|a|}{b}(secu\,tgu + \ln|secu+tgu|)+k = \frac{x}{2}\sqrt{a^2+b^2x^2}+\frac{a|a|}{2b}\ln\left(\frac{bx}{a}+\frac{\sqrt{a^2+b^2x^2}}{a}\right)+k$

Portanto:

$$\int \sqrt{a^2+b^2x^2}\,dx=\begin{cases}\frac{1}{2}|b|\,x^2+k, & se\ \ a=0\ \ e\ \ x\geq 0;\\ -\frac{1}{2}|b|\,x^2+k, & se\ \ a=0\ \ e\ \ x<0;\\ |a|\,x+k, & se\ \ a\neq 0\ \ e\ \ b=0;\\ \frac{x}{2}\sqrt{a^2+b^2x^2}+\frac{a|a|}{2b}\ln\left(\frac{bx}{a}+\frac{\sqrt{a^2+b^2x^2}}{a}\right)+k, & se\ \ a\neq 0\ \ e\ \ b\neq 0.\end{cases}$$

53. Se $a=0$:

$$\int \frac{1}{\sqrt{a^2+b^2x^2}}\,dx=\int \frac{1}{\sqrt{b^2x^2}}\,dx=\frac{1}{|b|}\int \frac{1}{|x|}\,dx=\begin{cases}\frac{\ln(x)}{|b|}+k & se\ \ x\geq 0;\\ -\frac{\ln(-x)}{|b|}+k & se\ \ x<0.\end{cases}$$

Se $a\neq 0$ com $b=0$:

$$\int \frac{1}{\sqrt{a^2+b^2x^2}}\,dx=\int \frac{1}{\sqrt{a^2}}\,dx=\frac{1}{|a|}\int dx=\frac{x}{|a|}+k$$

Se $a\neq 0$ com $b\neq 0$:

$$I=\int \frac{1}{\sqrt{a^2+b^2x^2}}\,dx=\frac{1}{|a|}\int \frac{1}{\sqrt{1+\left(\frac{bx}{a}\right)^2}}\,dx \quad Fazendo\ \frac{bx}{a}=tgu,\ -\frac{\pi}{2}<u<\frac{\pi}{2},\ dx=\frac{a\sec^2 u\,du}{b}:$$

$$I=\frac{\hat{a}}{b}\int \sec u\,du=\frac{\hat{a}}{b}\ln|\sec u+tgu|+k=\frac{\hat{a}}{b}\ln\left(\frac{bx}{a}+\frac{\sqrt{a^2+b^2x^2}}{a}\right)+k,\ \ onde\ \hat{a}=\frac{a}{|a|}.$$

Portanto:

$$\int \sqrt{a^2+b^2x^2}\,dx=\begin{cases}\frac{\ln(x)}{|b|}+k, & se\ \ a=0\ \ e\ \ x\geq 0;\\ -\frac{\ln(-x)}{|b|}+k, & se\ \ a=0\ \ e\ \ x<0;\\ \frac{x}{|a|}+k, & se\ \ a\neq 0\ \ e\ \ b=0;\\ \frac{\hat{a}}{b}\ln\left(\frac{bx}{a}+\frac{\sqrt{a^2+b^2x^2}}{a}\right)+k, & se\ \ a\neq 0\ \ e\ \ b\neq 0.\end{cases}$$

54. $x-1=tgu$: $\int \sqrt{x^2-2x+2}\,dx=\int \sqrt{(x-1)^2+1}\,dx=\int \sec^3 u\,du=\frac{1}{2}(\sec u\,tgu+\ln|\sec u+tgu|)+k$

$$\int \sqrt{x^2-2x+2}\,dx=\frac{1}{2}\left((x-1)\sqrt{x^2-2x+2}+\ln\left(\sqrt{x^2-2x+2}+x-1\right)\right)+k$$

