excellentia

P1 MAT 2455

USP

CÁLCULO III - POLI

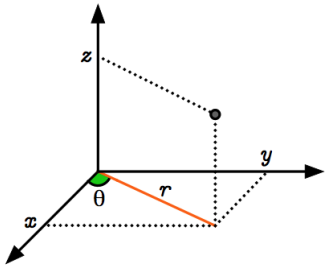
PROF. FABRÍCIO PIO

28.MAR.2015

NOME: _____

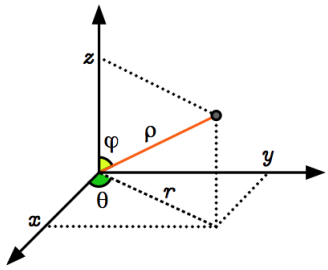
TEORIA

COORDENADAS POLARES / CILÍNDRICAS



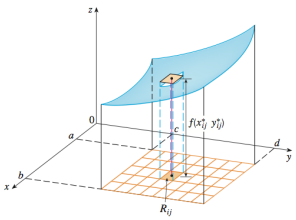
$$x = r\cos\theta$$
$$y = r\,\mathrm{sen}\,\theta$$
$$r^2 = x^2 + y^2$$
$$dA = r\,dr\,d\theta$$
$$dV = r\,dr\,d\theta\,dz$$

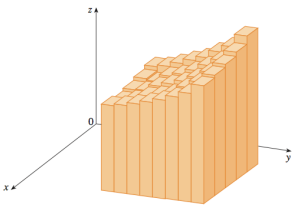
COORDENADAS ESFÉRICAS



$$r = \rho\,\mathrm{sen}\,\varphi$$
$$x = \rho\,\mathrm{sen}\,\varphi\cos\theta$$
$$y = \rho\,\mathrm{sen}\,\varphi\,\mathrm{sen}\,\theta$$
$$z = \rho\cos\varphi$$
$$\rho^2 = x^2 + y^2 + z^2$$
$$dV = \rho^2\,\mathrm{sen}\,\varphi\,d\rho\,d\theta\,d\varphi$$

PRISMAS RETANGULARES DE BASE $dx \times dy$ E ALTURA $f(x,y)$ CONSTITUEM O SÓLIDO A SER INTEGRADO.





TEOREMA DE FUBINI:

Se f é contínua no retângulo

$$R = \left\{(x,y) \mid a \le x \le b,\ c \le y \le d\right\}, \text{ então}$$

$$\iint_R f(x,y)\,dA = \int_a^b\int_c^d f(x,y)\,dy\,dx = \int_c^d\int_a^b f(x,y)\,dx\,dy$$

JACOBIANO — COORDENADAS POLARES

$$J = \begin{vmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial x}{\partial \theta} \\ \dfrac{\partial y}{\partial r} & \dfrac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\,\mathrm{sen}\,\theta \\ \mathrm{sen}\,\theta & r\cos\theta \end{vmatrix} = r$$

$$\therefore\ dA = dx\,dy = r\,dr\,d\theta$$

JACOBIANO — COORDENADAS CILÍNDRICAS

$$J = \begin{vmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial x}{\partial \theta} & \dfrac{\partial x}{\partial z} \\ \dfrac{\partial y}{\partial r} & \dfrac{\partial y}{\partial \theta} & \dfrac{\partial y}{\partial z} \\ \dfrac{\partial z}{\partial r} & \dfrac{\partial z}{\partial \theta} & \dfrac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\,\mathrm{sen}\,\theta & 0 \\ \mathrm{sen}\,\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$$

$$\therefore\ dV = dx\,dy\,dz = r\,dr\,d\theta\,dz$$

JACOBIANO — COORDENADAS ESFÉRICAS

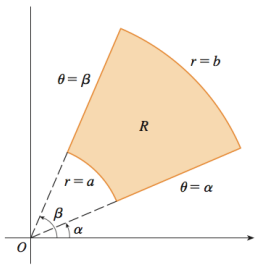
$$J = \begin{vmatrix} \dfrac{\partial x}{\partial \rho} & \dfrac{\partial x}{\partial \theta} & \dfrac{\partial x}{\partial \varphi} \\ \dfrac{\partial y}{\partial \rho} & \dfrac{\partial y}{\partial \theta} & \dfrac{\partial y}{\partial \varphi} \\ \dfrac{\partial z}{\partial \rho} & \dfrac{\partial z}{\partial \theta} & \dfrac{\partial z}{\partial \varphi} \end{vmatrix} =$$

