

Notas de Sistemas Dinâmicos
II (P2)

Paulo Akira

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As aproximações assintóticas para as curvas de resposta em frequência não são precisas para um valor com baixos valores de ω .

Frequência e Pico de ressonância:

Quando ocorrer a ressonância teremos a máxima dissipação de energia, ou seja, a maior intensidade.

$$|G(j\omega)| = \frac{1}{\sqrt{\left(\frac{1-\omega^2}{\omega n^2}\right)^2 + \left(\frac{2\xi\omega}{\omega n}\right)^2}} = g(\omega)$$

teremos ressonância quando tivermos o maior $|G(j\omega)|$, ou seja, a maior intensidade. ($g(\omega)$ ser mínimo)

$$\frac{dg(\omega)}{d\omega} = 0 \rightarrow \omega r = \omega n \sqrt{1-2\xi^2}, \text{ para } 0 < \xi < \frac{\sqrt{2}}{2} = 0,707$$

O módulo e a fase do pico são:

$$M_r = |G(j\omega_r)| = \frac{1}{2\xi\sqrt{1-\xi^2}} \quad |M_r|_{dB} = |G(j\omega_r)|_{dB} = -20 \log 2\xi\sqrt{1-\xi^2}$$

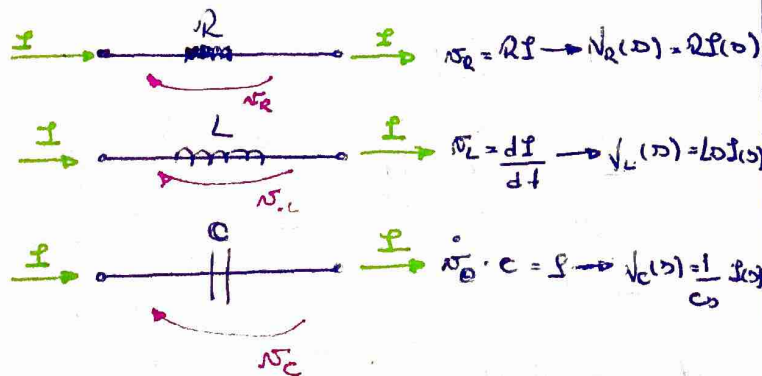
$$\angle G(j\omega_r) = -\arctan\left(\frac{\sqrt{1-2\xi^2}}{\xi}\right) = -90^\circ + \tan^{-1}\frac{\xi}{\sqrt{1-\xi^2}}$$

Fator zeros quadráticos:

Curvas de módulo e fase para zeros quadráticos podem ser obtidos invertendo-se o sinal das curvas de módulo e fase dos polos quadráticos. As principais diferenças são que os picos de ressonância são para baixo e as curvas de fase vão de 0° a 180° .

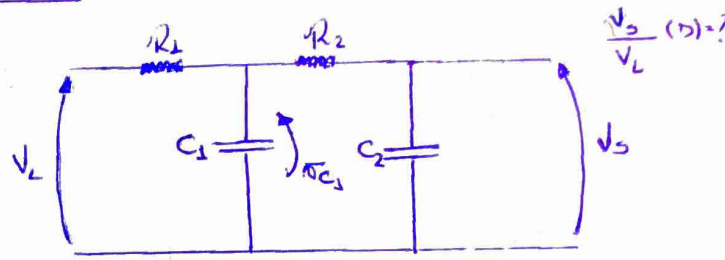
IV) Modelagem de Sistemas Elétricos COMEÇA AQUI!

Método de Impedâncias Complexas (Revisão)

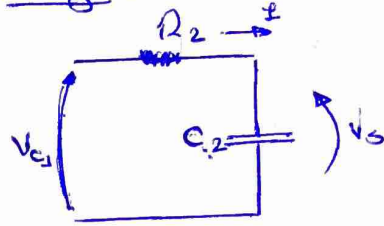


$N(s) = G(s) \cdot J(s)$ \rightarrow vale para quanto as condições iniciais são nulas! $\rightarrow R$ ou L ou $\frac{1}{Cs}$ (def. de função de transferência)

Exemplo:

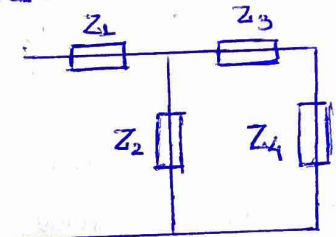


Solução:



$$\frac{V_{C1}(s)}{V_s} = \frac{1}{R_2 + \frac{1}{C_2 s}} = \frac{1}{R_2 C_2 s + 1} \Rightarrow V_s(s) = \frac{V_{C1}(s)}{R_2 C_2 s + 1}$$

V_C1(s):



$$Z_{eq} = \frac{Z_2(Z_3 + Z_4)}{Z_2 + Z_3 + Z_4} = \frac{1/C_1 s (R_2 + 1/C_2 s)}{1/C_1 s + R_2 + 1/C_2 s} = \frac{C_2 R_2 s + 1}{R_2 C_1 C_2 s^2 + (C_1 + C_2)s}$$

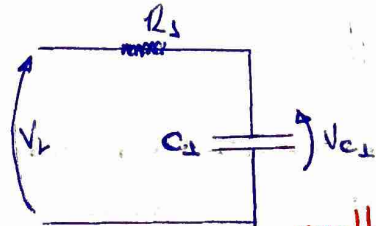
$$V_{C1}(s) = V_L(s) \cdot \frac{Z_{eq}(s)}{Z_1(s) + Z_{eq}(s)} = V_L(s) \cdot \frac{1}{R_2 R_1 C_1 C_2 s^2 + R_1 (C_1 + C_2)s + R_2 C_2 s + 1} \quad (2)$$

(2) \rightarrow (1):

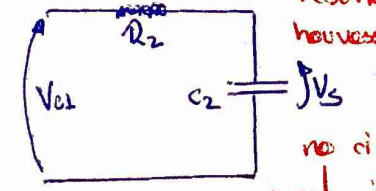
$$\frac{V_s}{V_L}(s) = \frac{1}{R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + R_2 C_2 s + R_1 C_2 s + 1}$$

$$\frac{V_s}{V_L}(s) = \frac{1}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_1 C_2 s}$$

$\neq G_1(s) G_2(s)$ \rightarrow Acoplamento (comportamento de um circuito é influenciado pelo circuito acoplado a ele)

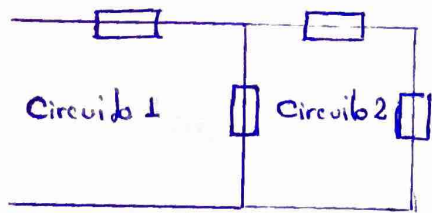


$$\frac{V_{C1}}{V_L} = \frac{1}{(R_1 C_1 s + 1)} = G_1(s)$$



A função de transferência resultante seria $G_1 \cdot G_2$ se não houvesse dissipação de potência no circuito 2, ou seja, sua impedância teria de ser infinita. $\frac{V_s}{V_{C1}} = \frac{1}{R_2 C_2 s + 1} = G_2(s)$

Quanto maior a impedância do circuito 2, maior é a corrente drenada do circuito 1 ($I_{C1} \downarrow, V_{C1} \downarrow$).
 Por outro lado, quanto maior impedância do circuito 2 face àquela do circuito 1, mais próximos estaremos da tensão de saída do capacitor 1 igual a tensão do circuito R_1, C_1 isolado.



Impedância do Circuito 2:

$$Z(s) = R_2 + \frac{1}{C_2 s}$$

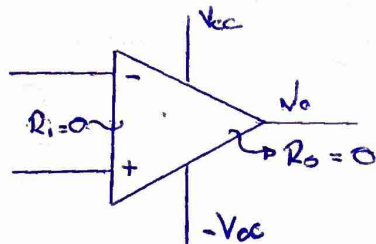
$R_1 = cte, C_2 \uparrow$

$R_2 \gg R_1 \Rightarrow$ acoplamento pequeno

$R_2 \downarrow \Rightarrow$ acoplamento aumenta

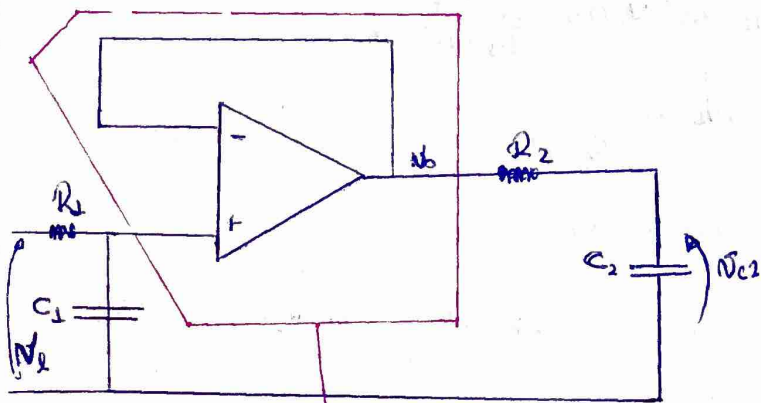
Circuitos acoplados em cascata (saída do circuito 1 é a entrada do circuito 2)

No amplificador operacional:

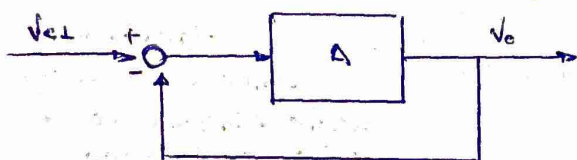


$$V_o = A(V_+ - V_-)$$

A muito grande



Amp Op seguidor



O amplificador operacional pode servir como um elemento de desacoplamento.

Resolvendo o diagrama de blocos: não adota o circuito 2

$$\frac{V_o}{V_{ei}}(s) = \frac{A}{1 + A}$$

$A \gg 1: \frac{V_o}{V_{ei}}(s) \approx 1$ } A saída do Amp Op não depende do que vem depois.

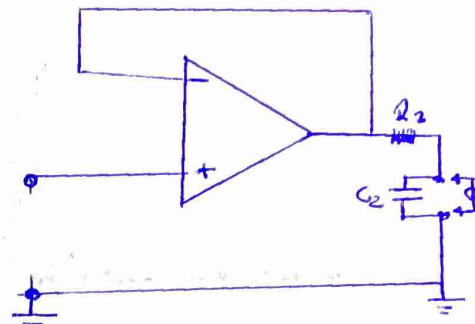
$$V_{e1}(s) = V_L(s) \cdot \frac{1}{R_1 C_1 s + 1} = G_1(s) \cdot V_L(s)$$

$$V_o(s) = V_{e2}(s) = V_o(s) \cdot \frac{1}{R_2 C_2 s + 1} = G_2(s) V_o(s) = G_2(s) V_{e1}(s) \Rightarrow V_o(s) = V_L(s) G_1(s) G_2(s)$$

$$G(s) = G_1(s) \cdot G_2(s)$$

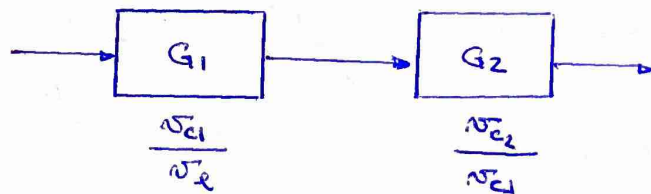
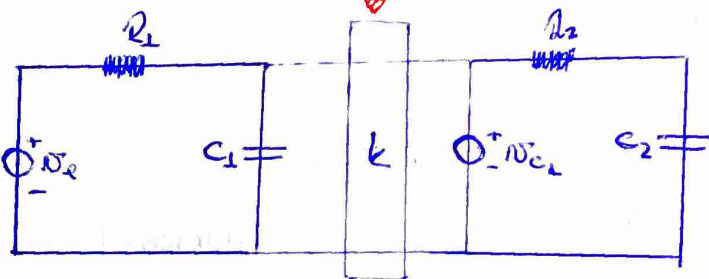
Aplicação:

Sensor de Medição de Tensão!



Amplificador desacoplado!