55. $\frac{x+1}{2}=senu\,,-\frac{\pi}{2}\le u\le\frac{\pi}{2}$: $\int\sqrt{3-2x-x^2}\,dx=\int\sqrt{2^2-(x+1)^2}\,dx=2\int\sqrt{1-\left(\frac{x+1}{2}\right)^2}\,dx=4\int\cos^2u\,du=$

$=2\int(\cos 2u+1)\,du=2\,senu\,cosu+2u+k=\left(\frac{x+1}{2}\right)\sqrt{3-2x-x^2}+2\,arcsen\left(\frac{x+1}{2}\right)+k$

56. $I=\int\frac{1}{(1+x^2)\sqrt{1-x^2}}\,dx=\frac{1}{2}\int\left(\frac{\sqrt{1-x^2}}{1+x^2}+\frac{1}{\sqrt{1-x^2}}\right)dx=\frac{1}{2}arcsenx+\frac{1}{2}\int\frac{\sqrt{1-x^2}}{1+x^2}\,dx$

$x=senu\,,\;\; dx=cosu\,du\quad I=\frac{1}{2}arcsenx+\frac{1}{2}\int\frac{1-sen^2u}{1+sen^2u}\,du=\frac{1}{2}arcsenx-\frac{u}{2}+\int\frac{1}{1+sen^2u}\,du=\int\frac{1}{1+sen^2u}\,du$

Observe que teria sido mais rápido substituir x por sen(u) no início. Falha minha.

$Multiplicando\ por\ cossec^2u\quad I=\int\frac{cossec^2u}{1+cossec^2u}\,du=\int\frac{cossec^2u}{2+cotg^2u}\,du\quad y=cotg(u),\;\; dy=-cossec^2(u)\,du$:

$=\int-\frac{1}{y^2+2}\,dy=-\frac{1}{2}\int\frac{1}{\left(\frac{y}{\sqrt{2}}\right)^2+1}\,dy=-\frac{\sqrt{2}}{2}arctg\left(\frac{y}{\sqrt{2}}\right)+k\,.Como\ y=\frac{1}{tgu}=\frac{\sqrt{1-sen^2u}}{senu}=\frac{\sqrt{1-x^2}}{x}$:

$\int\frac{1}{(1+x^2)\sqrt{1-x^2}}\,dx=-\frac{\sqrt{2}}{2}arctg\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right)+k$

57. $u=senx\Rightarrow du=cosx\,dx\quad\int\cos^3x\,dx=\int(1-u^2)\,du=u-\frac{u^3}{3}+k=senx-\frac{sen^3x}{3}+k$

58. $u=cosx\Rightarrow du=-senx\,dx\quad\int sen^5x\,dx=\int sen^4x\,senx\,dx=\int(1-\cos^2x)^2senx\,dx=-\int(u^2-1)^2du=$

$=-u+\frac{2u^3}{3}-\frac{u^5}{5}+k=-cosx+\frac{2\cos^3x}{3}-\frac{\cos^5x}{5}+k$

59. $u=senx\Rightarrow du=cosx\,dx\quad\int\frac{\cos^5x}{sen^3x}\,dx=\int\frac{(1-sen^2x)^2cosx}{sen^3x}\,dx=\int\frac{(1-u^2)^2}{u^3}\,du=\int\left(u^{-3}-\frac{2}{u}+u\right)du=$

$=-\frac{1}{2u^2}-2\ln|u|+\frac{u^2}{2}+k=\frac{sen^2x}{2}-2\ln|senx|-\frac{1}{2\,sen^2x}+k$

60. $\int sen^3\left(\frac{x}{2}\right)\cos^5\left(\frac{x}{2}\right)dx=\frac{1}{8}\int\left(2\,sen\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\right)^3\cos^2\left(\frac{x}{2}\right)dx=\frac{1}{8}\int sen^3x\frac{(1+cosx)}{2}\,dx=$

$=\int\left(\frac{sen^3x\,cosx}{16}+\frac{(1-\cos^2x)\,senx}{16}\right)dx=\frac{sen^4x}{64}+\int\frac{(1-\cos^2x)\,senx}{16}\,dx=\frac{sen^4x}{64}+\frac{\cos^3x}{48}-\frac{\cos x}{16}+k$

