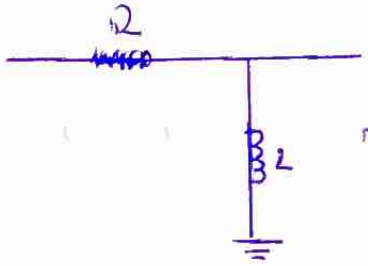
Notas de Eletrônica Analógica (P3)

Paulo Akira

2018.2



$$\Rightarrow \frac{\hat{V}_s}{\hat{V}_e} = \frac{j\omega}{j\omega + R/L}$$

→ Diagrama de Bode:

- Ganho K:

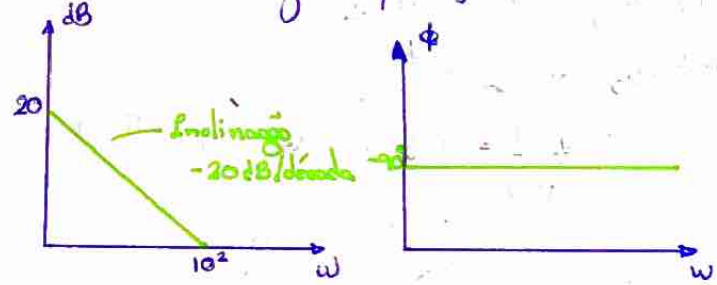
$G(j\omega) = K$

$$|K|_{dB} = 20 \log |K| \quad \begin{cases} \angle G(j\omega) = 0^\circ, & k > 0 \\ \angle G(j\omega) = 180^\circ, & k < 0 \end{cases}$$

- Fator Integral

$$G(j\omega) = \frac{1}{j\omega}$$

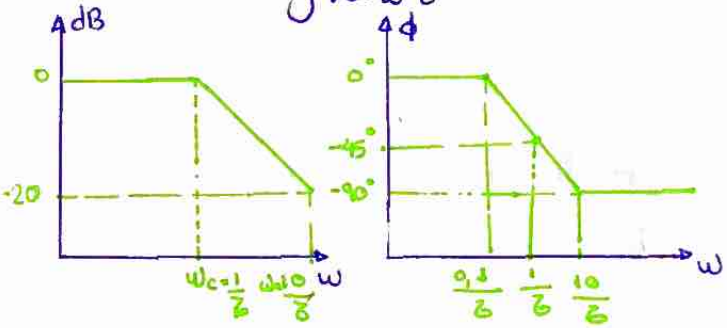
$$|G(j\omega)|_{dB} = -20 \log \omega \quad \& \quad \angle G(j\omega) = -90^\circ$$



- Fator de Primeira Ordem:

$$G(j\omega) = \frac{1}{1 + j\omega\tau}$$

$$|G(j\omega)|_{dB} = -20 \log \sqrt{1 + \omega^2\tau^2}$$

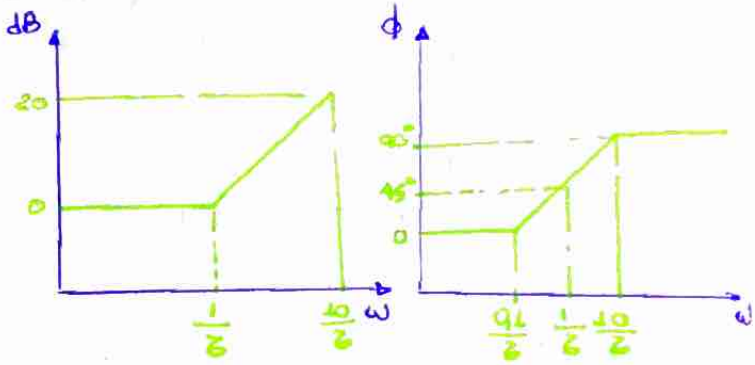


- Fatores Zeros de Primeira Ordem.