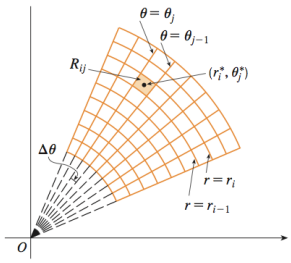
$$= \begin{vmatrix} \mathrm{sen}\,\varphi\cos\theta & -\rho\,\mathrm{sen}\,\varphi\,\mathrm{sen}\,\theta & \rho\cos\varphi\cos\theta \\ \mathrm{sen}\,\varphi\,\mathrm{sen}\,\theta & \rho\,\mathrm{sen}\,\varphi\cos\theta & \rho\cos\varphi\,\mathrm{sen}\,\theta \\ \cos\varphi & 0 & -\rho\,\mathrm{sen}\,\varphi \end{vmatrix} =$$

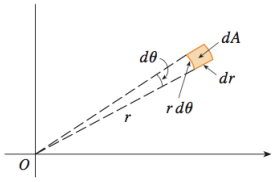
$$= \rho^2\,\mathrm{sen}\,\varphi$$

$$\therefore\ dV = dx\,dy\,dz = \rho^2\,\mathrm{sen}\,\varphi\,d\rho\,d\theta\,d\varphi$$

INTEGRAÇÃO EM COORDENADAS POLARES





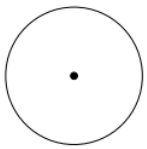


AO ADOTAR COORDENADAS POLARES, CADA REGIÃO É CONSIDERADA COMO UM RETÂNGULO DE DIMENSÕES $r\,d\theta \times dr$.

QUÁDRICAS

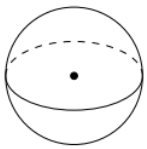
CIRCUNFERÊNCIA

$$x^2 + y^2 = r^2$$



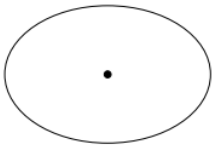
ESFERA

$$x^2 + y^2 + z^2 = r^2$$



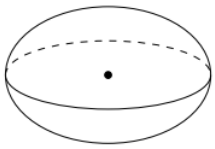
ELIPSE

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



ELIPSÓIDE

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

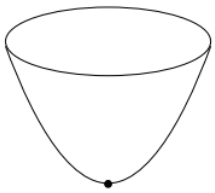


PARÁBOLA

$$ax^2 = by$$

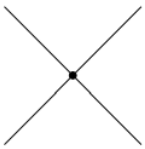
PARABOLÓIDE ELÍPTICO

$$ax^2 + by^2 = cz$$



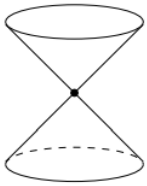
RETAS CONCORRENTES

$$\frac{x^2}{a^2} = \frac{y^2}{b^2}$$



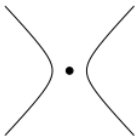
CONE DUPLO

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$



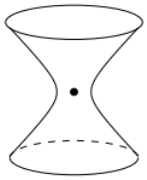
HIPÉRBOLE

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



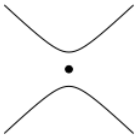
HIPERBOLÓIDE DE UMA FOLHA

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



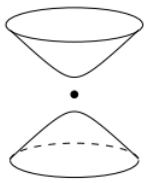
HIPÉRBOLE

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



HIPERBOLÓIDE DE DUAS FOLHAS

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



1. **P1 2010**

a) Calcule $\displaystyle\int_0^8 \left(\int_{\sqrt[3]{y}}^2 e^{x^4}\, dx \right) dy$.

b) Calcule o volume do sólido limitado pelas superfícies $z = 4 - \dfrac{x^2}{4} - y^2$, $z = 2y + 1$.

2. **P1 2009**

Calcule a massa do sólido dado por $x^2 + y^2 + z^2 \leq 4$ e $x^2 + y^2 \geq 1 + z^2$, $z \geq 0$ sendo a densidade dada por $\delta(x, y, z) = z^3$.

3. **T.WEB 2014**

Calcule $\iiint_V z\, dx\, dy\, dz$ sendo o sólido V determinado pelos pontos (x, y, z) tais que $z \leq 4 - x^2 - y^2$, $z \geq 0$ e interiores ao cilindro $(y-1)^2 + \dfrac{x^2}{2} = 1$. Esboce o sólido.

4. **P1 2013**

a) Calcule $\displaystyle\iint_B \frac{x}{y^2}\, dx\, dy$, onde

$$B = \left\{ (x, y) \in \mathbb{R}^2 \ : \ 1 \leq xy \leq 2,\ x^2 \leq y \leq 2x^2 \right\}$$

Sugestão: Considere a mudança de variáveis satisfazendo $u = \dfrac{y}{x^2}$ e $v = xy$.

b) Calcule a massa da região limitada pelo cone $z \geq \sqrt{x^2 + y^2}$ e pela esfera $x^2 + y^2 + z^2 = 2z$, sabendo que a densidade é dada por $\delta(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

5. **P2 2005**

a) Calcule a massa do fio dado pela intersecção da calha $y = x^2$ com o plano $z = x$ e tal que $0 \leq x \leq 1$, sendo a densidade dada por $\delta(x, y, z) = z$.