61.1 $\int \frac{1}{sen^5 x\cos^3 x}dx=\int \frac{sen^2 x+\cos^2 x}{sen^5 x\cos^3 x}dx=\int \frac{1}{sen^3 x\cos^3 x}dx+\int \frac{1}{sen^5 x\cos x}dx=$

$=\int \frac{sen^2 x+\cos^2 x}{sen^3 x\cos^3 x}dx+\int \frac{sen^2 x+\cos^2 x}{sen^5 x\cos x}dx=\int \frac{1}{sen\,x\cos^3 x}dx+2\int \frac{1}{sen^3 x\cos x}dx+\int \frac{cosx}{sen^5 x}dx=$

$=\int \frac{senx}{\cos^3 x}dx+3\int \frac{1}{sen\,x\cos x}dx+2\int \frac{cosx}{sen^3 x}dx+\int \frac{cosx}{sen^5 x}dx=$

$=\int \frac{senx}{\cos^3 x}dx+3\int \frac{senx}{\cos x}dx+3\int \frac{cosx}{sen\,x}dx+2\int \frac{cosx}{sen^3 x}dx+\int \frac{cosx}{sen^5 x}dx=$

$=\frac{1}{2\cos^2 x}+3\ln|tgx|-csc^2 x-\frac{1}{4sen^4 x}+k.$

61.2 $\int \frac{1}{sen^5 x\cos^3 x}dx=\int \frac{sec^8 x}{tg^5 x}dx=\int \frac{(sec^2 x)^3 sec^2 x}{tg^5 x}dx=\int \frac{(1+tg^2 x)^3 sec^2 x}{tg^5 x}dx=$

$=\int \frac{(1+3tg^2 x+3tg^4 x+tg^6 x)sec^2 x}{tg^5 x}dx=-\frac{1}{4tg^4 x}-\frac{3}{2tg^2 x}+3\ln|tgx|+\frac{tg^2 x}{2}+k.$

61.3 $\int \frac{1}{sen^5 x\cos^3 x}dx=\int \frac{csc^8 x}{ctg^3 x}dx=\int \frac{(csc^2 x)^3 csc^2 x}{ctg^3 x}dx=\int \frac{(1+ctg^2 x)^3 csc^2 x}{ctg^3 x}dx=$

$=\int \frac{(1+3ctg^2 x+3ctg^4 x+ctg^6 x)csc^2 x}{ctg^3 x}dx=\frac{1}{2ctg^2 x}+3\ln|tgx|-\frac{3}{2}ctg^2 x-\frac{ctg^4 x}{4}+k.$

61.4 $\int \frac{1}{sen^5 x\cos^3 x}dx=\int \frac{32\,senx\,cosx}{(16sen^4 x\cos^4 x)(2sen^2 x)}dx=\int \frac{16sen2x}{(sen^2 2x)^2(1-\cos(2x))}dx=$

$=-8\int \frac{1}{(1-u^2)^2(1-u)}du \quad (com\ \ u=\cos(2x)) \ =-8\int \frac{(1+u)}{(1-u^2)^2(1-u)(1+u)}du=$

$=-8\int \frac{1+u}{(1-u^2)^3}du=\int\left(\frac{-8u}{(1-u^2)^3}-\left(\frac{2}{(1-u^2)}\right)^3\right)du=\int\left(\frac{-8u}{(1-u^2)^3}-\left(\frac{1}{1+u}+\frac{1}{1-u}\right)^3\right)du=$

$=\frac{-2}{(1-u^2)^2}-\int\left(\frac{1}{(1+u)^3}+3\left(\frac{1}{1+u}+\frac{1}{1-u}\right)\frac{1}{1-u^2}+\frac{1}{(1-u)^3}\right)du=$

