

Resolução da parte 3 e 5 - Luta de cálculo 2

$$3.1) f(x,y) = x^2 + 4y^2$$

$$\text{Tensão } \nabla F(x,y) = \left(\frac{\partial F}{\partial x}(x,y); \frac{\partial F}{\partial y}(x,y) \right)$$

$$\nabla f(2,1) = (4, 8)$$

$$\text{s.t.: } (4,8) \cdot (x-2; y-1) = 0$$

$$4x - 8 + 8y - 8 = 0$$

$$x + 8y - 16 = 0$$

$$3.2) \text{ Sendo } f(x,y) = x^3 + 3xy + y^3 + 3x$$

$$g(x,y) = x^2 + xy + y^2$$

$$\text{Determinar } R \Rightarrow \nabla f(x_0, y_0) \cdot (x-x_0; y-y_0) = 0$$

$$\nabla f(1,2) = (3+6+3; 3+12) = (12, 15)$$

$$(12, 15) \cdot (x-1; y-2) = 0$$

$$12x - 12 + 15y - 30 = 0$$

$$\text{para que } x^2 + xy + y^2 = 7 \quad 4x + 5y - 14 = 0$$

$$\nabla g(1,2) = (4, 5)$$

$$(4, 5) \cdot (x-1, y-2) = 0$$

$$4x - 4 + 5y - 10 = 0$$

$$\text{Uma reta } 4 \cdot (x-1) + 5 \cdot (y-2) = 0$$

Outra:

$$4 \cdot (x+1) + 5 \cdot (y+2) = 0$$

NUNES

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3.8-) Sendo o plano π' : $z = F(x_0, y_0) + \frac{\partial F}{\partial x}(x_0, y_0) + \frac{\partial F}{\partial y}(x_0, y_0)$

$$z = -2x + 2y + 3$$

$$\therefore \frac{\partial F}{\partial x}(x_0, y_0) = -2 \quad \frac{\partial F}{\partial y}(x_0, y_0) = 2$$

Para que não possa ser curva de nível, o t a
ser determinado nenhuma dues $\in \mathbb{R}$

a) $\nabla f(x(t)) \cdot x'(t) = 0$ sendo t tal que $f(t) = (x_0, y_0)$
 $(-2, 2) \cdot (\frac{1}{t^2}, 1) = 0$

$$\frac{-2}{t^2} = -2 \rightarrow t = \pm 1$$

Pode

b) $(-2, 2) \cdot (t^4, -2t^2 + 3) = 0$

$$-2t^4 - 4t^2 + 6 = 0$$

$$t^4 + 2t^2 - 3 = 0$$

$$\Delta = 4 - 4 \cdot 1 \cdot -3 = 16$$

$$t^2 \Rightarrow -2 \pm 4 \quad \sqrt{t^2} = 1 \quad \therefore t = \pm 1 \quad \text{Pode}$$

$2 \quad (t^2)' = -3 \quad (\text{N}\ddot{\text{o}}\text{o})$

c) $(-2, 2) \cdot (2t, 8t^2 + 1) = 0$

$$-4t + 6t^2 + 2 = 0$$

$$3t^2 - 2t + 1 = 0$$

$$\Delta = 4 - 4 \cdot 3 \cdot 1 = -8 \quad \therefore t \notin \mathbb{R}, \text{ logo,}$$

não pode ser uma curva de nível.

3.4-)

a) $\gamma(t) = (\cotg(t); \sec^2(t))$

para que $\cotg(t) = 1$ e $\sec^2(t) = 2$
 $t = \pi/4$

$\gamma(t)$ contida em curva de nível c de f , nelo:

$f(\gamma(t)) = c$

↳ devolvendo: $\nabla f(\gamma(t)) \cdot \gamma'(t) = 0$

$\nabla f(1, 2) \cdot (-\cos t; 2 \sec^2 t, \tan t) = 0$

$\underbrace{(\frac{\partial f}{\partial x}(1, 2), \frac{\partial f}{\partial y}(1, 2))}_{\begin{matrix} a \\ b \end{matrix}} \cdot \underbrace{(-2, 4)}_{\begin{matrix} -2 \\ 4 \end{matrix}} = 0$

