

Questão 1

Turma A:

a) Decompondo em frações parciais obtemos $\frac{4x^2 + 13x + 16}{x(x^2 + 4x + 8)} = \frac{2}{x} + \frac{2x + 5}{x^2 + 4x + 8}$. Logo

$$\begin{aligned}\int \frac{4x^2 + 13x + 16}{x^3 + 4x^2 + 8x} dx &= \int \frac{4x^2 + 13x + 16}{x(x^2 + 4x + 8)} dx \\ &= \int \frac{2}{x} dx + \int \frac{2x + 5}{x^2 + 4x + 8} dx \\ &= \ln(x^2) + \int \frac{2x + 5}{x^2 + 4x + 8} dx \\ &= \ln(x^2) + \int \frac{2x + 4}{x^2 + 4x + 8} dx + \int \frac{1}{x^2 + 4x + 8} dx \\ &= \ln(x^2) + \ln(x^2 + 4x + 8) + \int \frac{1}{4 + (x + 2)^2} dx\end{aligned}$$

A última integral é resolvida via substituição $x + 2 = 2 \tan(\theta)$

$$\int \frac{1}{4 + (x + 2)^2} dx = \frac{1}{2} \int d\theta = \frac{1}{2} \arctan\left(\frac{x + 2}{2}\right) + c$$

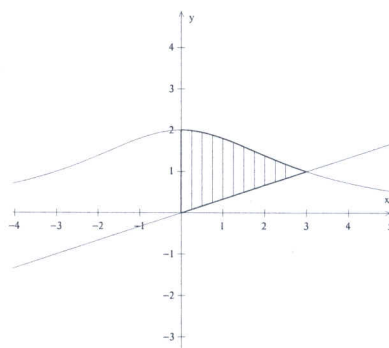
A solução é então

$$\int \frac{4x^2 + 13x + 16}{x^3 + 4x^2 + 8x} dx = \ln(x^2) + \ln(x^2 + 4x + 8) + \frac{1}{2} \arctan\left(\frac{x + 2}{2}\right) + c$$

b) Integrando por partes com $f = \ln(\cos(x))$ e $g' = \sec^2(x)$ temos

$$\begin{aligned}\int \sec^2(x) \ln(\cos(x)) dx &= \ln(\cos(x)) \tan(x) + \int \tan^2(x) dx \\ &= \ln(\cos(x)) \tan(x) + \int (\sec^2(x) - 1) dx \\ &= \ln(\cos(x)) \tan(x) + \tan(x) - x + c\end{aligned}$$

Questão 2. Seja R a região compreendida entre os gráficos de $f(x) = \frac{18}{9+x^2}$ e $g(x) = \frac{x}{3}$ para $x \in [0, 3]$, como mostra a figura abaixo:



(2,0) a) Calcule o volume do sólido obtido pela rotação de R em torno do eixo Ox .

(1,5) b) Calcule o volume do sólido obtido pela rotação de R em torno do eixo Oy .

$$a) V_x = V_1 - V_2$$

$$\begin{aligned}
 V_1 &= \int_0^3 \pi \left(\frac{18}{9+x^2} \right)^2 dx = 18^2 \pi \int_0^3 \frac{1}{(9+x^2)^2} dx \quad x = 3 \tan t \\
 &= 18^2 \pi \int_0^{\pi/4} \frac{3 \sec^2 t}{(9+9 \tan^2 t)^2} dt = 12 \pi \int_0^{\pi/4} \frac{1}{\sec^2 t} dt = \\
 &= 12 \pi \int_0^{\pi/4} \cos^2 t dt = 12 \pi \int_0^{\pi/4} \left(\frac{1}{2} + \frac{1}{2} \cos(2t) \right) dt = \\
 &= 12 \pi \left[\frac{1}{2} \frac{\pi}{4} + \frac{1}{2} \frac{\sin(2t)}{2} \right]_0^{\pi/4} = 12 \pi \left(\frac{\pi}{8} + \frac{1}{4} \right) \\
 &= \frac{3}{2} \pi^2 + 3 \pi \\
 V_2 &= \int_0^3 \pi \left(\frac{x}{3} \right)^2 dx = \frac{\pi}{9} \frac{x^3}{3} \Big|_0^3 = \pi \\
 V_x &= \frac{3}{2} \pi^2 + 3 \pi - \pi = \frac{3}{2} \pi^2 + 2 \pi
 \end{aligned}$$