Concluímos que:



Lembrando: Amp Op ideal

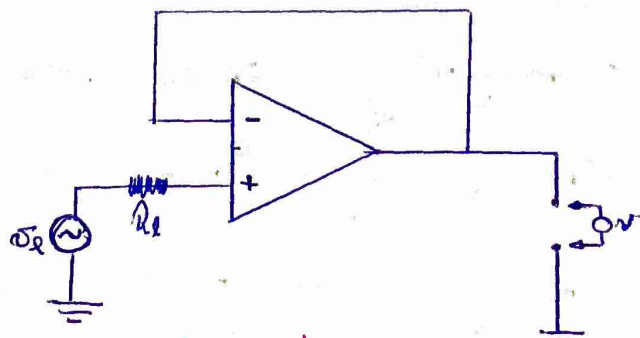
$$\Rightarrow V_o = A(V_+ - V_-)$$

$$\Rightarrow i_+ = i_- = 0$$

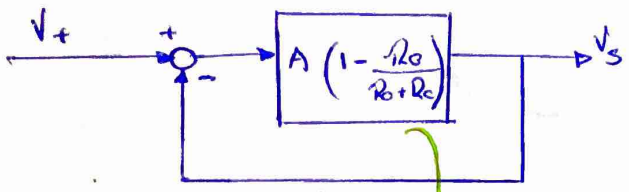
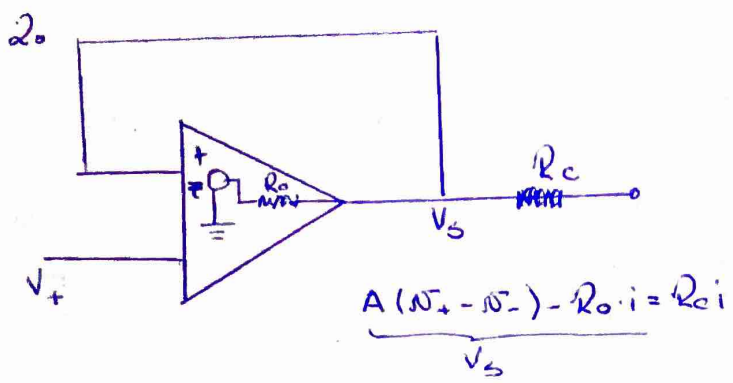
$$\Rightarrow A = cte \gg 1$$

Influência das resistências de entrada e saída

1.



$V = V_{ei} - R_e \cdot i$ } Quando maior $R_e \Rightarrow i \downarrow \Rightarrow V \rightarrow V_{ei}$ (14)

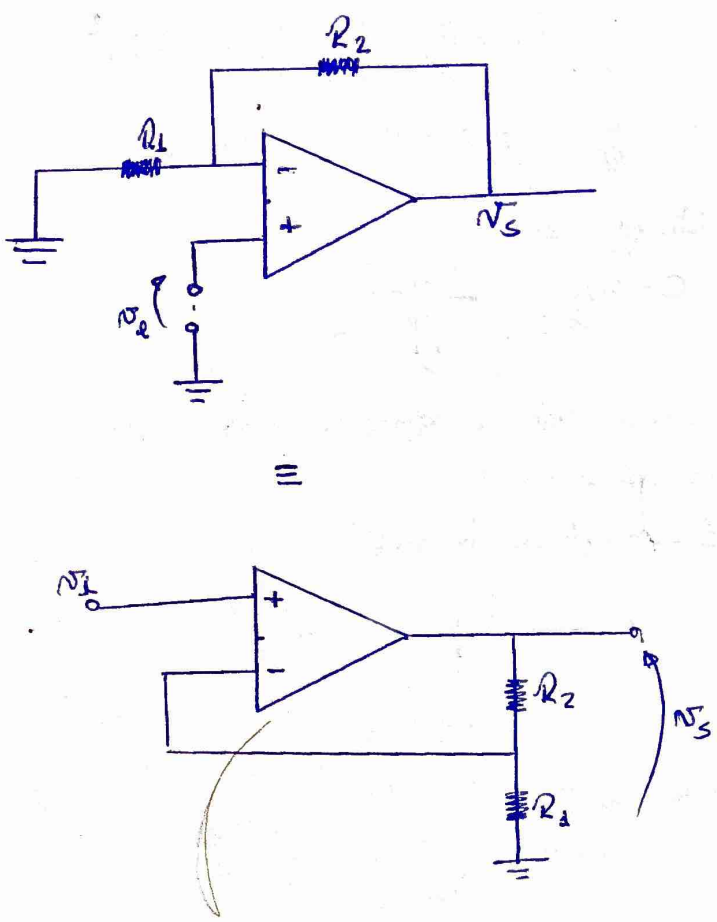


$R_0 \downarrow \rightarrow V_s \rightarrow A(V_+ - V_-)$

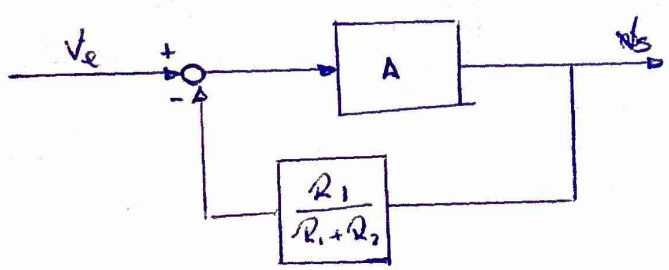
$V_+ - V_- = \frac{R_0 + R_c}{A}$

$\Rightarrow V_s = A R_c \frac{V_+ - V_-}{R_0 + R_c}$

Exemplo: circuito não-inversor



Lei de Ohm: $\frac{V_-}{R_1} = \frac{V_s}{R_1 + R_2} \Rightarrow V_- = V_s \cdot \frac{R_1}{R_1 + R_2}$



$$\frac{V_s}{V_e} = 1 + \frac{R_2}{R_1}$$

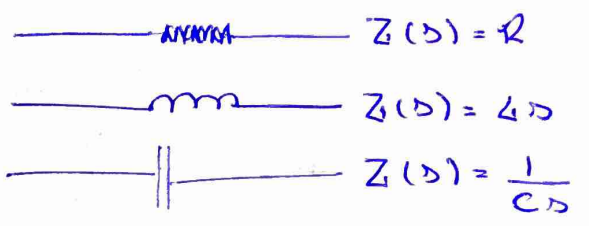
Sabemos que esta é a relação característica do amplificador não-inversor

$$\frac{V_s}{V_e} = \frac{A}{1 + \frac{A R_1}{R_1 + R_2}} = \frac{1}{\frac{1}{A} + \frac{R_1}{R_1 + R_2}} = 1 + \frac{R_2}{R_1}$$

$0, A \rightarrow \infty$

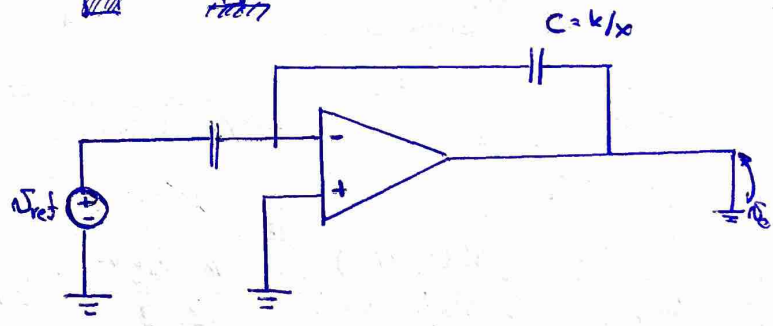
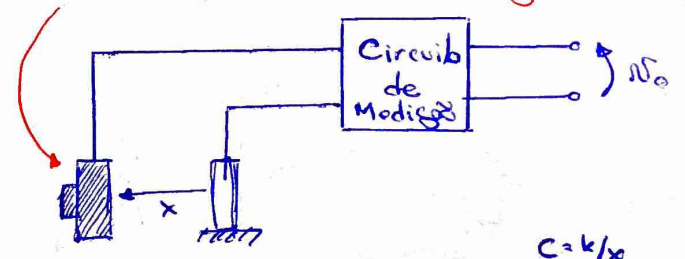
Combinando-se estes resultados com o conceito de impedâncias complexas, podemos construir outras operações com circuitos baseados no

Amp - Op.



Outra aplicação: linearização:

Sensor Capacitivo de Translação



$$V_s(s) = -\frac{1/C \cdot s}{1/C \cdot \text{ref} \cdot s} \cdot V_{\text{ref}}(s)$$

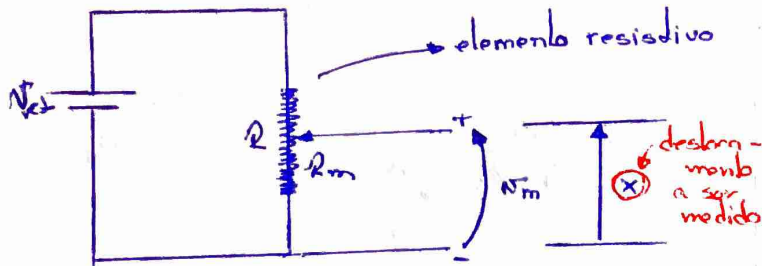
Para V_{ref} é diagrama de amplitude

$$V_s(s) = -\frac{V_{\text{ref}} \cdot C_{\text{ref}}}{k} \cdot \mathcal{L}(s)$$

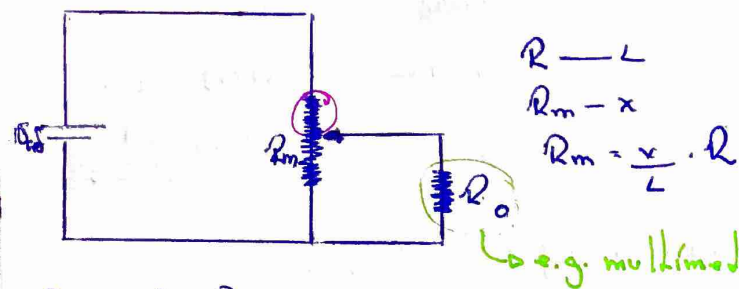
Explicação do Sensor Capacitivo a seguir!

→ Sensores de deslocamento linear e angular

• Potenciômetros



Na prática, temos o eixo de carga (o circuito de medida de tensão acoplado ao potenciômetro possui impedância finita).



$$R_{eq} = \frac{R_m \cdot R_0}{R_m + R_0} = \frac{R \frac{x}{L} \cdot R_0}{R \frac{x}{L} + R_0}$$

Lei de Ohm:

$$\frac{U_{fm}}{R_{eq}} = \frac{U_{ref}}{R(1-\frac{x}{L}) + R_{eq}} \Rightarrow U_{fm} = U_{ref} \left[\frac{R_{eq}}{R(1-\frac{x}{L}) + R_{eq}} \right] =$$

$$= U_{ref} \frac{1}{\frac{R(1-\frac{x}{L})}{R_{eq}} + 1} \quad (1)$$

$$\frac{R(1-\frac{x}{L})}{R_{eq}} = \frac{R(1-\frac{x}{L})}{\frac{R \frac{x}{L} \cdot R_0}{R \frac{x}{L} + R_0}} = \frac{(\frac{x}{L}) R + R_0}{\frac{x}{L} R_0} (1-\frac{x}{L})$$

$$= \left(\frac{R}{R_0} + \frac{L}{x} \right) \left(1-\frac{x}{L} \right) \quad (2)$$

Portanto,

$$U_{fm} = U_{ref} \cdot \frac{1}{\left(\frac{R}{R_0} + \frac{L}{x} \right) \left(1-\frac{x}{L} \right) + 1} = U_{ref} \frac{1}{\frac{R}{R_0} - \frac{R}{R_0} \frac{x}{L} + \frac{L}{x}}$$

$$U_{fm} = U_{ref} \cdot \frac{x}{L} \cdot \frac{1}{\left[\frac{R}{R_0} \left(\frac{x}{L} \right) \left(1-\frac{x}{L} \right) + 1 \right]}$$

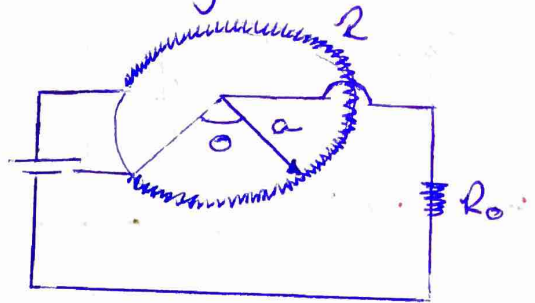
$$= U_{ref} \cdot \frac{x}{L} \left[\frac{R}{R_0} \left(\frac{x}{L} \right) \left(1-\frac{x}{L} \right) + 1 \right]^{-1}$$

que gostaríamos? $U_{fm} = kx$, $k = cte$

Para $\frac{R}{R_0} \rightarrow 0 \Rightarrow U_{fm} \rightarrow U_{ref} \frac{x}{L} = kx$

Portanto, é importante utilizarmos um circuito de medida com alta impedância. Se a impedância do sensor $\approx R_0$ alto!