$=-\frac{2}{(1-u^2)^2}+\frac{1}{2(1+u)^2}-\frac{1}{2(1-u)^2}-\frac{3}{2}\int\left(\frac{1}{1+u}+\frac{1}{1-u}\right)^2du=$

$=-\frac{2}{(1-u^2)^2}+\frac{1}{2(1+u)^2}-\frac{1}{2(1-u)^2}-\frac{3}{2}\left(\frac{-1}{1+u}+\frac{1}{1-u}\right)-\frac{3}{2}\int\left(\frac{1}{1+u}+\frac{1}{1-u}\right)du=$

$=-\frac{2}{(1-u^2)^2}+\frac{1}{2(1+u)^2}-\frac{1}{2(1-u)^2}+\frac{3}{2(1+u)}-\frac{3}{2(1-u)}-\frac{3}{2}\ln\left|\frac{1+u}{1-u}\right|+k$

Então:

$$\int \frac{1}{sen^5 x\cos^3 x}dx=-2cossec^2 2x+\frac{sec^4 x}{8}-\frac{3}{4}cossec^2 x+\frac{3}{4}sec^2 x-3\ln|ctgx|+k$$

62. $\int sen^4 x\,dx=\int\left(\frac{1-\cos(2x)}{2}\right)^2dx=\frac{1}{4}\int(1-2\cos(2x)+\cos^2(2x))dx=$

$=\frac{1}{4}\int\left(\frac{2}{2}-\frac{4\cos(2x)}{2}+\frac{1+\cos(4x)}{2}\right)dx=\frac{1}{8}\int(2-4\cos(2x)+1+\cos(4x))dx=$

$=\frac{1}{8}(3x-2sen(2x)+\frac{sen(4x)}{4})+k=\frac{3x}{8}-\frac{sen(2x)}{4}+\frac{sen(4x)}{32}+k$

63. $\int sen^2 x \cos^5 x\, dx = \int sen^2 x (1 - sen^2 x)^2 cosx\, dx = \int u^2 (u^4 - 2u^2 + 1) du = \frac{u^7}{7} - 2\frac{u^5}{5} + \frac{u^3}{3} + k =$

$= \frac{sen^7 x}{7} - 2\frac{sen^5 x}{5} + \frac{sen^3 x}{3} + k$

64. $\int sen^2 x \cos^4 x\, dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{8}\int (1 - \cos^2 2x + \cos 2x - \cos^3 2x) dx =$

$= \frac{x}{8} + \frac{sen 2x}{16} - \frac{1}{16}\int (1 + \cos 4x) dx - \frac{2}{16}\int (1 - sen^2 2x) \cos 2x\, dx =$

$= \frac{x}{16} + \frac{sen 2x}{16} - \frac{sen 4x}{64} - \frac{sen 2x}{16} + \frac{sen^3 2x}{48} + k = \frac{x}{16} - \frac{sen 4x}{64} + \frac{sen^3 2x}{48} + k$

Na página 387 do Vol.1 do Guidorizzi, está escrito: “Para o cálculo de integrais $\int sen^n x\, dx$ e $\int \cos^n x\, dx$, com $n \geq 5$, recomendamos utilizar as fórmulas de recorrência que serão estabelecidas no próximo exemplo”.

a) $\int sen^n x\, dx = -\frac{1}{n} sen^{n-1} x\, cosx + \frac{n-1}{n}\int sen^{n-2} x\, dx$.

b) $\int \cos^n x\, dx = \frac{1}{n}\cos^{n-1} senx + \frac{n-1}{n}\int \cos^{n-2} x\, dx$.