$-2a + 4b = 0$

$a = 2b$

ii) b) que $\delta'(v)$ este contida no gráfico de f , posterior afirmam que o vetor normal de f é perpendicular à tangente de $\delta'(v)$ (não devendo):

$(\frac{\partial f}{\partial x}(1, 2); \frac{\partial f}{\partial y}(1, 2); -1) \cdot \left(1; \frac{2v}{3\sqrt{v^2-1}}; \frac{3v^2-1}{3\sqrt{v^2-1}}\right)$

rendo $v=1$ (pono ter o ponto $(1, 2)$)

$(a; b; -1) \cdot \left(\frac{1}{3}; \frac{2}{3}; \frac{9}{6}\right)$

$\frac{a}{3} + \frac{2b}{3} - \frac{9}{6} = 0$

$a + 2b = 4$

$8b = 4 \rightarrow \boxed{b=1}$ $\boxed{a=1} \therefore \nabla f(1, 2) = \left(1; \frac{1}{2}\right)$

b) $\frac{\partial f}{\partial \vec{v}}(x_0, y_0) = \nabla f(x_0, y_0) \cdot \vec{v}$

$$\frac{\partial f}{\partial \vec{v}}(1, 2) = \left(1, \frac{1}{2}\right) \cdot \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = \frac{1}{2} + \frac{\sqrt{3}}{4} = \boxed{\frac{2+\sqrt{3}}{4}}$$

c) $\pi: z = \frac{(x_0, y_0)}{\partial x} + \frac{\partial f}{\partial x}(x_0, y_0) \cdot (x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0) \cdot (y - y_0)$

Sendo $(x_0, y_0) = (1, 2)$

Tema

$$z = 1 + x - 1 + \frac{y}{2} - 1$$

$$2z = 2x + y - 2$$

$$2x + y - 2 - 2z = 0$$

* $f(1, 2) = 1$, para p/ $v ='$, tem a curva $\sigma(t) = (t^2, t, 1)$

3.5-) Se $\nabla f(x, y)$ é tangente à $\gamma^*(t) = (t^2, t)$

em um ponto $p = \gamma(t_0)$; $\therefore \nabla f(x, y) \parallel \gamma'(t)$

\rightarrow Lembrando que $\gamma'(t)$ é tangente à $\gamma^*(t)$.

Logo, para $\exists \lambda \rightarrow \gamma(t_0) = (x_0, y_0)$

$$\frac{\partial f}{\partial x}(t_0^2, t_0) = \lambda \cdot 2t_0 \rightarrow 2t_0^2 = 2\lambda t_0 \quad \textcircled{I}$$

$$\lambda = t_0$$

$$\frac{\partial f}{\partial y}(t_0^2, t_0) = \lambda \cdot 1 \rightarrow 4t_0^3 = 2 \quad \textcircled{II}$$

\textcircled{I} em $\textcircled{II} \rightarrow 4t_0^3 = t_0$

$$\boxed{t_0 = \frac{1}{2}}$$

$$\nabla f\left(\frac{1}{4}, \frac{1}{2}\right) = \left(1, \frac{1}{2}\right) = \boxed{\left(\frac{1}{2}, \frac{1}{2}\right)}$$

$$X: \left(\frac{1}{4}, \frac{1}{2}\right) + \lambda \cdot (1, 1), \lambda \in \mathbb{R}$$

* Erro de cálculo

$$3.6) \quad \gamma(t) = (t, 2t^2, t^2)$$

$$\gamma'(t) = (1, 4t, 2t)$$

$\gamma^*(t)$ contida em $\{$ mo minc $c=4$ \therefore

$$\text{pt}, \quad \gamma'(t) = 0 \quad \} \text{ Normal } \square$$

então $(x,y) = (2,8)$ e $t=2$

$$\text{sendo } \frac{\partial f}{\partial x}(2,8) = a, \quad \frac{\partial f}{\partial y}(2,8) = b$$