• Deslocamento angular:



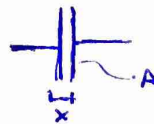
$$U_{fm} = U_{ref} \cdot \frac{a\theta}{a\theta_{max}} \left[1 + \frac{R}{R_0} \left(\frac{a\theta}{a\theta_{max}} \right) \left(1 - \frac{a\theta}{a\theta_{max}} \right) \right]^{-1}$$

$$= U_{ref} \frac{\theta}{\theta_{max}} \left[1 + \frac{R}{R_0} \left(\frac{\theta}{\theta_{max}} \right) \left(1 - \frac{\theta}{\theta_{max}} \right) \right]^{-1}$$

Para $\frac{R}{R_0} \rightarrow 0 \Rightarrow U_{fm} = k\theta$

• Sensor Capacitivo

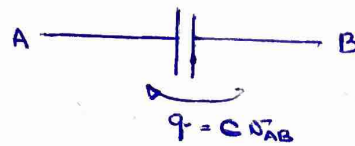
$$C = \frac{\epsilon A}{x}$$



A = área comum, sobreposta entre as placas

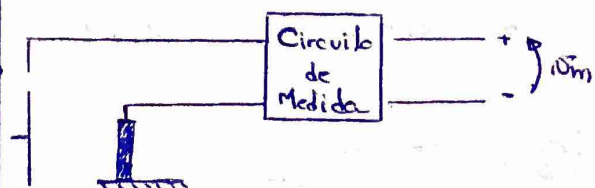
x = espessura

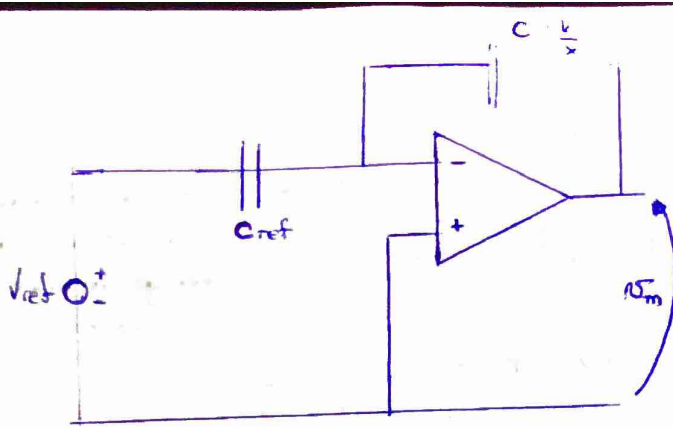
ε = constante do dielétrico



Para U_{AB} constante e C variável:

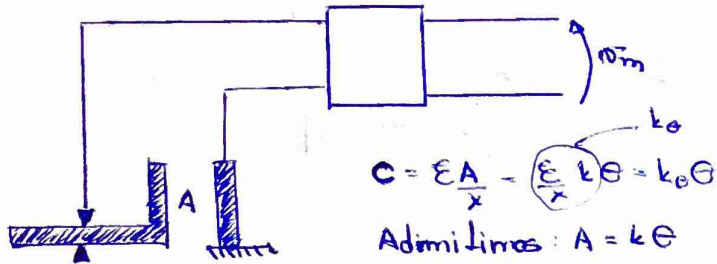
$$i = U_{AB} \cdot \frac{dC}{dt}$$





$$v_m = - \frac{C_{ref} \cdot v_{ref}}{k} \cdot x = k \cdot x$$

Deslocamento angular:



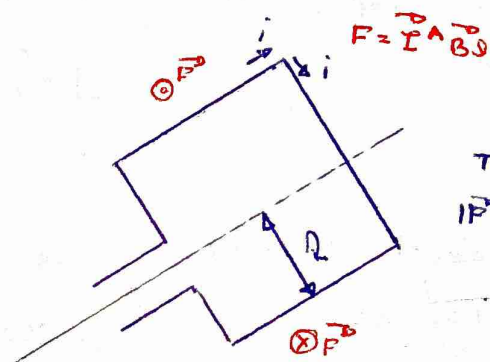
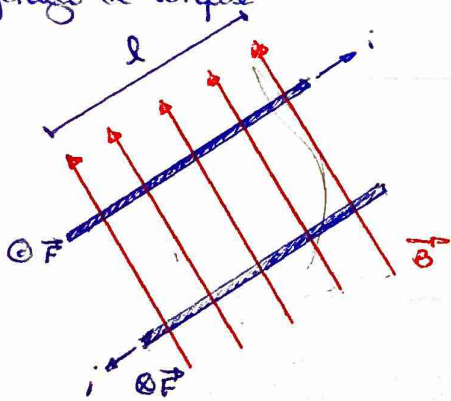
Podemos medir a velocidade angular:

$$i = v_{ref} \cdot \frac{dC}{dt} = v_{ref} k_e \frac{d\Theta}{dt} \Rightarrow \frac{d\Theta}{dt} = \frac{i}{k}$$

IV) Modelagem de Sistemas Eletromecânicos

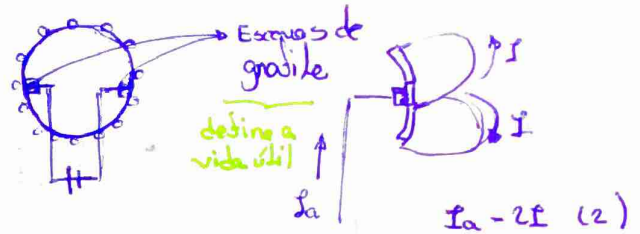
Motor CC com escovas de imã permanente:

geração de torque:

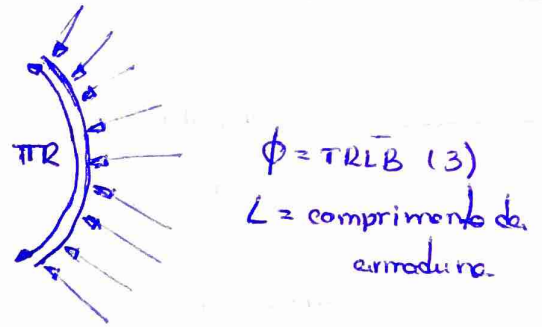


Para N condutores: $T = NR Bi l$ (1)

Corrente na armadura (conjunto de condutores)



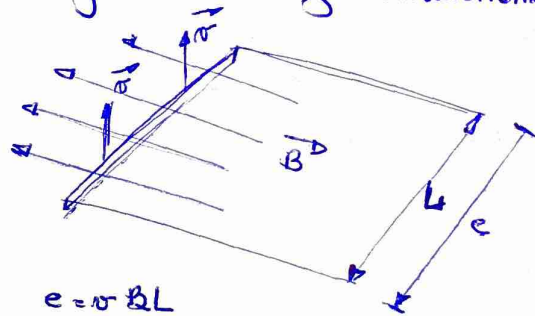
Fluxo na armadura



(2), (3) em (1)

$$T = \frac{N \phi}{2\pi} I_a = k_t I_a \Rightarrow T = k_t I_a$$

Surgimento da força contraeletromotriz



$e = v B l$
 (Lei de Lenz)

onde $v = \Omega R$

Por construção, para N condutores, temos $\frac{N}{2}$ arranjos em série de condutores e a força contraeletromotriz resultante é:

$$e = B R L \frac{N}{2} \Omega = \frac{B \pi R L N}{2\pi} \Omega = \frac{\phi N}{2\pi} \Omega$$

→ Motores elétricos em robótica

Desvantagens: alta rotação e baixo torque

Vantagens: alta exatidão, movimento contínuo, relações lineares facilitam controle.

Falores de seleção:

- fonte de alimentação
- condições ambientais

- consumo e manutenção
- controlabilidade

Partes do motor:

- Rotor
- Estator → Produção de Fluxo Magnético
- Sistema de Campo
- Armadura → produção de corrente

Obs: condutor em contato com escovas.

Motor de Passo: convertem pulsos elétricos em movimentos mecânicos que geram variações angulares discretas.

↳ conecta direto em sistemas digitais e pode trabalhar em malha aberta.

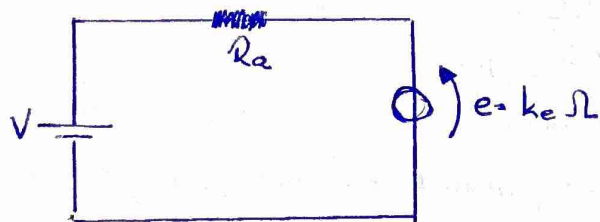
↳ não possui escovas, ⇒ imã permanente no rotor

Motor CC com Imã Permanente de Escovas:

Relações Fundamentais: $T = k_T \cdot I_a$, para unidades
 $e = k_e \cdot \Omega$
 coerentes: $k_T = k_e$.

Análise Estática:

$V = cte, \Omega = cte$

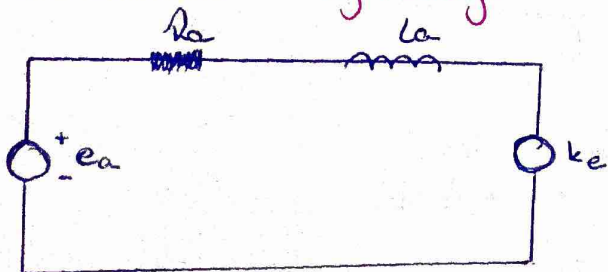


$$V = R_a I_a + k_e \Omega \Rightarrow I_a = \frac{V - k_e \Omega}{R_a}$$

O torque pode ser expresso por:

$$T = k_T \cdot I_a = \frac{k_T V}{R_a} - \frac{k_T k_e \Omega}{R_a} = K - K_m \Omega$$

Análise Dinâmica: *Liga / Desliga*

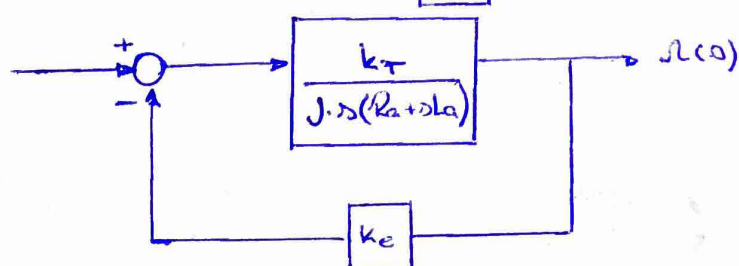
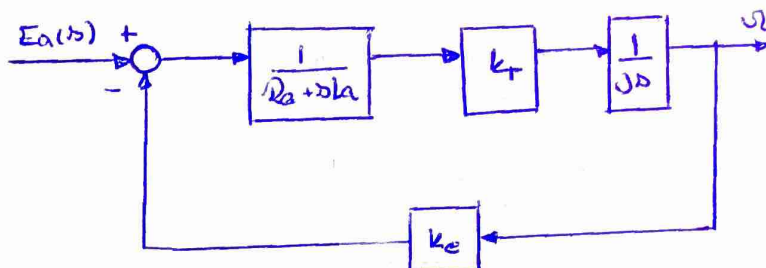


$$E_a(s) = R_a \cdot I_a(s) + L_a \cdot s \cdot I_a(s) + k_e \Omega(s) \Rightarrow$$

$$\Rightarrow I_a = \frac{E_a - k_e \Omega}{R_a + L_a \cdot s}$$

$T = k_T I_a(s) = \frac{k_T}{J \cdot s} \cdot \Omega(s)$ Momento de inércia da carga em relação ao eixo do motor.

Logo, $k_T \cdot \frac{E_a - k_e \Omega}{R_a + L_a \cdot s} \cdot \frac{1}{J \cdot s} = \Omega$



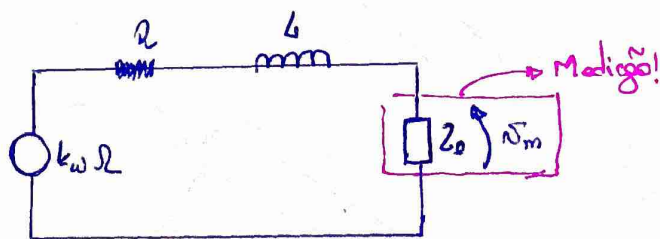
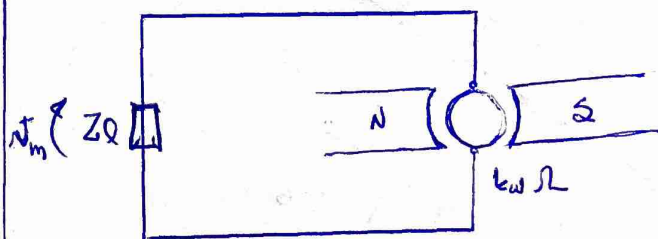
$$G(s) = \frac{k_T}{J \cdot s (L_a \cdot s + R_a) + k_e k_T}$$

Sistema de 1ª Ordem

Para $L=0$: $G(s) = \frac{k_T}{J \cdot s (R_a) + k_e k_T} = \frac{k}{T_s \cdot s + 1}$

$K = 1/k_e$ e $T = \frac{J R_a}{k_T k_e}$

→ Sensor de velocidade

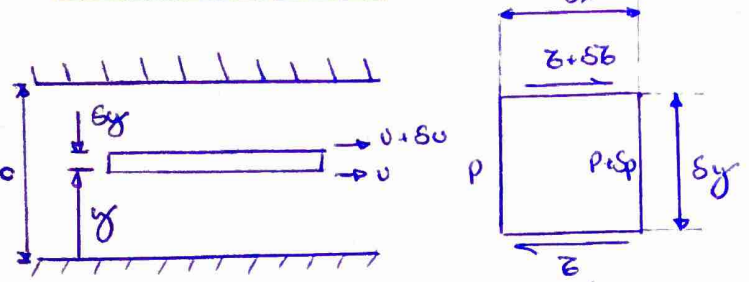


$$e = k_w \Omega(t) = R \cdot i(t) + L \frac{di(t)}{dt} + Z_0 \cdot i(t)$$

$$k_w \Omega(s) = R I(s) + L \cdot s I(s) + Z_0(s) I(s) =$$

$$= \left(\frac{R}{Z_0} + \frac{L \cdot s}{Z_0} + 1 \right) \Omega_m(s) \Rightarrow \Omega_m(s) = \frac{k_w \Omega(s)}{\left(\frac{R}{Z_0} + \frac{L \cdot s}{Z_0} + 1 \right)}$$