Usemos então a recomendação do livro para o exercício 65 da lista:

65. $\int \cos^6 (3x) dx = \frac{1}{3}\int \cos^6 u\, du = \frac{1}{3}\frac{1}{6}\cos^5 u\, senu + \frac{1}{3}\frac{5}{6}\int \cos^4 u\, du = \frac{\cos^5 u\, senu}{18} + \frac{5}{18}\int \cos^4 u\, du =$

$= \frac{\cos^5 u\, senu}{18} + \frac{5}{18}\frac{1}{4}\cos^3 u\, senu + \frac{5}{18}\frac{3}{4}\int \cos^2 u\, du = \frac{\cos^5 u\, senu}{18} + \frac{5\cos^3 u\, senu}{72} + \frac{5}{24}\frac{1}{2} cosu\, senu + \frac{5}{24}\frac{1}{2}\int dx =$

$= \frac{\cos^5 u\, senu}{18} + \frac{5\cos^3 u\, senu}{72} + \frac{5\, cosu\, senu}{48} + \frac{5u}{48} + k =$

$= \frac{\cos^5 3x\, sen 3x}{18} + \frac{5\cos^3 3x\, sen 3x}{72} + \frac{5\cos 3x\, sen 3x}{48} + \frac{5x}{16} + k$

Como na hora das provas nem sempre teremos fórmulas e fórmulas disponíveis, Batman sabe resolver qualquer integral por partes. Tente reduzir a produto de 2 funções em que você conheça a derivada de uma e a integral de outra (e que a integral desta ainda seja produto de senos e cossenos). Em certos momentos, faça substituições 1+cotg²x=cossec²x, 1+tg²x=sec²x e lembre as derivadas de secx, cossecx, tgx e cotgx. Veja o exercício a seguir:

66. $\int \frac{\cos^2 x}{sen^6 x} dx = \int cosx \frac{cosx}{sen^6 x} dx = senx \frac{cosx}{sen^6 x} - \int senx \left(-\frac{1}{sen^5 x} - \frac{6\, cosx}{sen^7 x}\right) dx =$

$= \frac{cosx}{sen^5 x} + \int \frac{6\, cosx}{sen^6 x} dx + \int \frac{1}{sen^4 x} dx = \frac{cosx}{sen^5 x} - \frac{6}{5\, sen^5 x} + \int \left(\frac{1}{sen^2 x} + \frac{\cos^2 x}{sen^4 x}\right) dx =$

$= \frac{cosx}{sen^5 x} - \frac{6}{5\, sen^5 x} + cotgx + \int \frac{\cos^2 x}{sen^4 x} dx.$

Mas:

$$\int \frac{\cos^2 x}{sen^4 x}dx = \int cosx \frac{cosx}{sen^4 x}dx = senx \frac{cosx}{sen^4 x} - \int senx\left(-\frac{1}{sen^3 x} - \frac{4\,cosx}{sen^5 x}\right)dx = \frac{cosx}{sen^3 x} + \int \left(cossec^2 x + \frac{4\,cosx}{sen^4 x}\right)dx$$

$$= \frac{cosx}{sen^3 x} - cotg^2 x - \frac{4}{3\,sen^3 x} + k$$

Então:

$$\int \frac{\cos^2 x}{sen^6 x}dx = \frac{cosx}{sen^5 x} - \frac{6}{5\,sen^5 x} + cotgx + \frac{cosx}{sen^3 x} - cotg^2 x - \frac{4}{3\,sen^3 x} + k$$

67. $\int \frac{1}{sen^2 x \cos^4 x}dx = \int (1 + ctg^2 x)\frac{1}{\cos^4 x}dx = \int \frac{1}{sen^2 x \cos^2 x}dx + \int sec^4 x\,dx = 2\,tg(2x) + \int sec^4 x\,dx.$

$\int sec^4 x\,dx = \int sec^2 x\,sec^2 x\,dx = tgx\,sec^2 x - 2\int tg^2 x\,sec^2 x\,dx = tgx\,sec^2 x - 2\int (sec^2 x - 1)sec^2 x\,dx =$

$= 2\,tgx + tgx\,sec^2 x - 2\int sec^4 x\,dx \Leftrightarrow \quad \int sec^4 x\,dx = \frac{2}{3}tgx + \frac{tgx\,sec^2 x}{3} + k.$