$$(a, b, -1) \cdot (1, 8, 4) = 0$$

$$\textcircled{1} \quad a + 8b = 0$$

$$\textcircled{2} \quad z: (a, b) \cdot (x-2, y-8) = 0$$

sendo $(x,y) = (1,-4) \rightarrow$ pertence à se

$$(a, b) \cdot (-1, -12) = 0$$

$$-a - 12b = 0$$

$$a = -12b \textcircled{3}$$

$$\textcircled{4} \quad -12b + 8b = 0 \rightarrow -4b = 0$$

$$b = -1 \quad e \quad a = 12$$

$$\therefore \nabla f(2,8) = \underbrace{(12, -1)}$$

$$\mathcal{P}: z = f(2,8) + \frac{\partial f}{\partial x}(2,8) \cdot (x-2) + \frac{\partial f}{\partial y}(2,8) \cdot (y-8)$$

$$z = 4 + 12x - 24 - y + 8 - 4$$

$$z = \underbrace{12x - y - 12}$$

$$\textcircled{5} \quad f(2,8) = 4, \quad \text{pdm } \nabla f(2,8) = (12, -1)$$

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$$3.7-) f(\gamma(t)) = 1 \quad \vec{m} = \left(\frac{\partial F(x_0, y_0)}{\partial x}; \frac{\partial F(x_0, y_0)}{\partial y}; -1 \right)$$

Sabendo que γ contém $(1, 1, \frac{1}{2})$ e $(4, 1, 2)$, definir o vetor \vec{p} paralelo relativo $\vec{p} = (3, 0, \frac{3}{2})$

Já que \vec{m} é normal à γ : $\vec{m} \cdot \vec{p} = 0$

$$\text{Logo, } (a, b, -1) \cdot (3, 0, \frac{3}{2}) = 0$$

$$3a = \frac{3}{2} \rightarrow \boxed{a = \frac{1}{2}}$$

Além disso, por $\gamma'(t)$ serem um f :

$$f(\gamma(t)) = 1 \rightarrow \text{derivando: } \nabla f(x_0, y_0), \gamma'(t) = 1$$

$$(a, b) \cdot (-\frac{1}{t^{2}}, 1) = 0$$

$$b = \frac{a}{t^{2}} \rightarrow b = \frac{1}{2t^{2}}$$

Agora definindo $\bar{\sigma} = (4-x_0, 1-y_0, 1)$ Mícl 1

$$\hookrightarrow f(x_0, y_0) = 1$$

$$(a, b, -1) \cdot (4-x_0, 1-y_0, 1) = b \quad | -$$

$$4a - a \cdot x_0 + b - b \cdot y_0 - 1 = 0 \quad | t/t = t_0$$

$$2 - \frac{x_0}{2} + b - b \cdot y_0 - 1 = 0 \quad | \begin{matrix} y_0 = t_0 \\ x_0 = 1 + \frac{1}{t_0} \end{matrix}$$

Logo da modificação (e multiplicando por -1)

$$\frac{x_0}{2} - b + b \cdot y_0 - 1 = 0 \rightarrow \frac{t_0 + 1}{2} - \frac{1}{2t_0^2} + 1 - \frac{t_0}{2t_0^2} - 1 = 0$$

$$\frac{t_0 + 2}{2t_0} - \frac{1}{2t_0^2} = 1$$

$$(t_0+2) \cdot 2t_0^2 - \frac{2t_0^2}{2t_0} = 2t_0^2$$

$$4t_0 + 1 - t_0^2 + 2t_0 - 1 - 2t_0^2 = 0$$

$$-t_0^2 + 2t_0 - 1 = 0$$

$$t_0^2 - 2t_0 + 1 = 0$$

$$\Delta = 4 - 4 \cdot 1 = 0$$

$$(t_0+2) \cdot 2t_0^2 + 1 = 0$$

$$t_0 = \frac{2}{2} - \frac{1}{2} \rightarrow b = \frac{1}{2}, a = \frac{1}{2}$$

$$\mathcal{P}: Z = f(x_0, y_0) + a \cdot (x - x_0) + b \cdot (y - y_0)$$

$$\textcircled{*} \quad f(x_0, y_0) = 1 \quad x_0 = 1 + 1 = 2 \quad y_0 = 1$$

$$Z = 1 + \frac{x}{2} - 1 + \frac{y}{2} - \frac{1}{2}$$

$$2Z = 2 + x - 2 + y - 1$$

$$\mathcal{P}: \underbrace{x+y-2z=1}$$

3.8-) Continuidade:

$$\lim_{(x,y) \rightarrow (0,0)} \sqrt[3]{x^2y} = 0 \quad (\text{abre}) \quad \text{é contínua, por } \quad \bullet = f(0,0)$$

Se é contínua em $(0,0)$, tm todas as derivadas direcionais em $(0,0)$.