V) Sistemas Hidráulicos



$$F_R = [\rho - (\rho + \rho_p)] \delta y \delta z + [(z + \delta z) - z] \delta x \delta y$$

Em RP, $F_R = 0$:

$$-\rho_p \delta y \delta z + \delta z \delta x \delta y = 0 \Rightarrow$$

$$\Rightarrow -\rho_p \delta y + \delta z \delta x = 0 \Rightarrow$$

$$\Rightarrow \frac{\partial p}{\partial x} = \frac{\partial z}{\partial y}$$

Aplicando a Lei de Tensão de Cisalhamento:

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \Rightarrow$$

$$\Rightarrow \frac{\partial p}{\partial x} y = \mu \frac{\partial u}{\partial y} + C_1 \Rightarrow$$

$$\Rightarrow \left(\frac{\partial p}{\partial x} \right) \frac{y^2}{2} = \mu u + C_1 y + C_2$$

cc: $u = 0$ para $y = 0 \Rightarrow C_2 = 0$

$$u = V$$
 para $y = c \Rightarrow C_1 = \left(\frac{\partial p}{\partial x} \right) \frac{c}{2} - \frac{\mu V}{c}$

$$u = \left(\frac{\partial p}{\partial x} \right) \frac{y^2}{2\mu} - \frac{y}{\mu} \left[\left(\frac{\partial p}{\partial x} \right) \frac{c}{2} - \frac{\mu V}{c} \right] =$$

$$= \left(\frac{\partial p}{\partial x} \right) \frac{1}{2\mu} (y^2 - cy) + \frac{yV}{c}$$

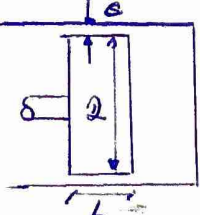
Calculo da vazão:

$$Q = \int_0^c u b dy = b \left[\frac{\partial p}{\partial x} \cdot \frac{1}{2\mu} \left(\frac{y^3}{3} - \frac{cy^2}{2} \right) + \frac{y^2 V}{2c} \right]_0^c =$$

$$= b \left[- \left(\frac{\partial p}{\partial x} \right) \frac{c^3}{12\mu} + \frac{Vc}{2} \right]$$

Amortecedor: $b = \pi D$

$$Q = - \frac{\pi D^2}{4} \cdot v_p = \pi D \left[- \left(\frac{\partial p}{\partial x} \right) \frac{c^3}{12\mu} + \frac{Vc}{2} \right]$$



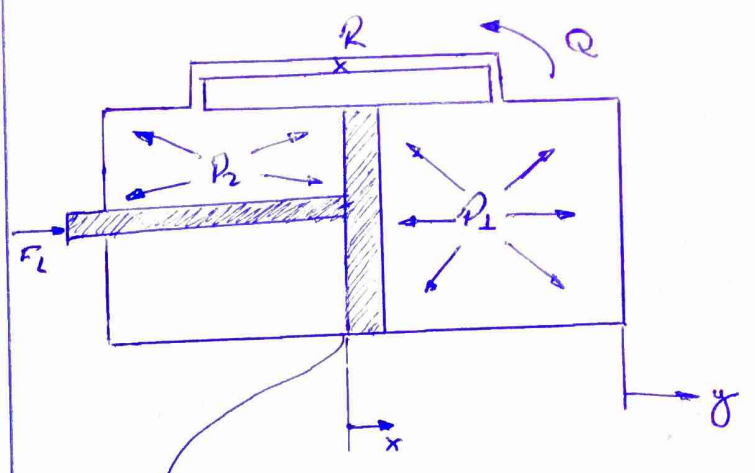
é T uniforme \Rightarrow var. de pressão de.

$$-\frac{\partial p}{\partial x} = \frac{\Delta p}{L} \therefore \pi D \left[- \frac{\Delta p}{L} \frac{c^3}{12\mu} + \frac{Vc}{2} \right] = - \frac{\pi D^2}{4} v_p \Rightarrow$$

$$\Delta p = Q \frac{\mu L 12}{\pi D} \left(\frac{1 - 2c}{D} \right)$$

$\Delta p \cdot A_p = F_L \rightarrow 2^a$ Lei (desprezar cisalhamento)

\rightarrow Amortecimento viscoso.



$A =$ área transversal do pistão.

Inércia do pistão desprezível

$$① F_L = A(p_1 - p_2) \quad (2^a \text{ Lei de Newton})$$

Equação da Continuidade

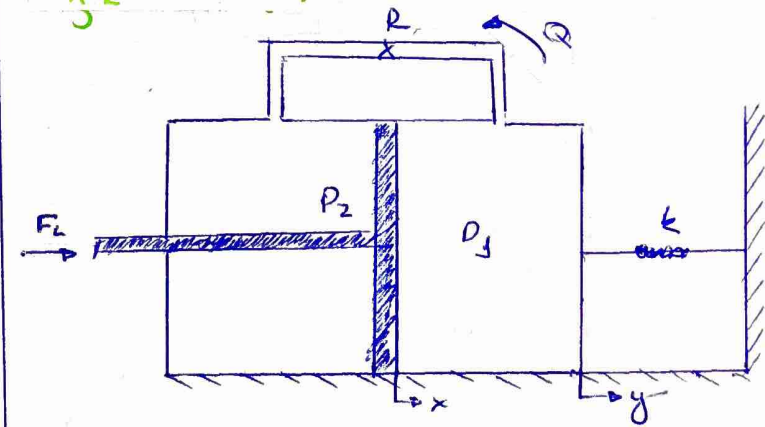
$$② p \cdot Q dt = A \cdot p (dx - dy) \Rightarrow Q = A(\dot{x} - \dot{y})$$

Escoamento laminar no conduto:

$$③ Q = \frac{p_1 - p_2}{R} \quad \text{Resistência ao escoamento: } R = \frac{p_1 - p_2}{Q}$$

$$F_L = A(p_1 - p_2) = ARQ = ARA(\dot{x} - \dot{y}) = A^2 R(\dot{x} - \dot{y})$$

$$F_L = b(\dot{x} - \dot{y}) \Rightarrow \text{Torção do amortecedor (resposta): } F_a = -b(\dot{x} - \dot{y})$$



Desprezando a inércia do conjunto, vem:

$$F_L - ky = 0 \Rightarrow b(\dot{x} - \dot{y}) - ky = 0$$

Admitindo-se condições iniciais nulas, aplicamos a transformada de Laplace.

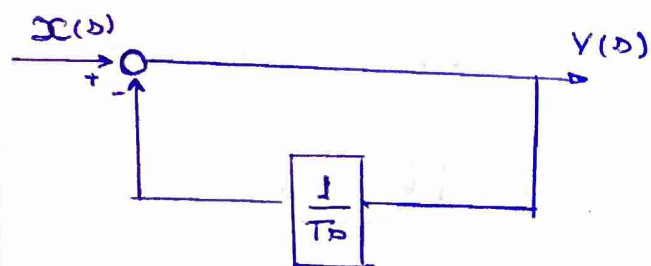
$$b(sX(s) - Y(s)) - kY(s) = 0 \Rightarrow$$

$$\Rightarrow bs \mathcal{X}(s) = (bs+k) Y(s) \Rightarrow$$

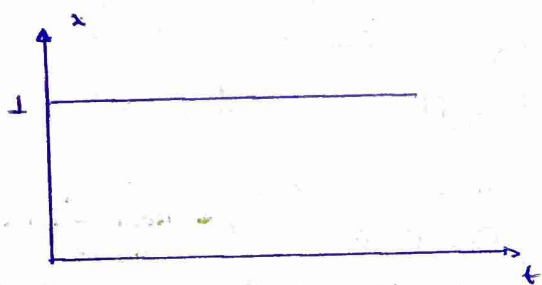
$$\Rightarrow \frac{Y(s)}{\mathcal{X}(s)} = \frac{bs}{bs+k} = \frac{1}{1 + \frac{1}{\frac{b}{k}s}} = \frac{1}{1 + \frac{1}{T_D s}}$$

$$= \frac{1}{1 + \frac{1}{T_D s}} \quad , T_D = \frac{b}{k} = \frac{A^2 R}{k}$$

$$\frac{Y(s)}{\mathcal{X}(s)} = \frac{1}{1 + \frac{1}{T_D s}}$$



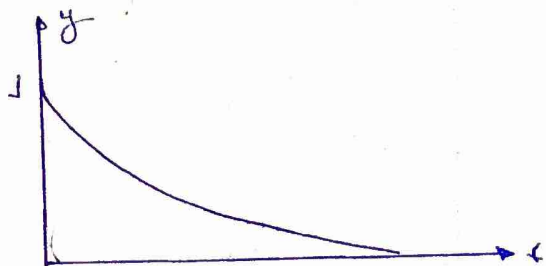
para $\mathcal{X}(s) = \frac{1}{s}$:



$$Y(s) = \frac{1}{s} \cdot \frac{1}{1 + \frac{1}{T_D s}} = \frac{1}{s} \cdot \frac{s T_D}{s T_D + 1} = \frac{T_D}{s T_D + 1}$$

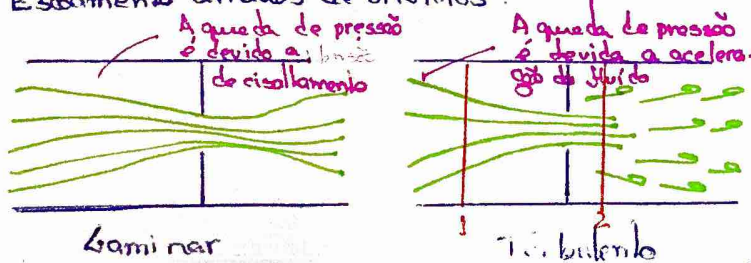
$$= T_D \cdot \frac{1}{s T_D + 1} \Rightarrow$$

$$\Rightarrow y(t) = \frac{1}{dt} T_D [1 - e^{-t/T_D}] = e^{-t/T_D}$$



→ Servo válvula

Escoamento através de orifícios:



A região onde a área, ou jato é mínima (segundo) é chamada de "vena contracta". Entre 1 e 2, o escoamento pode ser admitido potencial ocorrendo segundo linhas de corrente.

Vale a relação de Bernoulli:

$$\frac{\rho}{2} v_1^2 + P_1 + \rho g h_1 = \frac{\rho}{2} v_2^2 + P_2 + \rho g h_2 \Rightarrow$$

$$\Rightarrow \rho \frac{v_1^2}{2} + P_1 = P_2 + \rho \frac{v_2^2}{2} \quad (1)$$

Além disso, vale a equação da continuidade:

$$A_1 v_1 = A_2 v_2 \quad (2)$$

$$\frac{P_1 - P_2}{\rho} = \frac{v_2^2}{2} - \frac{v_1^2}{2} = \frac{v_2^2}{2} \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right] \Rightarrow$$

$$\Rightarrow v_2 = \frac{1}{\sqrt{1 - \left(\frac{A_2}{A_1} \right)^2}} \cdot \sqrt{\frac{2(P_1 - P_2)}{\rho}} \Rightarrow$$

$$\Rightarrow Q_2 = A_2 v_2 = A_2 \frac{1}{\sqrt{1 - \left(\frac{A_2}{A_1} \right)^2}} \cdot \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

Condições de ajuste:

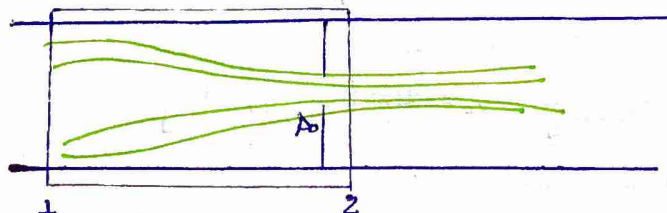
1) A velocidade do jato é um pouco menor devido a efeitos viscosos (aparece o coeficiente de velocidade) $\rightarrow c_v$

2) A_2 é diferente da área do orifício, A_o .