EXERCÍCIOS DE FIXAÇÃO

6. **LANG**

Calcule o valor das seguintes integrais iteradas:

a) $\displaystyle\int_0^2\int_1^3 (x+y)\,dx\,dy$ b) $\displaystyle\int_0^2\int_1^{x^2} y\,dy\,dx$

c) $\displaystyle\int_1^3\int_0^1 \sqrt{x}\,dx\,dy$ d) $\displaystyle\int_0^{\pi}\int_0^{x} x\,\text{sen}\,y\,dy\,dx$

e) $\displaystyle\int_1^2\int_y^{y^2} dx\,dy$ f) $\displaystyle\int_0^{\pi}\int_0^{\text{sen}\,x} y\,dy\,dx$

7. **LANG**

Calcule as integrais seguintes e esboce a região de integração em cada caso.

a) $\displaystyle\int_1^2\int_{x^2}^{x^3} x\,dy\,dx$ b) $\displaystyle\int_1^{-1}\int_x^{2x} e^{x+y}\,dy\,dx$

c) $\displaystyle\int_0^2\int_1^3 |x-2|\,\text{sen}\,y\,dx\,dy$ d) $\displaystyle\int_0^{\frac{\pi}{2}}\int_{-y}^{y} \text{sen}\,x\,dx\,dy$

e) $\displaystyle\int_{-1}^1\int_0^{|x|} dy\,dx$ f) $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\cos y} x\,\text{sen}\,y\,dx\,dy$

8. **LANG**

Calcule a integral da função

$$f(x,y) = \frac{1}{(x^2+y^2)^3}$$

na região entre as duas circunferências de raio 2 e raio 3, centradas na origem.

9. **STEWART**

Use polar coordinates to find the volume of the given solid.

a) Under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \leq 9$.

b) Inside de sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

c) A sphere of radius a.

d) Bounded by the polarid $z = 10 - 3x^2 - 3y^2$ and the plane $z = 4$.

e) Above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

f) Bounded the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

g) Inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$.

10. **STEWART**

Use a double integral to find the area of the region.

a) One loop of the rose $r = \cos 3\theta$.

b) The region enclosed by the cardioid $r = 1 - \text{sen}\,\theta$.

c) The region enclosed by the lemniscate $r^2 = 4\cos 2\theta$.

d) The region inside the circle $r = 4\,\text{sen}\,\theta$ and outside the circle $r = 2$.

11. **LANG**

Calcule a área limitada pela curva $r^2 = 2a^2\cos 2\theta$.

12. **STEWART**

Evaluate the given integral by changing do polar coordinates.

a) $\displaystyle\iint_R x\,dA$, where R is the disk with center the origin and radius 5.

b) $\displaystyle\iint_R y\,dA$, where R is the region in the first quadrant bounded by the circle $x^2 + y^2 = 9$ and the lines $y = x$ and $y = 0$.

c) $\displaystyle\iint_R x\,dA$, where R is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 25$.

d) $\displaystyle\iint_R \sqrt{x^2+y^2}\,dA$, where $R = \{(x,y) \mid 1 \leq x^2 + y^2 \leq 9, y \geq 0\}$.

e) $\displaystyle\iint_D e^{-x^2-y^2}\,dA$, where D is the region bounded by the semicircle $x = \sqrt{4-y^2}$ and the y-axis.

f) $\displaystyle\iint_D \frac{1}{(1+x^2+y^2)^{\frac{3}{2}}}\,dA$, where D is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 16$.

g) $\displaystyle\iint_D (x^2+y^2)\,dA$, where D is the region bounded by the spirals $r = \theta$ and $r = 2\theta$ for $0 \leq \theta \leq 2\pi$.

13. **STEWART**

Evaluate the iterated integral by converting to polar coordinates.

a) $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\,dy\,dx$ b) $\displaystyle\int_{-a}^a\int_0^{\sqrt{a^2-y^2}} (x^2+y^2)^{\frac{3}{2}}\,dx\,dy$

c) $\displaystyle\int_0^2\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2y^2\,dx\,dy$ d) $\displaystyle\int_0^2\int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dy\,dx$

14. **STEWART**

Evaluate the triple integral:

a) $\displaystyle\iiint_E z\,dV$, where E is bounded by the planes $x = 0, y = 0, z = 0, y + z = 0, y + z = 1$ and $x + z = 1$.

b) $\displaystyle\iiint_E (x+2y)\,dV$, where E is bounded by the parabolic cylinder $y = x^2$ and the planes $x = z$, $x = y$, and $z = 0$.