Diferenciabilidade em $(0,0)$

$$\lim_{(h,k) \rightarrow (0,0)} E(h,k) = 0$$

$$(h,k) \rightarrow (0,0) \quad \|E(h,k)\|$$

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$$E(h,k) = f(x_0+h, y_0+k) - F(x_0, y_0) - \frac{\partial f}{\partial x}(x_0, y_0)h - \frac{\partial f}{\partial y}(x_0, y_0)k$$

Anote:

$$\left\{ \begin{array}{l} \frac{\partial f}{\partial x}(0,0) = \lim_{R \rightarrow 0} \frac{f(x_0+R, y_0) - f(x_0, y_0)}{R} = 0 \\ \frac{\partial f}{\partial y}(0,0) = \lim_{R \rightarrow 0} \frac{f(x_0, y_0+R) - f(x_0, y_0)}{R} = 0 \end{array} \right.$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{R \rightarrow 0} \frac{f(x_0, y_0+R) - f(x_0, y_0)}{R} = 0$$

$$\lim_{(h,k) \rightarrow 0, d((h,k), (0,0))} \frac{f(h,k) - f(0,0)}{\sqrt{h^2+k^2}} = \lim_{(h,k) \rightarrow (0,0)} \frac{\sqrt[3]{h^2+k^2}}{\sqrt{h^2+k^2}}$$

Seja $\gamma_1(t) = (t, 0)$ uma curva paramétrica à f e

$\gamma_2(t) = (t, t)$ também.

Se o limite acima existir, então $\lim_{t \rightarrow 0} \gamma_1(t) = \lim_{t \rightarrow 0} \gamma_2(t)$

Vamos ver:

$$\lim_{t \rightarrow 0} \gamma_1(t) = \lim_{t \rightarrow 0} \frac{\sqrt[3]{t^2 \cdot 0}}{\sqrt{t^2}} = 0 = L_1$$

$$\lim_{t \rightarrow 0} \gamma_2(t) = \lim_{t \rightarrow 0} \frac{\sqrt[3]{t^3}}{\sqrt{2t^2}} = \lim_{t \rightarrow 0} \frac{t}{t\sqrt{2}} = \frac{\sqrt{2}}{2} = L_2$$

$L_1 \neq L_2 \therefore$ Não existe, i.e. não é diferenciável

* Lembrando que p/ ser diferenciável a "limite" tem que dar zero.

$$3.9) \text{ a) } \nabla f(x,y) = (e^{-x}; -x \cdot e^{-x} + 3)$$

$$\nabla f(1,0) = (1, 2) = \underline{\text{direção}}$$

o somando derivado direcional: $\frac{\partial F}{\partial \vec{v}}(x,y) = \nabla f(x,y) \cdot \vec{v}$

Máxima: $\vec{J} = \frac{\nabla f(x,y)}{\|\nabla f(x,y)\|}$

$$\vec{U} = \frac{(1,2)}{\sqrt{5}} = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$\frac{\partial F}{\partial \vec{v}}(1,0) = (1,2) \cdot \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) = \frac{1}{\sqrt{5}} + \frac{4}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

$$\text{direção} = (1,2) \quad \frac{\partial F}{\partial \vec{v}}(1,0) = \sqrt{5}$$

b) $f(x,y) = \ln(x^2 + y^2)$

$$\nabla f(x,y) = \left(\frac{2x}{x^2+y^2}; \frac{2y}{x^2+y^2}\right)$$

$$\nabla f(1,2) = \left(\frac{2}{5}; \frac{4}{5}\right)$$

$$\vec{v} = \left(\frac{2}{5}, \frac{4}{5}\right) \cdot \frac{1}{\sqrt{20}} \cdot 5 = \left(\frac{2}{2\sqrt{5}}, \frac{4}{2\sqrt{5}}\right) = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$\frac{\partial F}{\partial \vec{v}}(1,2) = \left(\frac{2}{5}, \frac{4}{5}\right) \cdot \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) = \frac{2}{5\sqrt{5}} + \frac{2}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$\text{direção} = \left(\frac{2}{5}, \frac{4}{5}\right) \quad \frac{\partial F}{\partial \vec{v}}(1,2) = \frac{2}{\sqrt{5}}$$

$$\rightarrow \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

3.10-) Dinego o é gradient.