$$G(j\omega) = 1 + j\omega\tau$$

$$|G(j\omega)|_{dB} = 20 \log \sqrt{1 + \omega^2\tau^2}$$

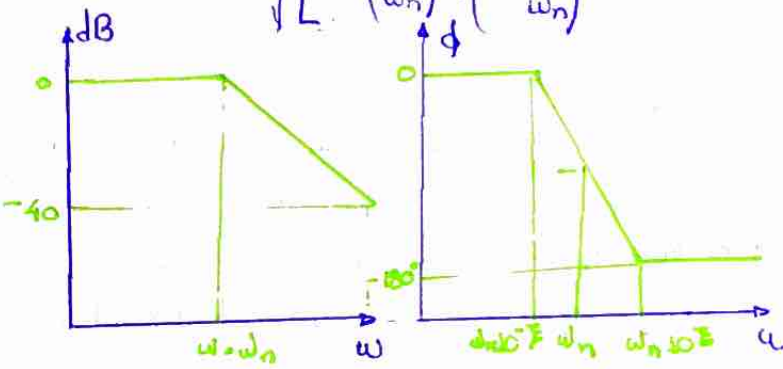
$$\angle G(j\omega) = \angle(1 + j\omega\tau) = \operatorname{atg} \omega\tau$$



- Fatores Polos Quadráticos:

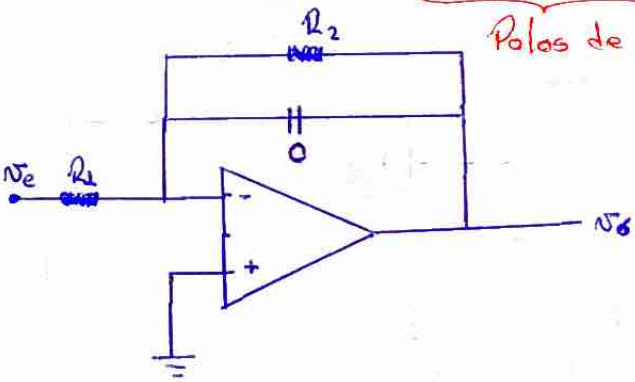
$$G(j\omega) = \frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\xi\frac{j\omega}{\omega_n} + 1}$$

$$|G(j\omega)|_{dB} = -20 \log \sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}$$



→ Filtros Passa Baixa de Primeira Ordem

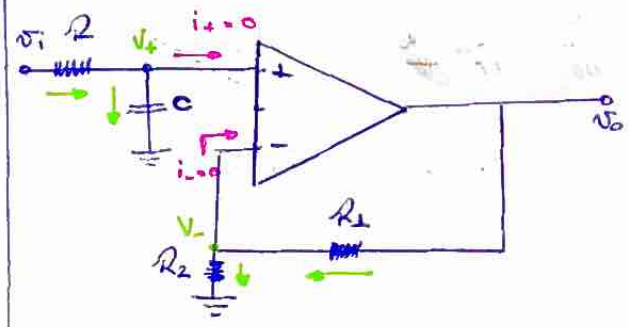
Polos de 1ª Ordem



$$Z_{eq} = \frac{R_2}{1 + j\omega R_2 C}$$

$$\frac{\hat{V}_e}{R_1} = \frac{-\hat{V}_s}{Z_{eq}} \Rightarrow \frac{\hat{V}_s}{\hat{V}_e} = -\frac{Z_{eq}}{R_1} = -\frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2 C}$$

Exercício:



$$\frac{\hat{V}_i - \hat{V}_+}{R} = j\omega C\,\hat{V}_+ \Rightarrow \hat{V}_i - \hat{V}_+ = j\omega RC\,\hat{V}_+ \Rightarrow$$

$$\Rightarrow \hat{V}_+(1+j\omega RC) = \hat{V}_i \Rightarrow \hat{V}_+ = \frac{\hat{V}_i}{1+j\omega RC}$$

$$\frac{\hat{V}_o - \hat{V}_-}{R_1} = \frac{\hat{V}_-}{R_2} \Rightarrow \hat{V}_o - \hat{V}_- = \frac{R_1}{R_2}\hat{V}_- \Rightarrow$$