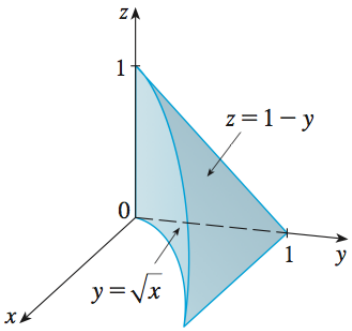
c) $\displaystyle\iiint_E x\,dV$, where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$.

15. **STEWART**

The figure shows the region of integration for the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z)\, dz\, dy\, dx$$

Rewrite this integral as an equivalent iterated integral in the five other orders.

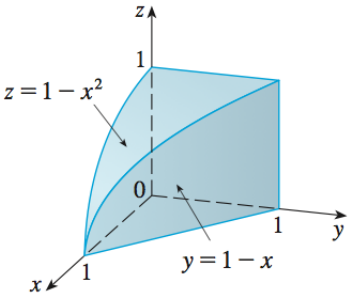


16. **STEWART**

The figure shows the region of integration for the integral

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x, y, z)\, dy\, dz\, dx$$

Rewrite this integral as an equivalent iterated integral in the five other orders.



EXERCÍCIOS DA LISTA

17.

Escreva as duas integrais iteradas correspondentes à integral dupla $\iint_D f(x, y)\, dx\, dy$, onde D é a região do plano limitada pelas curvas $y = -x^2 + x + 2$ e $x - 2y + 1 = 0$.

18.

Calcule as seguintes integrais, invertendo a ordem de integração:

a) $\displaystyle\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy$

b) $\displaystyle\int_0^3 \int_{y^2}^9 y \cos(x^2)\, dx\, dy$

c) $\displaystyle\int_0^1 \int_{\arcsin y}^{\frac{\pi}{2}} \cos x \sqrt{1 + \cos^2 x}\, dx\, dy$

19.

Calcule as integrais iteradas:

a) $\displaystyle\int_0^1 \int_0^z \int_0^y xyz\, dx\, dy\, dz$

b) $\displaystyle\int_0^\pi \int_0^2 \int_0^{\sqrt{4-x^2}} z \sin y\, dx\, dz\, dy$

20.

Calcule a massa da região R limitada por:

a) $z(x^2 + y^2) = 2$, $z = 0$, $x^2 + y^2 = 1$, $x^2 + y^2 = 2$, com $x \geq 0$ e $y \leq 0$ e $\delta(x, y, z) = 1$

b) $x^2 + y^2 = 1 + z^2$, $x^2 + y^2 = 4$ com $\delta(z, y, z) = |z|$

21.

Calcule a integral $\displaystyle\iiint_E x\, dx\, dy\, dz$, onde

$$E = \left\{(x, y, z) \ : \ \frac{x^2}{4} + \frac{y^2}{9} + z^2 \leq 1, x \geq 0\right\}$$

22.

Calcule $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx$.

23.

Calcule a seguinte integral: $\displaystyle\int_0^1 \int_y^{\sqrt[3]{y}} \operatorname{sen}(x^2)\, dx\, dy$.

24.

Determine o volume do sólido limitado pelas superfícies $\dfrac{(x-1)^2}{2} + \dfrac{y^2}{3} = 1$; $z = x^2 + y^2$ e $z = 0$.

25.

Calcule a integral $\displaystyle\iiint_S \sqrt{y}\, dx\, dy\, dz$ sendo S o sólido limitado pelas superfícies $y = x^3$; $x = y^2$ e $y = z^2$.

26.

Calcule as seguintes integrais de linha ao longo da curva indicada:

a) $\int_\gamma x\, ds$, $\gamma(t) = (t^3, t)$, $0 \leq t \leq 1$.

b) $\int_\gamma xy^4 ds$, γ é a semi-circunferência $x^2 + y^2 = 16$, $x \geq 0$.

c) $\int_\gamma xyz\, ds$, $\gamma : x = 2t, y = 3\,\text{sen}\, t, z = 3\cos t$, $0 \leq t \leq \pi/2$.

d) $\int_\gamma xy^2 z\, ds$, γ é o segmento de reta de $(1, 0, 1)$ a $(0, 3, 6)$.

27.

a) Determine a massa de um arame cujo formato é o da curva $\gamma(t) = (2t, t^2, t^2)$, onde $0 \leq t \leq 1$, e a densidade de massa em cada ponto é $\delta(x, y, z) = x$.

b) Um cabo delgado é dobrado na forma de um semi-círculo $x^2 + y^2 = 4$, $x \geq 0$. Se a densidade é $\delta(x, y) = x^2$, determine a massa e o centro de massa do cabo.

c) Determine a massa e o centro de massa de um fio no espaço com o formato da hélice $x = 2\,\text{sen}\, t$, $y = 2\cos t$, $z = 3t$, $0 \leq t \leq 2\pi$, se a densidade é uma constante k.

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