$$\nabla f(x, y) = (2x - 2, 2y - 4) = (1, 1)$$

$$\begin{aligned} 2x - 2 &= 1 \\ 2y - 4 &= 1 \end{aligned} \quad \left\{ \begin{array}{l} 2x - 2 = 2y - 4 \\ \downarrow \end{array} \right.$$

$$\begin{aligned} R: 2x - 2y &= -2 \\ 1 + x - y &= 0 \end{aligned}$$

→ Tocar ou parar dessa reta

$$3.11-) f(t) = (t+1, -t^2)$$

$$\nabla f(-1, -4) = (2, \frac{\partial f(-1, -4)}{\partial y})$$

$$P(-1, -4) \quad t = -2$$

$$f(\gamma(t)) = c \Rightarrow \nabla f(-1, -4) \cdot \gamma'(-2) = 0$$

$$(2, a) \cdot (1, 4) = 0$$

$$2 + 4a = 0$$

$$a = -\frac{1}{2}$$

$$\frac{\partial f(-1, -4)}{\partial y} = (2, -\frac{1}{2}), (\frac{3}{5}, \frac{4}{5})$$

$$\frac{\partial f(-1, -4)}{\partial y} = \frac{6}{5} - \frac{2}{5} = \boxed{\frac{4}{5}}$$

3.12-) I enunciado nos põe que $\nabla^*(\mathbf{r})$ é
 $\int_{\Omega} \nabla^*(\mathbf{v}) \cdot \nabla f \, d\sigma$, logo
 $\int_{\Omega} \nabla^*(\mathbf{v}) \cdot \nabla f \, d\sigma = 0$

$$\begin{aligned} f(\mathbf{x}(\mathbf{r})) &= c \Rightarrow \vec{n} \cdot \nabla f(\mathbf{r}) = 0 \quad \text{I} \\ f(\mathbf{x}(\mathbf{r})) &= c \Rightarrow \vec{n} \cdot \nabla f(\mathbf{r}) = 0 \quad \text{II} \end{aligned} \quad \left. \begin{array}{l} \text{Vira } \vec{n} \\ \text{para } \\ \text{em espaço} \end{array} \right.$$

$$\frac{\partial}{\partial t} \left(\frac{1}{2}, -\frac{1}{2} \right) \rightarrow t = 1 \quad \therefore U = -\frac{1}{2}$$

$$\nabla f\left(\frac{1}{2}, -\frac{1}{2}\right) = \left(\underbrace{\frac{\partial f}{\partial x}\left(\frac{1}{2}, -\frac{1}{2}\right)}_a, \underbrace{\frac{\partial f}{\partial y}\left(\frac{1}{2}, -\frac{1}{2}\right)}_b \right)$$

$$\text{I} \quad (a, b, -1) \cdot \left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) = 0$$

$$\frac{-a+b}{2} - \frac{1}{2} = 0 \rightarrow b-a = 1 \quad b = a+1$$

$$\text{II} \quad (a, b, -1) \cdot \left(1, 1, -\frac{1}{U^2} \right) = 0$$

$$a+b - 1 + \frac{1}{U^2} = 0$$

$$\begin{cases} a+b = -3 & \text{II} \\ b-a = 1 & \text{I} \end{cases}$$

$$2b = -4$$

$$\boxed{b = -2} \quad \boxed{a = -1}$$

$$\frac{\partial f}{\partial U}\left(\frac{1}{2}, -\frac{1}{2}\right) = (-1, -2) \cdot \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) = -\frac{\sqrt{2}}{2} - \frac{2\sqrt{2}}{2} = \boxed{-\frac{3\sqrt{2}}{2}}$$