Então:

$$\int \frac{1}{sen^2 x \cos^4 x}dx = 2\,tg(2x) + \frac{2}{3}tgx + \frac{tgx\,sec^2 x}{3} + k.$$

68. $x = \cos 2u, -\frac{\pi}{4} < u < \frac{\pi}{4} \qquad \int \sqrt{\frac{1-x}{1+x}}dx = -2\int sen\,2u\sqrt{\frac{1-\cos 2u}{1+\cos 2u}}du = -2\int sen(2u)\,tg(u)\,du =$

$= -4\int sen^2 u\,du = -2\int (1 - \cos 2u)\,du = -2u + sen(2u) + k = -\arccos(x) + \sqrt{1-x^2} + k =$

$= \frac{\pi}{2} - arccosx + \sqrt{1-x^2} + k = arcsenx + \sqrt{1-x^2} + k.$

Seguindo a sugestão:

69. $u = x^{\frac{1}{6}} \Rightarrow dx = 6u^5\,du \qquad \int \frac{1}{\sqrt{x} - \sqrt[3]{x}}dx = \int \frac{6u^5}{u^3 - u^2}du.$ Efetuando a divisão polinomial:

$$\frac{u^5}{u^3 - u^2} \equiv \frac{(u^3 - u^2)(u^2 + u + 1) + u^2}{u^3 - u^2} \equiv u^2 + u + 1 + \frac{u^2}{u^2(u-1)} \equiv u^2 + u + 1 + \frac{1}{u-1} \quad com \quad u \neq 0.$$

$$\int \frac{1}{\sqrt{x} - \sqrt[3]{x}}dx = 2u^3 + 3u^2 + 6u + 6\ln|u-1| + k = 2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln\left|\sqrt[6]{x} - 1\right| + k$$

70. $\dfrac{x+1}{x^2(x^2+4)^2}\equiv\dfrac{(x+1)}{16}\left(\dfrac{1}{x^2}-\dfrac{x^2+8}{(x^2+4)^2}\right)\equiv\dfrac{(x+1)}{16}\left(\dfrac{1}{x^2}-\dfrac{1}{(x^2+4)}-\dfrac{4}{(x^2+4)^2}\right)$ Então:

$$\int\frac{x+1}{x^2(x^2+4)^2}dx=\int\left(\frac{1}{16x}+\frac{1}{16x^2}-\frac{x}{16(x^2+4)}-\frac{1}{16(x^2+4)}-\frac{x}{4(x^2+4)^2}-\frac{1}{4(x^2+4)^2}\right)dx=$$

$$=\frac{\ln|x|}{16}-\frac{1}{16x}-\frac{\ln(x^2+4)}{32}-\frac{arctg\left(\frac{x}{2}\right)}{32}+\frac{1}{8(x^2+4)}-\frac{1}{64}\int\frac{1}{\left(\left(\frac{x}{2}\right)^2+1\right)^2}dx$$

$x=2\,tgu\Rightarrow dx=2\,sec^2u\,du$ $\quad\int\dfrac{1}{((x/2)^2+1)^2}dx=2\int\cos^2u\,du=\int(1+\cos 2u)\,du=u+senu\,cosu+k=$

$$=arctg\left(\frac{x}{2}\right)+\frac{(x/2)}{1+(x/2)^2}+k=arctg\left(\frac{x}{2}\right)+\frac{2x}{x^2+4}+k$$

Portanto:

$$\int\frac{x+1}{x^2(x^2+4)^2}dx=\frac{\ln|x|}{16}-\frac{1}{16x}-\frac{\ln(x^2+4)}{32}-\frac{arctg\left(\frac{x}{2}\right)}{32}+\frac{1}{8(x^2+4)}-\frac{arctg(x/2)}{64}-\frac{x}{32(x^2+4)}+k$$