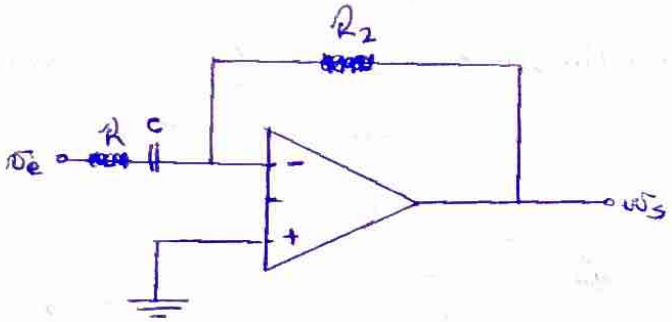
$$\Rightarrow \hat{V}_o = \left(1+\frac{R_1}{R_2}\right)\hat{V}_- \Rightarrow \hat{V}_- = \frac{\hat{V}_o R_2}{R_1+R_2}$$

$$\hat{V}_+ = \hat{V}_- \Rightarrow \frac{\hat{V}_i}{1+j\omega RC} = \frac{\hat{V}_o R_2}{R_1+R_2} \Rightarrow$$

$$\Rightarrow \frac{\hat{V}_o}{\hat{V}_i} = \underbrace{\frac{R_1+R_2}{R_2}\,\frac{1}{1+j\omega RC}}_{\text{Filtro Passa Baixa.}}$$

→ Filtro Passa Alta de Primeira Ordem

Zeros de 1ª Ordem



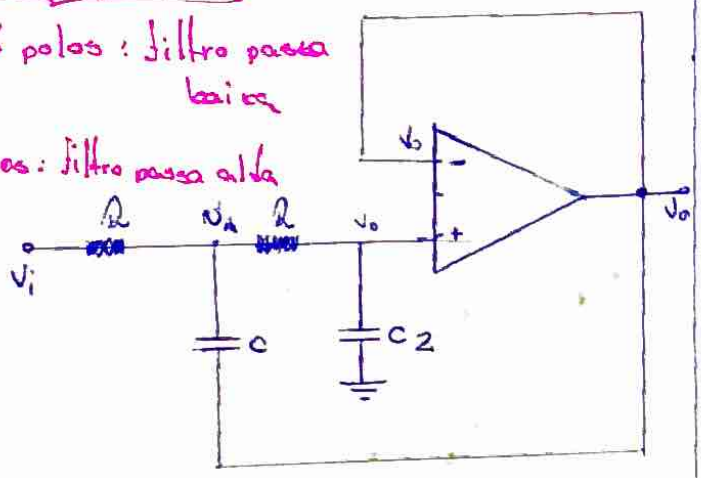
$$Z_{eq} = R_1 + \frac{1}{j\omega C}$$

$$\frac{\hat{V}_e}{Z_{eq}} = \frac{-\hat{V}_s}{R_2} \Rightarrow \frac{\hat{V}_o}{\hat{V}_e} = -R_2 \cdot \frac{1}{R_1 + \frac{1}{j\omega C}}$$

→ Filtro de 2ª Ordem

→ só polos : filtro passa baixa

→ zeros : filtro passa alta



$$\frac{V_i - V_A}{R} = \frac{V_A - V_o}{R} + (V_A - V_o)C\,s \quad (1)$$

$$\frac{V_A - V_o}{R} = V_o C\,s \Rightarrow V_A = V_o(1+RC_2 s) \quad (2)$$

De (1):

$$\frac{V_i}{R} = V_A\left(\frac{2}{R} + Cs\right) - V_o\left(\frac{1}{R} + C_1 s\right) \quad (3)$$

De (2) em (3):

$$\frac{V_i}{R} = V_o(1+RC_2 s)\left(\frac{2}{R} + Cs\right) - V_o\left(\frac{1}{R} + Cs\right)$$

$$\Rightarrow V_i = V_o\left[1 + 2RC_2 s + R^2 C C_2 s^2\right] \Rightarrow$$

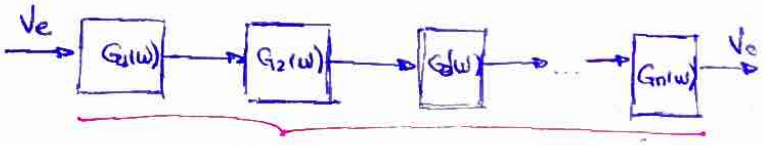
$$\Rightarrow \frac{V_o}{V_i} = \underbrace{\frac{1}{R^2 C C_2 s^2 + 2RC_2 s + 1}}_{\text{2 polos .. filtro passa baixa.}}$$

XI) Projeto de Filtros:

→ Função de Transferência:

$$\frac{V_o(j\omega)}{V_e(j\omega)} = \underbrace{G(j\omega)}_{\substack{P(j\omega) \rightsquigarrow \text{polos} \\ Q(j\omega) \rightsquigarrow \text{zeros}}} = \underbrace{|G(j\omega)|}_{\text{amplitude}}\,\underbrace{e^{j\phi(\omega)}}_{\text{fase}}$$

Para fazer filtros de ordem maior podemos cascatear filtros de ordem menor!



$$G_1 \cdot G_2 \cdot G_3 \cdots G_n$$

Os circuitos devem estar desacoplados!