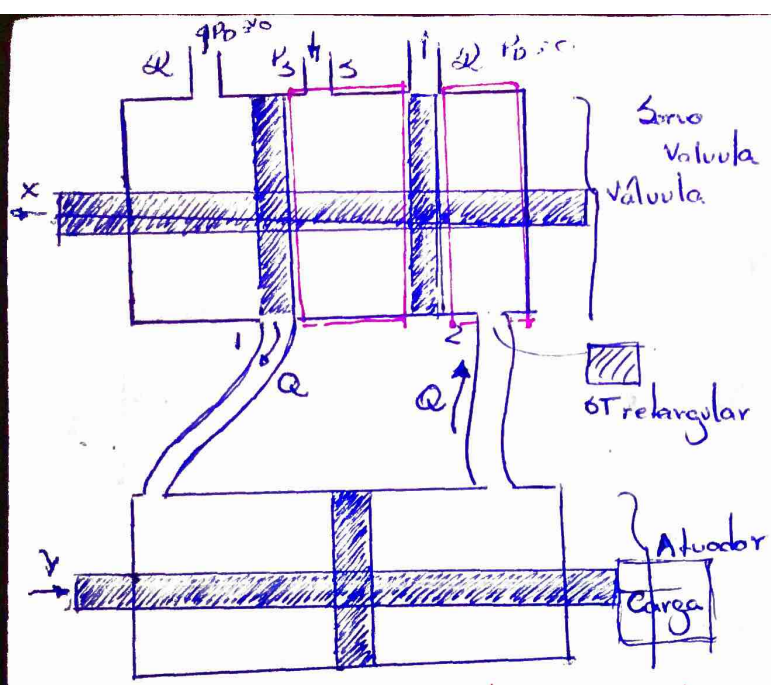
Aparece o coeficiente de contração: c_c

$$\therefore Q_2 = c_c A_o \sqrt{\frac{2(P_1 - P_2)}{\rho}} \quad , A_2 = c_c A_o$$

onde, $c_c = c_c \cdot c_v$ Coef de descarga para orifícios pontiagudos, regime turbulento e $A_o \ll A_1$.
Vamos admitir $c_c = 0,6$



$$Q_2 = c_c \cdot A_o \cdot \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$



Escoamento em RP: \rightarrow É um amplificador de potência.

Trecho 1: No entre 1 e 2

$$P_3 + \frac{\rho}{2} v_3^2 = P_1 + \frac{\rho}{2} v_1^2$$

$$Q_1 = C_d A_1 \sqrt{\frac{2}{\rho} (P_3 - P_1)}, \quad A_1 = Lx$$

↳ largura do orifício

$$\rightarrow Q_1 = C_d Lx \sqrt{\frac{2}{\rho} (P_3 - P_1)} \rightarrow$$

$$\rightarrow Q_1 = C_v x \sqrt{(P_3 - P_2)}, \quad C_v = C_d L \sqrt{\frac{2}{\rho}}$$

Trecho 2: No entre 2 e Q

$$Q_0 = C_d \cdot Lx \sqrt{\frac{2}{\rho} (P_2 - P_0)} = C_v x \sqrt{P_2}$$

A princípio, admite-se escoamento incompressível, sem vazamento, onde

$$Q_1 = Q_0 = Q$$

$$P_2 = P_1 - P_2$$

$$P_2 = P_3 - \left(\frac{Q}{C_v x}\right)^2 \quad \text{e} \quad P_2 = \left(\frac{Q}{C_v x}\right)^2$$

Logo,

$$Q = C_v x \sqrt{\frac{1}{2} (P_3 - P_2)} \quad (1)$$

Atuador: $Q = A_p \dot{Y}$ (2)

$$F_L = P_1 A_p - P_2 A_p = A_p (P_1 - P_2) = A_p \cdot P_2 \rightarrow$$

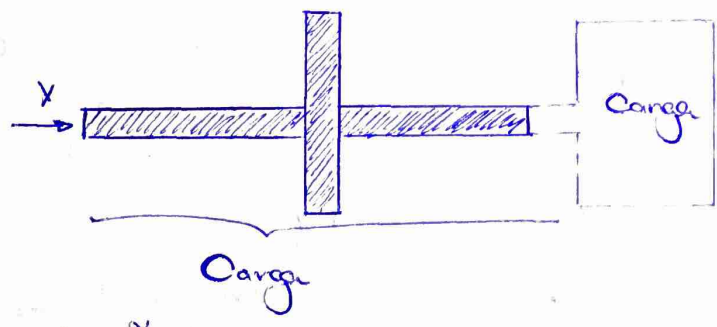
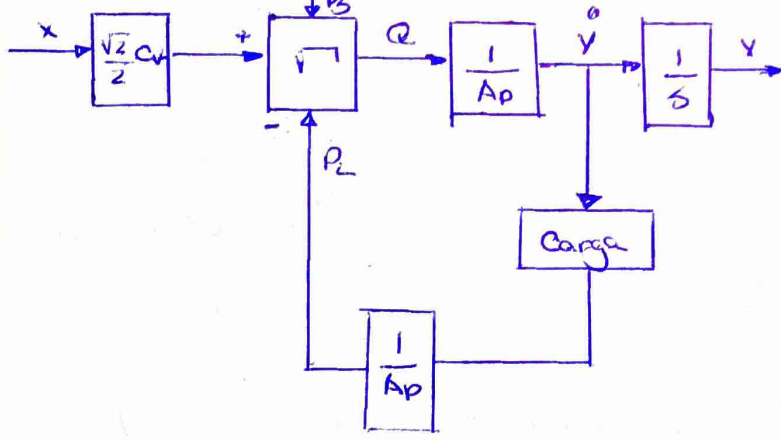
$$\rightarrow P_2 = \frac{F_L}{A_p} \quad (3)$$

Fórmula do êmbolo desprezível!

Aplicando (1), (2), em (3):

$$A_p \dot{Y} = C_v x \sqrt{\frac{1}{2} (P_3 - P_2)} \Rightarrow$$

$$\Rightarrow \dot{Y} = \frac{C_v}{A_p} \sqrt{\frac{1}{2} (P_3 - P_2)} x \quad (4)$$



Linearização:

$$Q = C_v x \sqrt{\frac{1}{2} (P_3 - P_2)} \Rightarrow$$

$$\rightarrow Q = \bar{Q} + C_v \sqrt{\frac{1}{2} (P_3 - P_2)} \left[(x - x_0) - \frac{C_v x_0}{2} \left[\frac{1}{2} (P_3 - P_2) \right]^{-1/2} (P_2 - P_{20}) \right]$$

$$q = k_1 x - k_2 \Delta P_2$$

$$q = Q - \bar{Q} = A_p \cdot (\dot{Y} - \dot{Y}_0) = A_p \cdot \dot{j}$$

$$x = x - x_0$$

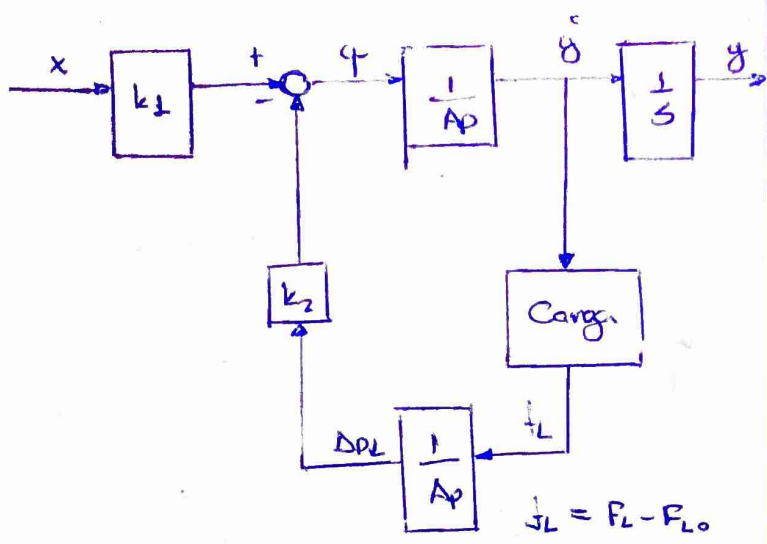
$$\Delta P_2 = P_2 - P_{20}$$

$$k_1 = C_v \left[\frac{1}{2} (P_3 - P_2) \right]^{1/2}$$

$$k_2 = \frac{C_v x_0}{2} \left[\frac{1}{2} (P_3 - P_2) \right]^{-1/2}$$

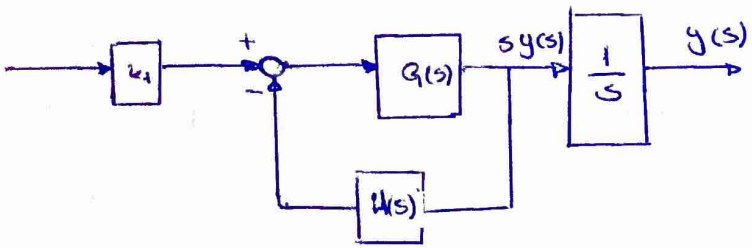
Utilizamos:

$$Q = Q_0 + \frac{\partial Q}{\partial x} \Big|_{x=x_0} (x - x_0) + \frac{\partial Q}{\partial P_2} \Big|_{P_2=P_{20}} (P_2 - P_{20})$$



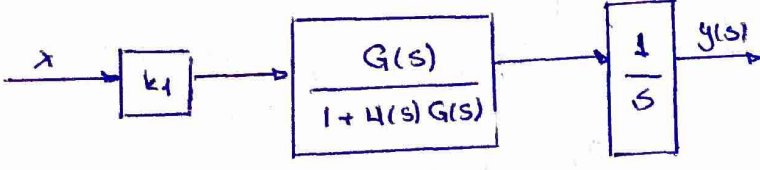
Carga: consideremos o sistema massa-amortecedor:

$$m\ddot{y} + b\dot{y} = m(\ddot{y} + \ddot{y}_0) + b(\dot{y} + \dot{y}_0) = (ms^2 + bs)y(s) = (mb + b)s y(s)$$

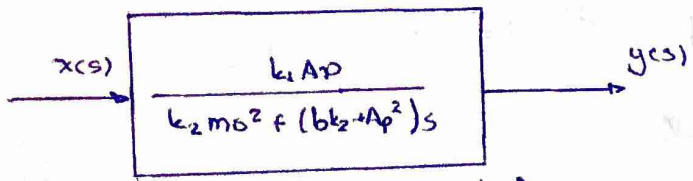


$$G(s) = \frac{1}{Ap}; U(s) = (ms + b) \frac{1}{Ap} \cdot k_2$$

O diagrama de blocos pode ser reduzido:



$$\frac{G(s)}{1 + U(s)G(s)} = \frac{Ap}{Ap^2 + (ms + b)k_2}$$



$$\frac{k_1 A}{k_2 ms^2 + (bk_2 + Ap^2)s} = \frac{1}{s} \cdot \frac{k_1 Ap}{bk_2 + Ap^2} = \frac{1}{s} \cdot \frac{k}{Ts + 1}$$

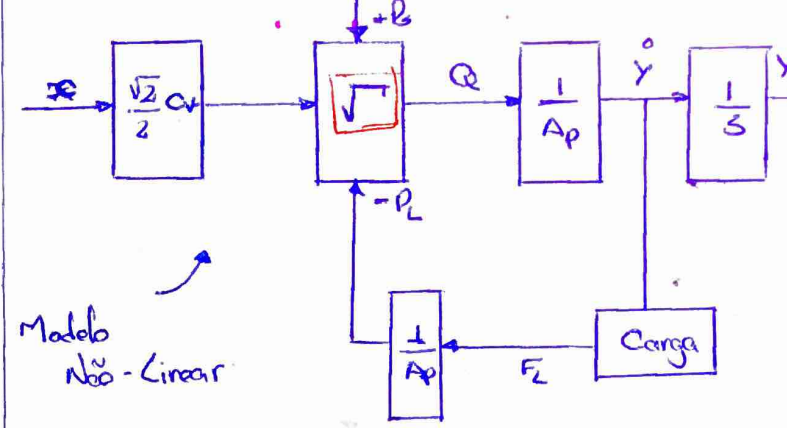
$$K = \frac{k_1 Ap}{bk_2 + Ap^2}; T = \frac{mk_2}{bk_2 + Ap^2}$$

para $\frac{mk_2}{bk_2 + Ap^2} \ll 1$, simplifica-se a expressão!