71. $\displaystyle\int\frac{arctgx}{x^2}dx=-\frac{arctgx}{x}+\int\frac{1}{x(1+x^2)}dx=-\frac{arctgx}{x}+\int\left(\frac{1}{x}-\frac{x}{(1+x^2)}\right)dx=$

$$=-\frac{arctgx}{x}+\ln|x|-\frac{\ln(1+x^2)}{2}+k$$

72. $\displaystyle\int\frac{x^2}{\sqrt{2x-x^2}}dx=\int\frac{(x-1)^2+2x-1}{\sqrt{2x-x^2}}dx=\int\left(\frac{(x-1)^2}{\sqrt{2x-x^2}}+\frac{2x}{\sqrt{2x-x^2}}-\frac{1}{\sqrt{2x-x^2}}\right)dx=$

$$=\int\left(\frac{(x-1)^2}{\sqrt{1-(x-1)^2}}-\frac{2-2x}{\sqrt{2x-x^2}}+\frac{1}{\sqrt{1-(x-1)^2}}\right)dx=$$

$$=\frac{arcsen(x-1)}{2}-\frac{(x-1)\sqrt{2x-x^2}}{2}-2\sqrt{2x-x^2}-arcsen(x-1)+k=$$

$$=-\frac{arcsen(x-1)}{2}-\frac{(x-1)\sqrt{2x-x^2}}{2}-2\sqrt{2x-x^2}+k$$

73. $\dfrac{4x^2-3x+3}{(x^2-2x+2)(x+1)}\equiv\dfrac{2x-1}{x^2-2x+2}+\dfrac{2}{x+1}\equiv\dfrac{2x-2}{x^2-2x+2}+\dfrac{2}{x+1}+\dfrac{1}{(x-1)^2+1}$.

Então: $\displaystyle\int\frac{4x^2-3x+3}{(x^2-2x+2)(x+1)}dx=\ln(x^2-2x+2)+2\ln|x+1|+arctg(x-1)+k$

74. Mais uma equação paramétrica. Ver “Litvinenko e Mordkovich” se interessar o assunto. Apenas usarei a fatoração básica que é acostumada ao aluno para resolver, sem se preocupar com buscar uma solução “artística”.
Vamos primeiro desmanchar para o denominador:

$$\frac{1}{[(x-c)(x-d)]^2}=\left[\frac{1}{c-d}\left(\frac{1}{x-c}-\frac{1}{x-d}\right)\right]^2=\frac{1}{(c-d)^2}\left[\frac{1}{(x-c)^2}+\frac{1}{(x-d)^2}-\frac{2}{(c-d)}\left(\frac{1}{x-c}-\frac{1}{x-d}\right)\right]$$

É claro que essa fatoração só existe para $c\neq d$. Isso nos leva a ter que analisar o caso $c=d$.

Para $c\neq d$:

$$E(x)=\frac{(x-a)(x-b)}{[(x-c)(x-d)]^2}=\frac{1}{(c-d)^2}\left[\frac{(x-a)(x-b)}{(x-c)^2}+\frac{(x-a)(x-b)}{(x-d)^2}-2\frac{(x-a)(x-b)}{(c-d)}\left(\frac{1}{x-c}-\frac{1}{x-d}\right)\right]=$$

$$=\frac{1}{(c-d)^2}\left[2+\frac{(2c-a-b)x+ab-c^2}{(x-c)^2}+\frac{(2d-a-b)x+ab-d^2}{(x-d)^2}-\frac{2}{c-d}\left(\frac{(c-a-b)x+ab}{x-c}-\frac{(d-a-b)x+ab}{x-d}\right)\right]$$

Observe que:

$$\begin{cases}(2c-a-b)x+ab-c^2=(2c-a-b)(x-c)+(c+a)(c+b)\\(2d-a-b)x+ab-d^2=(2d-a-b)(x-d)+(d+a)(d+b)\\(c-a-b)x+ab=(c-a-b)(x-c)+(c-a)(c-b)\\(d-a-b)x+ab=(d-a-b)(x-d)+(d-a)(d-b)\end{cases}$$