→ Projeto de Filtro de 1ª Ordem.

$$G(j\omega) = k_o \cdot \frac{\omega_o}{j\omega + \omega_o} = k_o\omega_o \frac{1}{\omega^2 + \omega_o^2}\cdot(\omega_o - j\omega)$$

$$|G(j\omega)| = \frac{k_o\omega_o}{\sqrt{\omega^2+\omega_o^2}} \therefore |G_{dB}(j\omega)| =$$

$$= 20\log(k_o) - 20\log\left[\left(\frac{\omega}{\omega_o}\right)^2 + 1\right]^{1/2} =$$

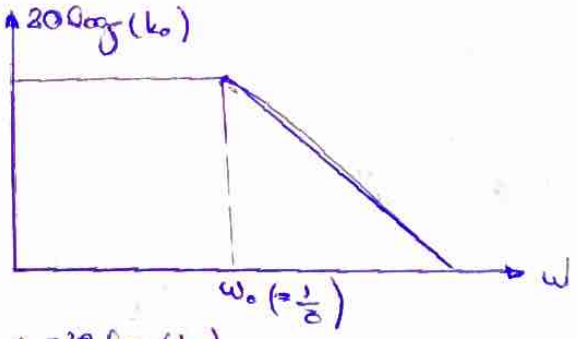
$$\simeq 20\log(k_o) - 20\log\left(\frac{\omega}{\omega_o}\right)$$

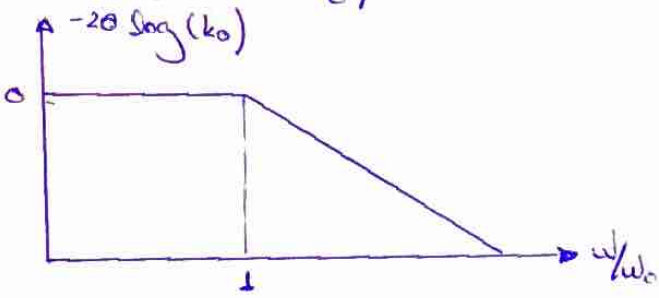
$\omega >> \omega_o$:

$|G(j\omega)|_{dB} \cong 20\log(k_0) - 20\log(\omega) + 20\log(\omega_0)$

$|G(j\omega)|_{dB} \cong 20\log(k_0\omega_0) - 20\log(\omega)$

$\phi = \operatorname{arctg}\left(-\frac{\omega}{\omega_0}\right)$





→ Projeto de Filtro de 2ª Ordem

$$G(j\omega) = k_0 \cdot \frac{\omega_0^2}{(j\omega)^2 + \alpha\omega_0 j\omega + \omega_0^2} \rightarrow$$

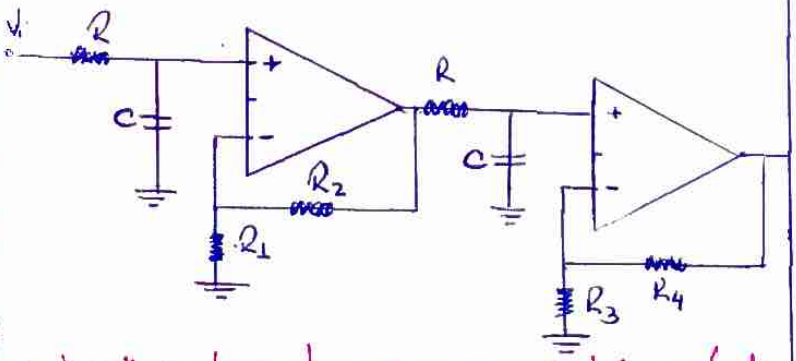
$$\rightarrow G(j\omega) = \frac{k_0}{\left(j\frac{\omega}{\omega_0}\right)^2 + \alpha j\left(\frac{\omega}{\omega_0}\right) + 1}$$

$$P_{1,2} = \frac{-\alpha\omega_0 \pm \sqrt{(\alpha\omega_0)^2 - 4\omega_0^2}}{2} = \omega_0 \cdot \frac{-\alpha \pm \sqrt{\alpha^2 - 4}}{2}$$

a) $\alpha = 2$, $P_1 = P_2 = -\omega_0$

$$G(j\omega) = \frac{k_0\omega_0^2}{(j\omega+\omega_0)(j\omega+\omega_0)} = \underbrace{\frac{\sqrt{k_0}}{1 + j\left(\frac{\omega}{\omega_0}\right)} \cdot \frac{\sqrt{k_0}}{1 + j\left(\frac{\omega}{\omega_0}\right)}}$$