$$b) y = \frac{x}{3} \iff x = 3y$$

$$y = \frac{18}{9+x^2}, x \geq 0 \iff 9+x^2 = \frac{18}{y} \quad x \geq 0 \iff$$

$$x = \sqrt{\frac{18}{y} - 9}$$

$$V_y = V_3 + V_4$$

$$V_3 = \int_0^1 \pi (3y)^2 dy = 9\pi \frac{y^3}{3} \Big|_0^1 = 3\pi$$

$$V_4 = \int_1^2 \pi \left(\sqrt{\frac{18}{y} - 9} \right)^2 dy = \pi \int_1^2 \left(\frac{18}{y} - 9 \right) dy =$$

$$= \pi \left(18 \ln y \Big|_1^2 - 9y \Big|_1^2 \right) = \pi (18 \ln 2 - 9)$$

$$V_y = 3\pi + \pi(18 \ln 2 - 9) = \pi (18 \ln 2 - 6) = 6\pi (3 \ln 2 - 1)$$

Questão 3.

I) Sejam $f(x) = \sqrt[5]{x}$ e $x_0 \neq 0$.

$$p_3(x) = x_0^{\frac{1}{5}} + \frac{1}{5}x_0^{-\frac{4}{5}}(x - x_0) - \frac{4}{5^2 \cdot 2}x_0^{-\frac{9}{5}}(x - x_0)^2 + \frac{36}{5^3 \cdot 3!}x_0^{-\frac{14}{5}}(x - x_0)^3$$

$$E(x) = -\frac{21}{5^4}\bar{x}^{-\frac{19}{5}}(x - x_0)^4$$

(1,0) b) Usando o item (a) para $x_0 = 32$, encontre um valor aproximado para $\sqrt[5]{34}$ e decida se o erro, em módulo, é inferior a $\frac{1}{5^2 \cdot 2^{15}}$.

$$p_3(x) = 2 + \frac{1}{5 \cdot 2^4}(x - 32) - \frac{1}{5^2 \cdot 2^8}(x - 32)^2 + \frac{3}{5^3 \cdot 2^{13}}(x - 32)^3$$

$$\sqrt[5]{34} \approx p_3(34) = 2 + \frac{1}{5 \cdot 2^4}(34 - 32) - \frac{1}{5^2 \cdot 2^8}(34 - 32)^2 + \frac{3}{5^3 \cdot 2^{13}}(34 - 32)^3$$

$$\sqrt[5]{34} \approx 2 + \frac{1}{40} - \frac{1}{1600} + \frac{3}{128000} = 2 + 0,025 - 0,000625 + 0,0000234 = 2,0243984.$$

$$E(x) = -\frac{21}{5^4 \bar{x}^{\frac{19}{5}}} \cdot (x - 32)^4 \Rightarrow E(34) = -\frac{21}{5^4 \bar{x}^{\frac{19}{5}}} \cdot (34 - 32)^4 = -\frac{21}{25 \cdot 5^2 \bar{x}^{\frac{19}{5}}} \cdot 2^4$$

Como $\bar{x} > 32$, pois $32 < \bar{x} < 34$, temos que $\frac{1}{\bar{x}} < \frac{1}{32} = \frac{1}{2^5}$.

$$\text{Portanto, } |E(34)| = \left| -\frac{21}{25 \cdot 5^2 \bar{x}^{\frac{19}{5}}} \cdot 2^4 \right| = \frac{21}{25} \cdot \frac{1}{5^2} \cdot \frac{1}{\bar{x}^{\frac{19}{5}}} \cdot 2^4 < \frac{21}{25} \cdot \frac{1}{5^2} \cdot \frac{2^4}{2^{19}} < \frac{1}{5^2 \cdot 2^{15}}.$$

(1,5) II) Seja $F(x) = \int_{\sin x}^{\cos x} e^{2x+t^2} dt$. Calcule $F'(x)$ e determine $F'(\frac{\pi}{4})$.

$$F'(x) = 2e^{2x} \cdot \int_{\sin x}^{\cos x} e^{t^2} dt + e^{2x} [e^{\cos^2 x} \cdot (-\sin x) - e^{\sin^2 x} \cdot \cos x]$$

$$F'(\frac{\pi}{4}) = 2 \cdot 0 + e^{\frac{\pi}{2}} \left[e^{\frac{1}{2}} \cdot \left(-\frac{\sqrt{2}}{2}\right) - e^{\frac{1}{2}} \cdot \frac{\sqrt{2}}{2} \right] = -e^{\frac{\pi}{2} + \frac{1}{2}} \sqrt{2}$$