anterior para

$$\frac{y(s)}{x(s)} = \frac{k}{s}$$

Resumo: servo-válvula



Modelo Não-Linear

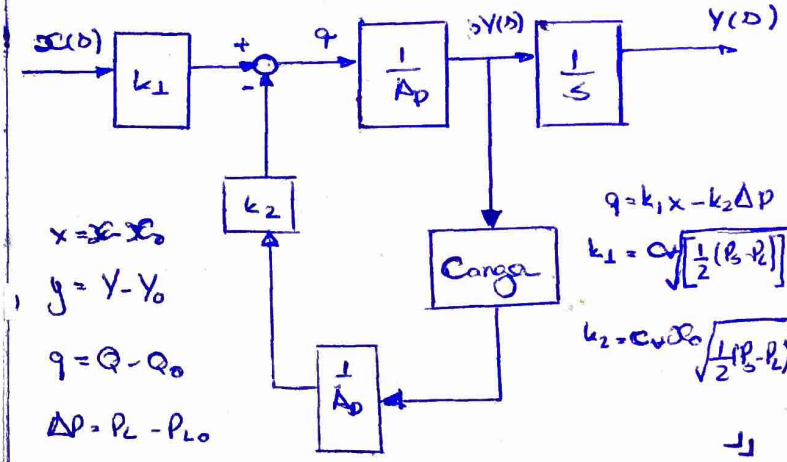
$$Q = C_v \sqrt{\frac{1}{2}(P_3 - P_2)}$$

$$Q = Ap \dot{y}$$

$$P_L = P_3 - P_2$$

$$F_L = P_L Ap$$

Modelo Linear



$$x = x - x_0$$

$$y = Y - Y_0$$

$$q = Q - Q_0$$

$$\Delta P = P_L - P_{L0}$$

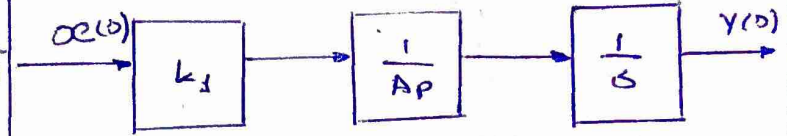
$$k_1 = C_v \sqrt{\frac{1}{2}(P_3 - P_2)}$$

$$k_2 = C_v \sqrt{\frac{1}{2}(P_3 - P_2)}$$

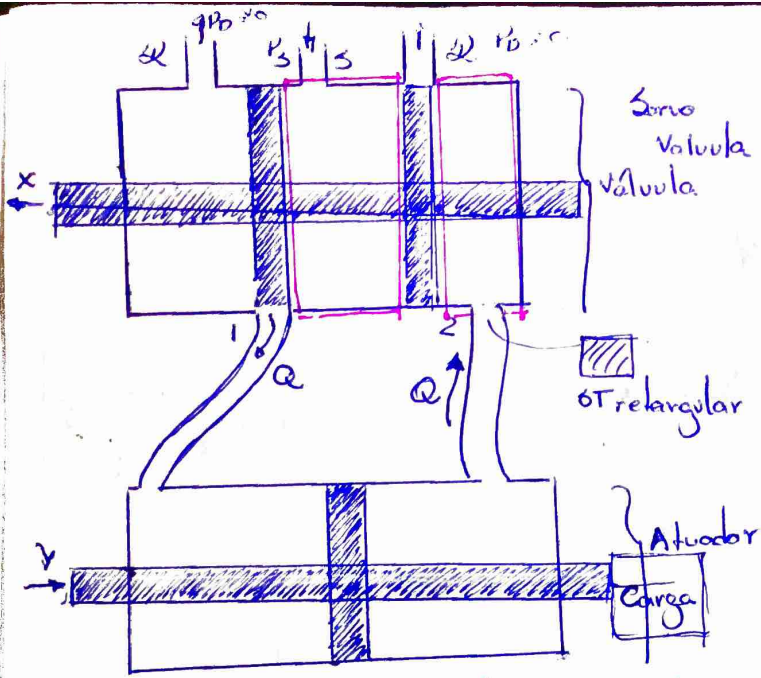
Caso especial:

Estado da carga pequeno $x_0 = 0 \Rightarrow$ vamos negligenciar k_2

$$k_2 = C_v \sqrt{\frac{1}{2}(P_3 - P_2)} = 0$$



$$\frac{Y(s)}{X(s)} = \frac{k_1 / Ap}{s} = \frac{k}{s}$$



Escoamento em RP:

→ É um amplificador de potência.

Trecho 1: No entre 1 e 2

$$P_3 + \frac{\rho}{2} v_3^2 = P_1 + \frac{\rho}{2} v_1^2$$

$$Q_1 = C_d A_1 \sqrt{\frac{2}{\rho} (P_3 - P_1)}, \quad A_1 = L x$$

$$\rightarrow Q_1 = C_d L x \sqrt{\frac{2}{\rho} (P_3 - P_1)} \rightarrow$$

$$\rightarrow Q_1 = C_v x \sqrt{(P_3 - P_2)}, \quad C_v = C_d L \sqrt{\frac{2}{\rho}}$$

Trecho 2: No entre 2 e 0

$$Q_0 = C_d \frac{L x}{A_2} \sqrt{\frac{2}{\rho} (P_2 - P_0)} = C_v x \sqrt{P_2}$$

A princípio, admite-se escoamento incompressível, sem vazamentos, onde

$$Q_1 = Q_0 = Q$$

$$P_2 = P - P_3$$

$$P_1 = P_3 - \left(\frac{Q}{C_v x}\right)^2 \quad e \quad P_2 = \left(\frac{Q}{C_v x}\right)^2$$

Logo,

$$Q = C_v x \sqrt{\frac{1}{2} (P_3 - P_2)} \quad (1)$$

Atuador: $Q = A_p \dot{y}$ (2)

$$F_L = P_1 A_p - P_2 A_p = A_p (P_1 - P_2) = A_p P_L \rightarrow$$

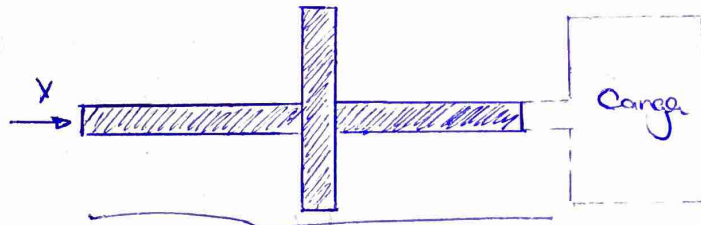
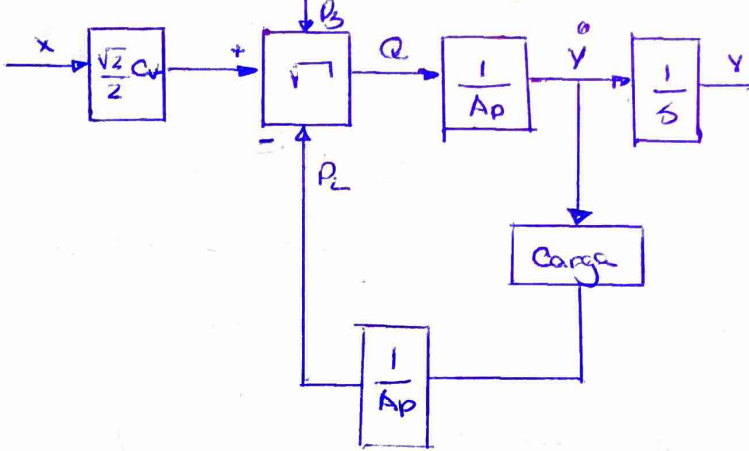
$$\rightarrow P_L = \frac{F_L}{A_p} \quad (3)$$

Força na carga
Inércia do êmbolo desprezível!

Aplicando (3), (2), em (1):

$$A_p \dot{y} = C_v x \sqrt{\frac{1}{2} \left(\frac{P_3 - P_L}{A_p}\right)} \Rightarrow$$

$$\Rightarrow \dot{y} = \frac{C_v}{A_p} \sqrt{\frac{1}{2} (P_3 - P_L)} x \quad (4)$$



Carga

Linearização:

$$Q = C_v x \sqrt{\frac{1}{2} (P_3 - P_L)} \Rightarrow$$

$$\rightarrow Q - \bar{Q} = C_v \sqrt{\frac{1}{2} (P_3 - P_L)} \left[(x - x_0) - \right.$$

$$\left. - \frac{C_v x_0}{2} \left[\frac{1}{2} (P_3 - P_L) \right]^{-1/2} (P_L - P_{L0}) \right]_{P_L = P_{L0}} \Rightarrow$$

$$q = k_1 x - k_2 \Delta P_L$$

$$q = Q - \bar{Q} = A_p (\dot{y} - \dot{y}_0) = A_p \dot{y}$$

$$x = x - x_0$$

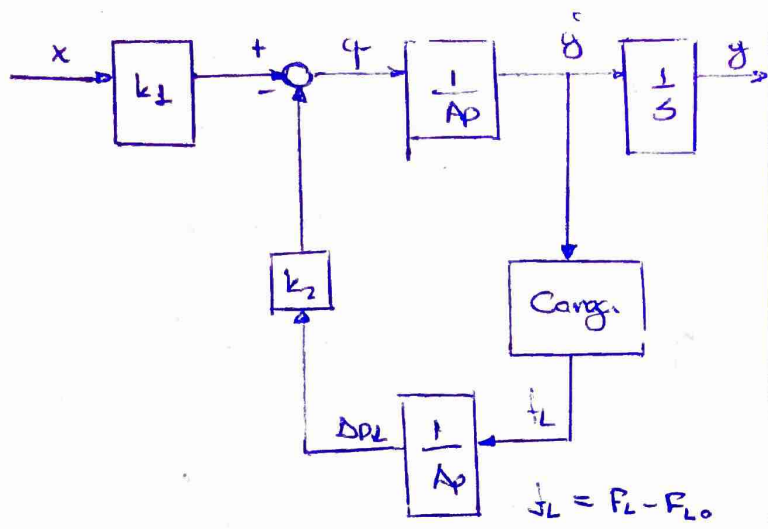
$$\Delta P_L = P_L - P_{L0}$$

$$k_1 = C_v \left[\frac{1}{2} (P_3 - P_L) \right]^{1/2}$$

$$k_2 = \frac{C_v x_0}{2} \left[\frac{1}{2} (P_3 - P_L) \right]^{-1/2}$$

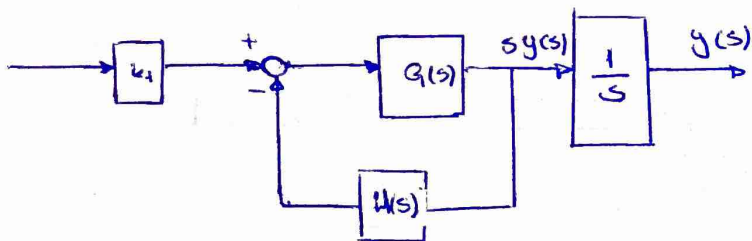
Utilizamos:

$$Q = Q_0 + \frac{\partial Q}{\partial x} \Big|_{x=x_0} (x - x_0) + \frac{\partial Q}{\partial P_L} \Big|_{P_L=P_{L0}} (P_L - P_{L0})$$



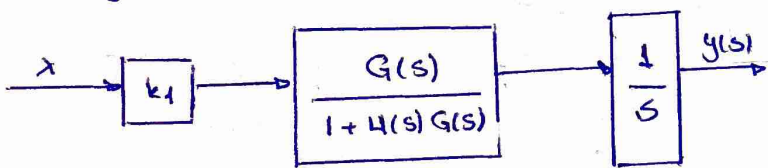
Carga: consideremos o sistema massa-amortecedor:

$$m\ddot{y} + b\dot{y} = m(\ddot{y} + \ddot{y}_0) + b(\dot{y} + \dot{y}_0) = (ms^2 + bs)y(s) = (mb + b)s y(s)$$

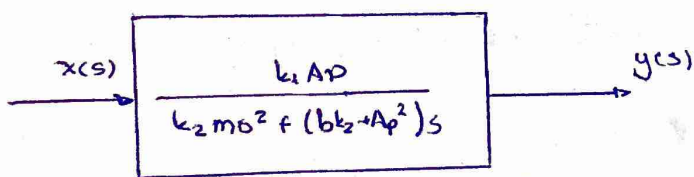


$$G(s) = \frac{1}{Ap}; U(s) = (ms + b) \frac{1}{Ap} \cdot k_2$$

O diagrama de blocos pode ser reduzido:



$$\frac{G(s)}{1 + U(s)G(s)} = \frac{Ap}{Ap^2 + (ms + b)k_2}$$



$$\frac{k_1 A}{k_2 m s^2 + (b k_2 + A^2) s} = \frac{1}{s} \cdot \frac{\frac{k_1 A p}{b k_2 + A^2}}{\left(\frac{m k_2}{b k_2 + A^2} s + 1\right)} = \frac{1}{s} \frac{k}{T s + 1}$$