Com isso:

$$E=\frac{1}{(c-d)^2}\left[\frac{2c-a-b}{x-c}+\frac{2d-a-b}{x-d}+\frac{(c+a)(c+b)}{(x-c)^2}+\frac{(d+a)(d+b)}{(x-d)^2}-\frac{2}{c-d}\left(\frac{(c-a)(c-b)}{x-c}-\frac{(d-a)(d-b)}{x-d}\right)\right]$$

Integrando E obtemos:

$$\frac{1}{(c-d)^2}\left[(2c-a-b)\ln|x-c|+(2d-a-b)\ln|x-d|-\frac{(c+a)(c+b)}{(x-c)}-\frac{(d+a)(d+b)}{(x-d)}-\frac{2}{c-d}(A(x))\right]+k,$$

$$A(x)=(c-a)(c-b)\ln|x-c|-(d-a)(d-b)\ln|x-d|$$

Se $c\neq d$, para termos somente funções racionais queremos que os coeficientes dos logaritmos sejam 0, isto é:

$$\begin{cases}2c-a-b=2\dfrac{(c-a)(c-b)}{c-d}\Leftrightarrow ad+bd=-ac-bc+2ab+2cd\\2d-a-b=2\dfrac{(d-a)(d-b)}{d-c}\Leftrightarrow ac+bc=-ad-bd+2ab+2cd\end{cases}\Rightarrow(a+b)(c+d)=2(ab+cd).$$

Agora analisemos o caso em que $c=d$:

$$E(x)=\frac{(x-a)(x-b)}{(x-c)^4}$$

Pelo teorema da página 371 do Guidorizzi Vol.1, temos que existem constantes A, B, C e D tais que:

$$\begin{cases} \dfrac{x-a}{(x-c)^2}=\dfrac{A}{x-c}+\dfrac{B}{(x-c)^2} \\ \dfrac{x-b}{(x-c)^2}=\dfrac{C}{x-c}+\dfrac{D}{(x-c)^2} \end{cases}$$ Multiplicando as duas equações, temos que:

$$E(x)=\frac{AC}{(x-c)^2}+\frac{BC+AD}{(x-c)^3}+\frac{BD}{(x-c)^4}$$ E integrando E(x):

$$\int E(x)\,dx=-\frac{AC}{(x-c)}-\frac{BC+AD}{2(x-c)^2}-\frac{BD}{3(x-c)^3}+k$$ Ou seja, sempre que $c\neq d$, a integral da função dada é uma composição de funções racionais, mesmo quando AC=BD=BC+AD=0, pois função racional é toda função quociente $\frac{p(x)}{q(x)}$ de funções polinomiais em que $q(x)\neq 0$. Logo, temos como resposta:

$$c=d \quad ou \quad (a+b)(c+d)=2(ab+cd) \quad .$$

Note que os coeficientes buscados que resultaram na primeira rel aparecem logo no início, então com uma boa argumentação textual parte do algebrismo poderia ser poupada caso necessário.

75.

Veja que:

$$d/dx\left(\frac{x}{cosx}\right)=\frac{cosx+xsenx}{\cos^2 x}$$ e que $$d/dx\left(\frac{-1}{cosx+xsenx}\right)=\frac{xcosx}{(cosx+xsenx)^2}.$$ Então:

$$\int \frac{x^2}{(cosx+xsenx)^2}dx=\int \frac{xcosx}{(cosx+xsenx)^2}\frac{x}{cosx}dx=-\frac{x}{cosx(cosx+xsenx)}+\int sec^2 x\,dx=$$

$$=-\frac{x}{cosx(cosx+xsenx)}+tgx+k=\frac{senx-xcosx}{cosx+xsenx}+k$$

Erratas, sugestões, auxílio em formatações, envio de soluções ou simplesmente bate-papo: *estudospoli@gmail.com*

Resolução feita por *Alfred Pennysworth*. Se tiver outra lista com muitos pedidos para resolver, *feedback* e interesse, postarei outra resolução (dentro do possível). Bom semestre a todos!