Podemos implementar como dois filtros de 1ª ordem associados.



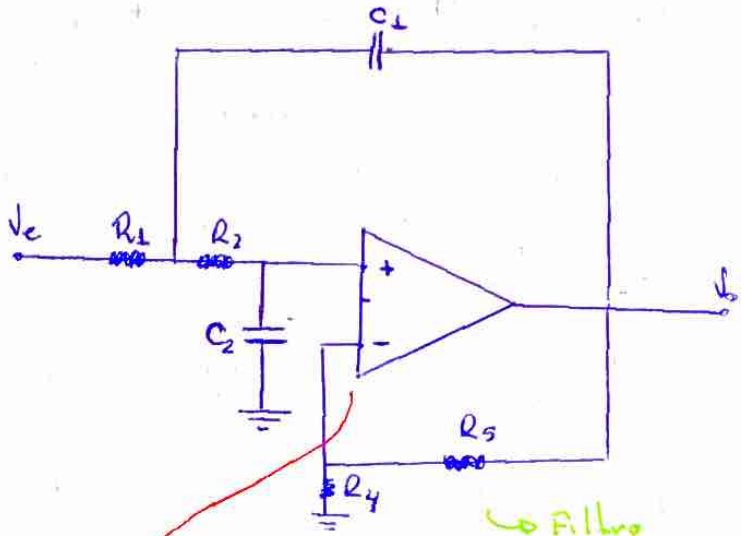
A frequência de corte ocorre quando há queda de 3dB ∴ quando a amplitude cai a $A/\sqrt{2}$

$\omega_0 = \frac{1}{RC}$

b) Filtro Sallen-Key

Tipo de filtro de 2ª Ordem usado para implementar filtro passa-baixa ou passa-alta

Principal vantagem é que sua implementação é simples.

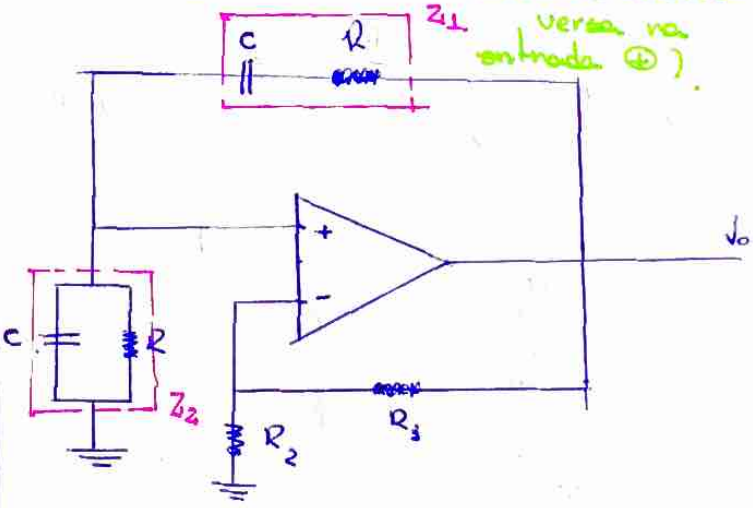


Amplificador não-inversor!

↳ Filtro passa-baixa (p/ implementar um filtro passa-alta basta trocar as resistências por capacitores e vice-versa na entrada ⊕).