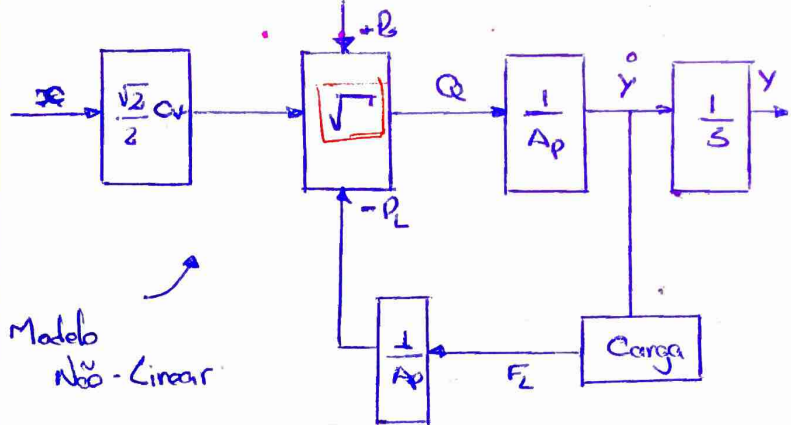
$$K = \frac{k_1 A p}{b k_2 + A^2}; T = \frac{m k_2}{b k_2 + A^2}$$

para $\frac{m k_2}{b k_2 + A^2} \ll 1$, simplifica-se a expressão

anterior para:

$$\frac{y(s)}{x(s)} = \frac{k}{s}$$

Resumo: servo-válvula



Modelo Não-Linear

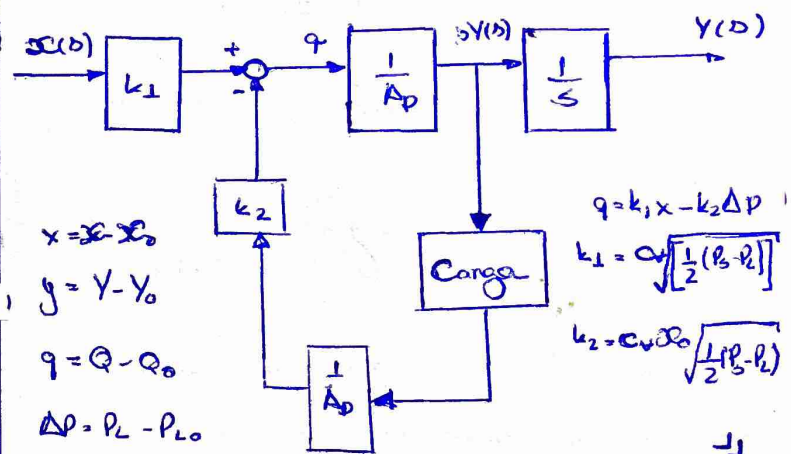
$$Q = C_v \sqrt{\frac{1}{2}(P_3 - P_2)}$$

$$Q = A_p \dot{y}$$

$$P_L = P_3 - P_2$$

$$F_L = P_L A_p$$

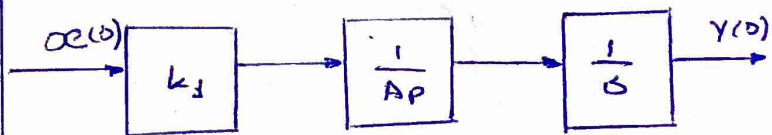
Modelo Linear



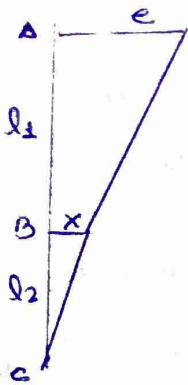
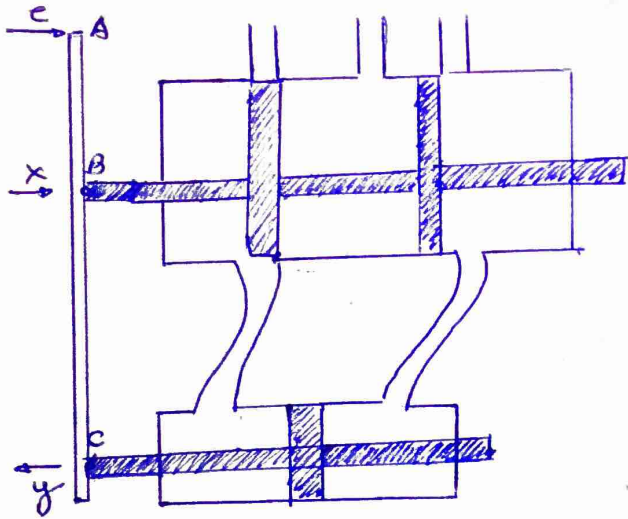
Caso especial:

Efeito da carga pequeno $x_0 = 0 \Rightarrow$ vamos negligenciar k_2

$$k_2 = C_v x_0 \sqrt{\frac{1}{2}(P_3 - P_2)} = 0$$



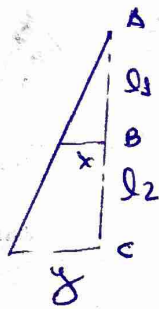
$$\frac{Y(s)}{X(s)} = \frac{k_1 / A_p}{s} = \frac{k}{s}$$



Pequeñas deflexiones

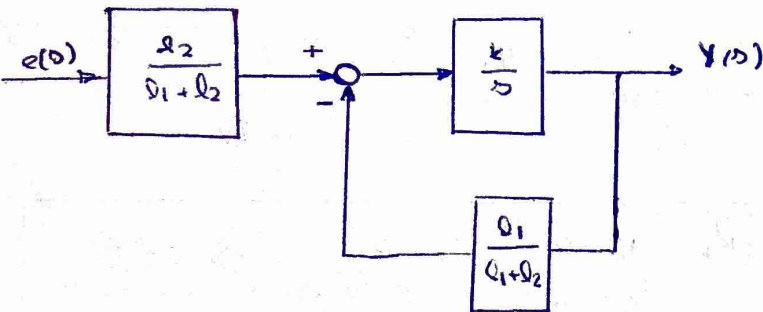
$$\frac{e}{l_1+l_2} = \frac{x}{l_2} \Rightarrow$$

$$\Rightarrow x = \frac{e \cdot l_2}{l_1+l_2} \quad (1)$$



$$\frac{x}{l_2} = \frac{y}{l_1+l_2} \Rightarrow$$

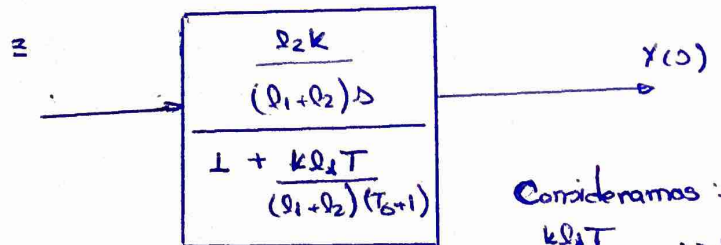
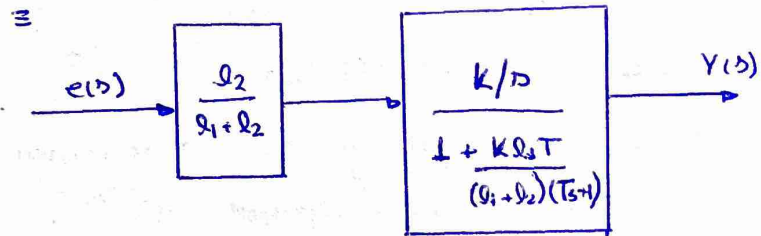
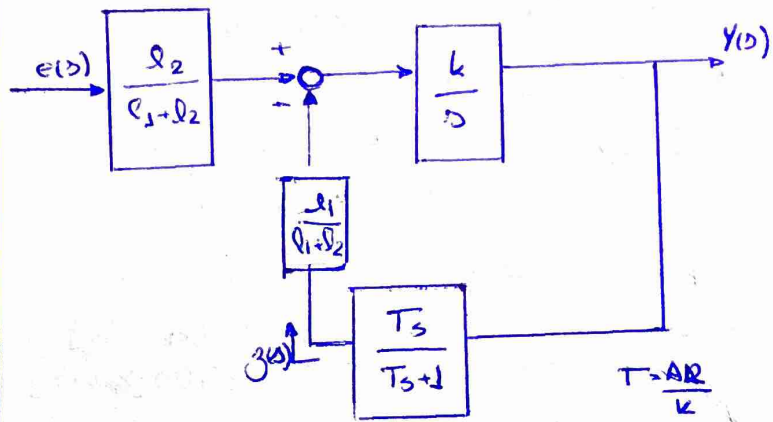
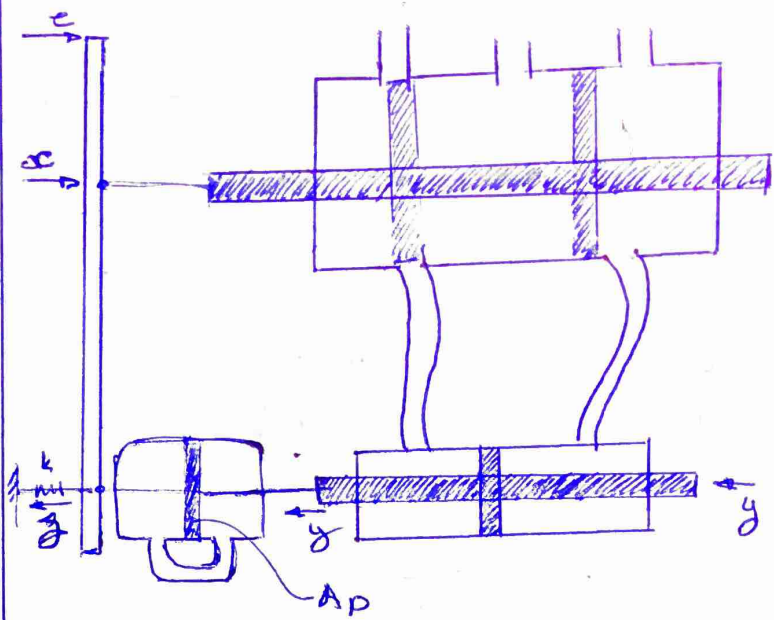
$$\Rightarrow |x| = \frac{l_2}{l_1+l_2} y \quad (2)$$



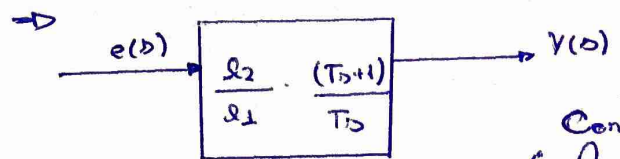
$$\frac{Y(s)}{e(s)} = \frac{k \cdot l_2}{s(l_1+l_2) + k l_1} = \frac{l_2/l_1}{s(l_1+l_2)/k l_1}$$

Consideramos: $\frac{l_1+l_2}{2 l_1} \ll 1$

$$\therefore \frac{Y(s)}{e(s)} = \frac{l_2}{l_1} \rightarrow \text{Control Proporcional}$$



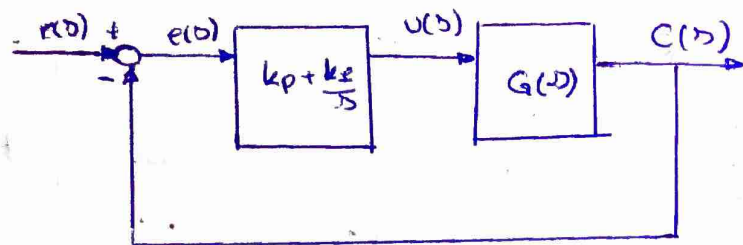
Consideramos: $\frac{k l_2 T}{(l_1+l_2)(T_s+1)} \gg 1$



Control Proporcional Integral

$$\therefore \frac{Y(s)}{e(s)} = \frac{l_2}{l_1} + \frac{l_2}{l_1} \cdot \frac{1}{T_s} = k_p \left(\frac{k_i}{s} \right)$$

Efecto de este tipo de control:



$$\frac{C(s)}{r(s)} = \frac{G(s) [k_p + k_f/s]}{1 + G(s) [k_p + k_f/s]}$$

$$= \frac{G(s) (k_p s + k_f)}{s + G(s) (k_p s + k_f)}$$

Suponha uma entrada igual a um degrau unitário e $G(s)$ represente um sistema "tipo 0", ou seja, $G(0) = k$.

$$\rightarrow C(s) = \frac{1}{s} \cdot \frac{G(s) [k_p s + k_f]}{s + G(s) [k_p s + k_f]}$$

Supondo, o sistema estável, temos

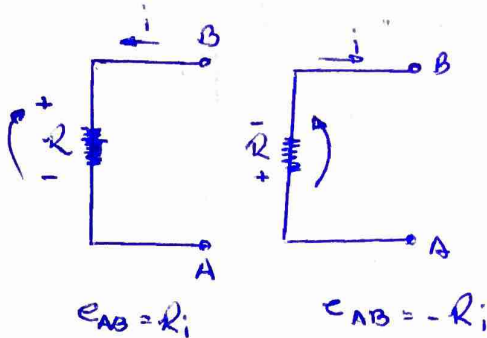
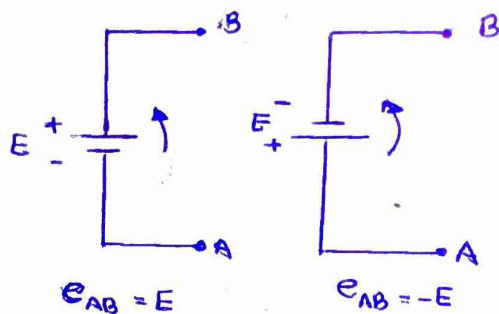
$$\lim_{t \rightarrow \infty} c(t) = \lim_{s \rightarrow 0} s C(s) = \lim_{s \rightarrow 0} \frac{G(s) [k_p s + k_f]}{s + G(s) [k_p s + k_f]} = \frac{G(0) k_f}{G(0) k_f} = 1$$

$$\lim_{t \rightarrow \infty} \text{Erro}(t) = r(t) - c(t) = 0$$

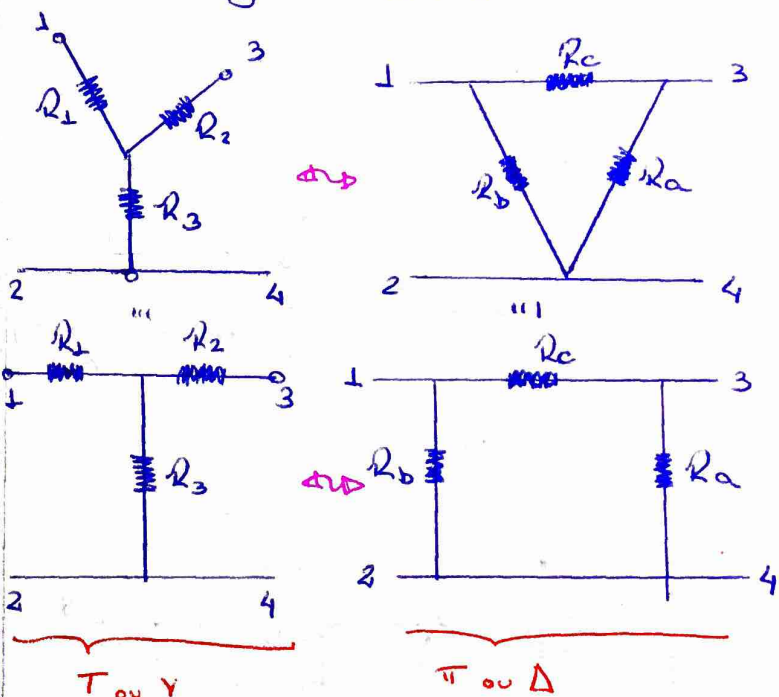
∴ o eixo da ação integrativa é o de retirar o erro em RP para o sistema tipo "0".

Exerc) Amplificadores Operacionais

Convenção:



Transformação Y-Δ ou T-π



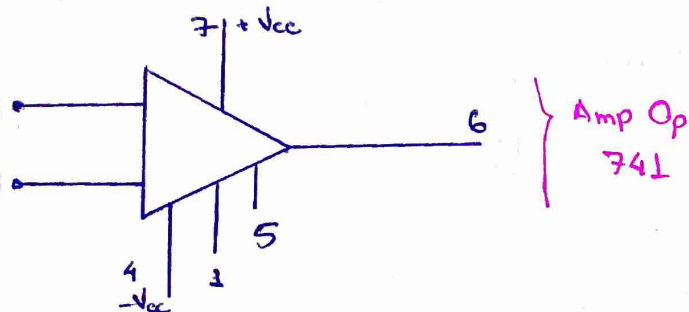
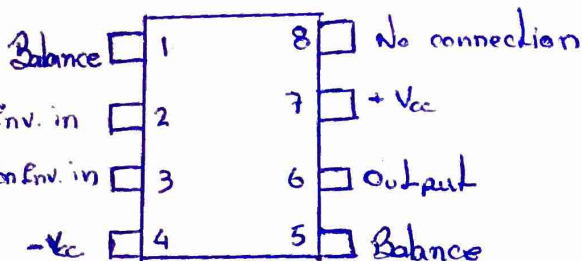
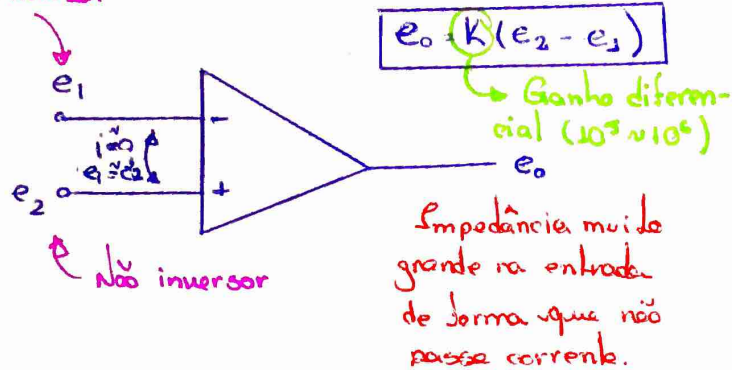
$$R_1 = \frac{R_b \cdot R_c}{R_a + R_b + R_c}; \quad R_2 = \frac{R_a \cdot R_c}{R_a + R_b + R_c}; \quad R_3 = \frac{R_a \cdot R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_2}; \quad R_b = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_2}$$

$$R_c = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_3}$$

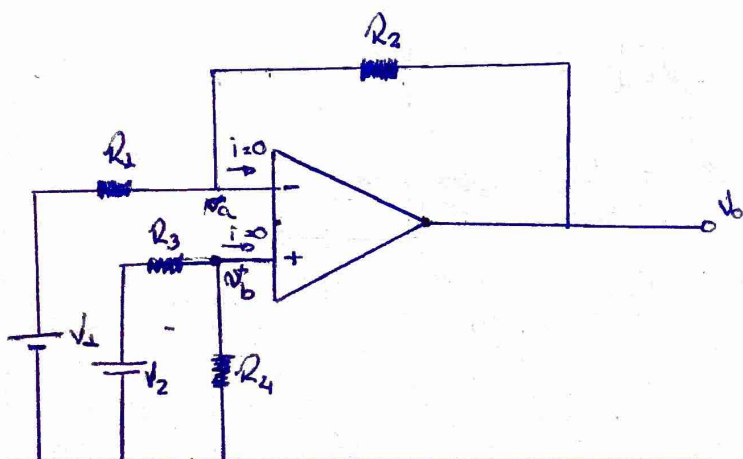
Modelagem de Amplificadores Operacionais

Inversor



* Ver Amp Op inversor, não inversor e somador no caderno de Eletrônica Analógica (PMR 3208) para a P2.

Amplificador Diferencial ou Substrator:



$$Nó A: \frac{V_1 - V_A}{R_1} = \frac{V_A - V_0}{R_2}$$

$$V_0 = \left(\frac{R_2}{R_1} + 1\right) V_A - \frac{R_2}{R_1} V_1$$

$$Nó B: \frac{V_2 - V_0}{R_3} = \frac{V_0 - 0}{R_4} \Rightarrow V_0 = \frac{R_4}{R_3 + R_4} V_2$$

Como $V_A = V_B$:

$$V_0 = \frac{R_2 (1 + R_3/R_4)}{R_3 (1 + R_3/R_4)} V_2 - \frac{R_2}{R_1} V_1$$

Usando:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow V_0 = \frac{R_2}{R_1} (V_2 - V_1)$$

Capacitores:

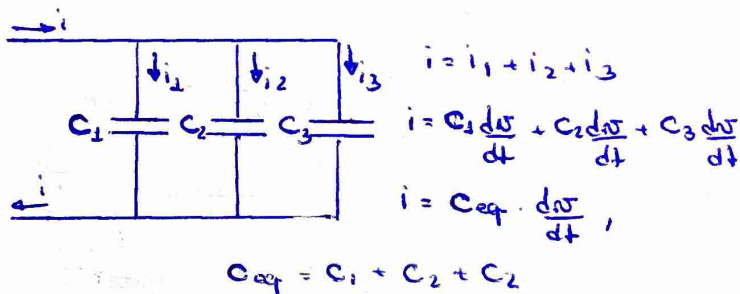
$$C = \frac{\epsilon A}{d}$$

ϵ → permissividade
 A → Área da placa
 d → Distância entre as placas

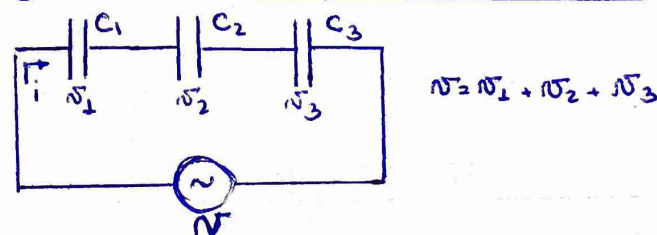
$$q = C \cdot V ; i = \frac{dq}{dt} = C \frac{dV}{dt}$$

Logo,
$$V = \frac{1}{C} \int i dt + V(0)$$

$$P = V \cdot i = V C \frac{dV}{dt}$$



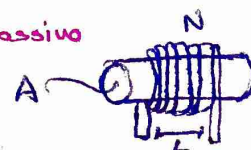
Logo, em paralelo, $C_{eq} = C_1 + C_2 + \dots + C_n$.



$$V = \frac{1}{C_1} \int i dt + \frac{1}{C_2} \int i dt + \frac{1}{C_3} \int i dt = \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right] \int i dt = C_{eq} \cdot \int i dt$$

Logo, em série, $C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$

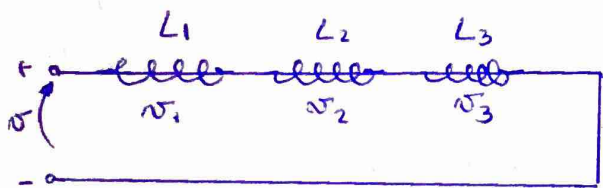
Indutores: ← elemento passivo



$v = L \frac{di}{dt}$, onde $L = \frac{N^2 \mu A}{l}$

Indutância (Henry)

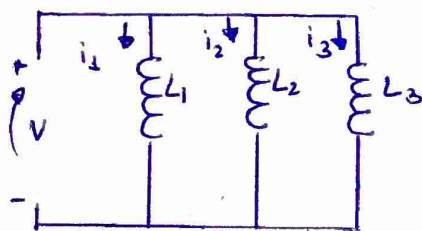
$\frac{di}{dt} = \frac{v}{L} \rightarrow i = \frac{1}{L} \int v dt$



$v = v_1 + v_2 + v_3$

$v = (L_1 + L_2 + L_3) \frac{di}{dt}$

Logo, em série, $L = L_1 + L_2 + \dots + L_n$

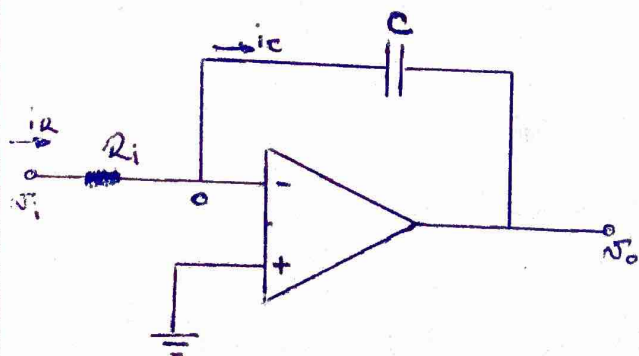


$i = i_1 + i_2 + i_3 = \left[\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right] \cdot \int v dt$

Logo, em paralelo, $L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}}$

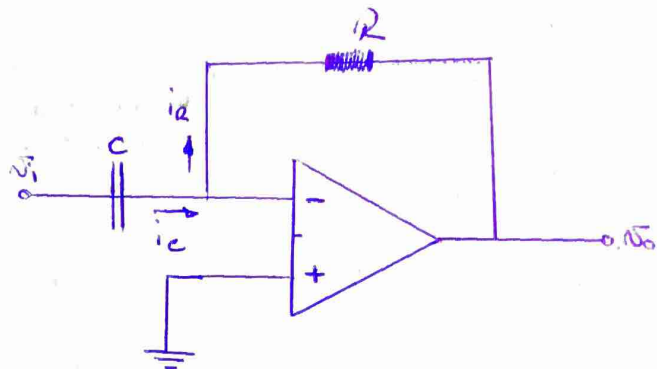
Amp Op Integrador:

Substituir a resistência de realimentação por um capacitor.



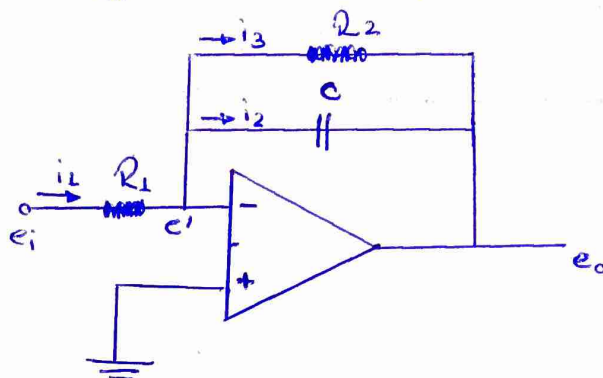
$i_2 = i_c \rightarrow \frac{v_i}{R_i} = -C \frac{dv_o}{dt} \rightarrow v_o = -\frac{1}{RC} \int v_i dt$

Amp Op diferenciador:



$i_2 = i_c \rightarrow \frac{v_o}{R} = -C \frac{dv_i}{dt} \rightarrow v_o = -RC \frac{dv_i}{dt}$

→ Função de Transferência:



$i_1 = \frac{e_i - e'}{R_1} = \frac{e_i}{R_1}$; $i_2 = \frac{d}{dt}(e' - e_o) \cdot C = -C \frac{de_o}{dt}$

$i_3 = \frac{e' - e_o}{R_2} = -\frac{e_o}{R_2}$

$i_1 = i_2 + i_3 \Rightarrow \frac{e_i}{R_1} = -C \frac{de_o}{dt} - \frac{e_o}{R_2}$

$\int \left[\frac{e_i}{R_1} \right] = \int \left[-C \frac{de_o}{dt} \right] + \int \left[-\frac{e_o}{R_2} \right] \rightarrow$

$\rightarrow \frac{E_i(s)}{R_1} = -C s E_o(s) - \frac{E_o(s)}{R_2} \rightarrow$

$\rightarrow G(s) = \frac{E_o(s)}{E_i(s)} \rightarrow G(s) = -\frac{R_2}{R_1} \frac{1}{(R_2 C s + 1)}$