XII) Tópicos Adicionais:

→ O oscilador de Wien



$$Z_1 = R + \frac{1}{j\omega C}$$

$$Z_2 = \frac{R \cdot 1/j\omega C}{R + \frac{1}{j\omega C}} = \frac{\frac{R}{j\omega C}}{\frac{j\omega RC}{j\omega C} + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC}$$

$$V_+ = \frac{Z_2}{Z_1 + Z_2} V_0 = \frac{\frac{R}{1+j\omega RC}}{\left(R + \frac{1}{j\omega C}\right) + \left(\frac{R}{1+j\omega RC}\right)} \cdot V_0$$

Simplificando: $V_+ = \dfrac{1}{3 + \left(\omega RC - \frac{1}{\omega RC}\right)j} V_2$

Quando não há defasagem:

V_2 é real!

$$\omega RC - \frac{1}{\omega RC} = 0 \Rightarrow \omega = \frac{1}{RC}$$

Para uma onda senoidal: $\omega = 2\pi f$

$$2\pi f = \frac{1}{RC} \Rightarrow \boxed{f = \frac{1}{2\pi RC}}$$

Nestas condições: $\underbrace{V_+ = \frac{1}{3} V_1 \Leftrightarrow V_0 = 3V_+}_{\text{Ganho de 3}}$

Quem impõe o ganho são os resistores R_1 e R_2

Para manter a oscilação!

$\begin{cases} > 3 \Rightarrow \text{sinal distorce} \\ < 3 \Rightarrow \text{não há oscilação (não satura)} \end{cases}$

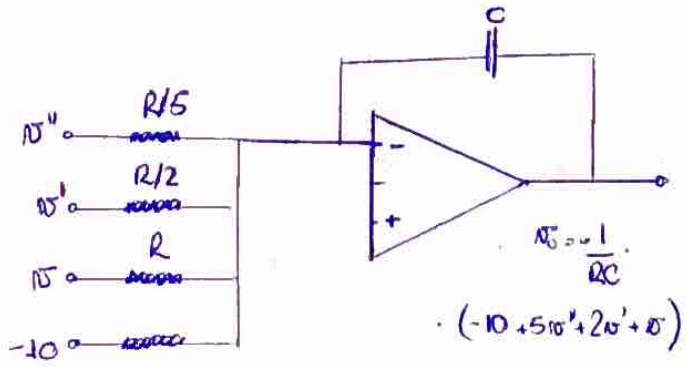
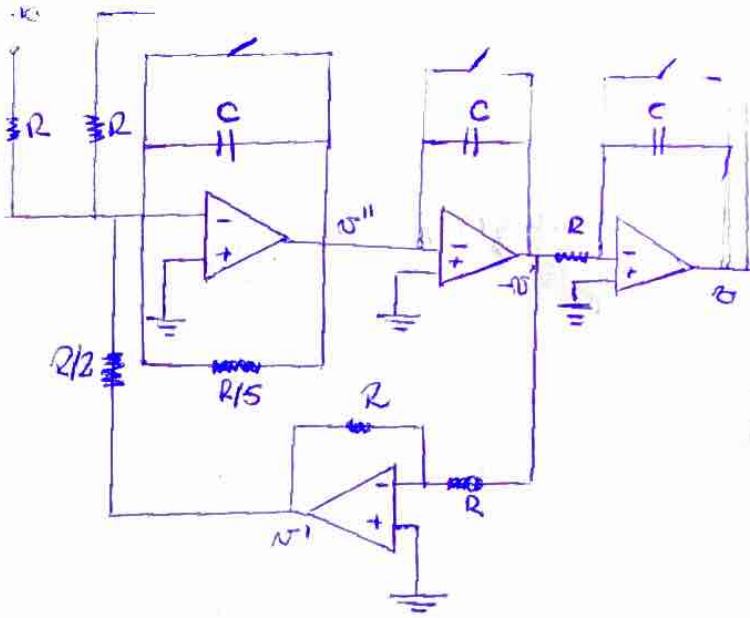
$$G = 1 + \frac{R_1}{R_2} \Rightarrow 3 = 1 + \frac{R_1}{R_2} \Rightarrow \frac{R_1}{R_2} = 2 \Rightarrow R_1 = 2R_2$$

A grande vantagem dos osciladores de Wien é que eles produzem sinais com poucos ruídos.

→ Resolução de EDOs com Amp Ops:

$$\frac{d^3v}{dt^3} + 5\frac{d^2v}{dt^2} + 2\frac{dv}{dt} + v - 10 = 0$$

$$v'' = \int v'''\,dt = -\int(-10 + 5v'' + 2v' + v)\,dt$$



Com essa montagem, devemos cascatear outros Amp Ops para integrar este resultado. Note que necessitaremos de realimentação.