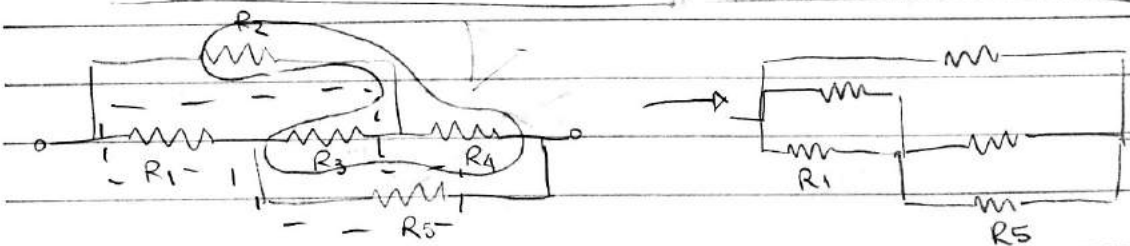
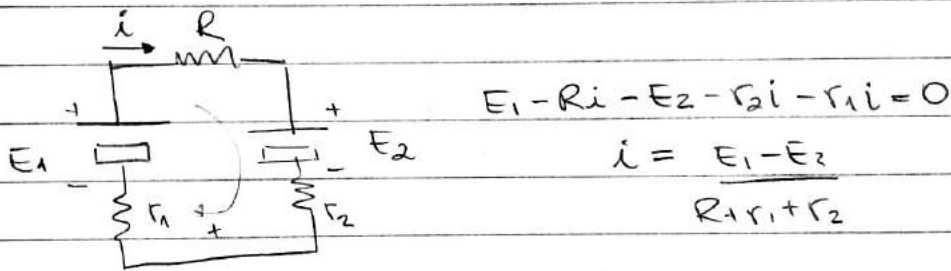
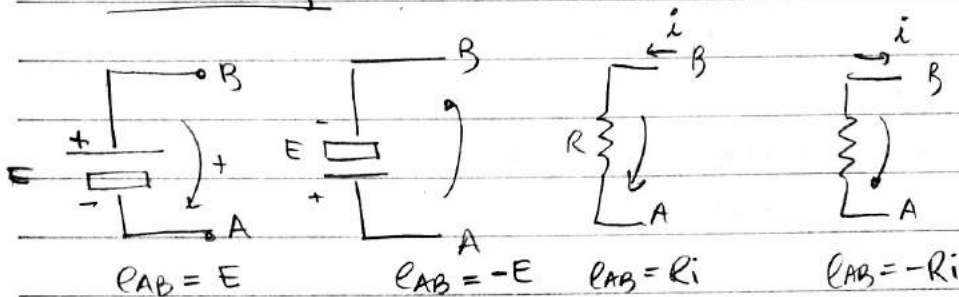


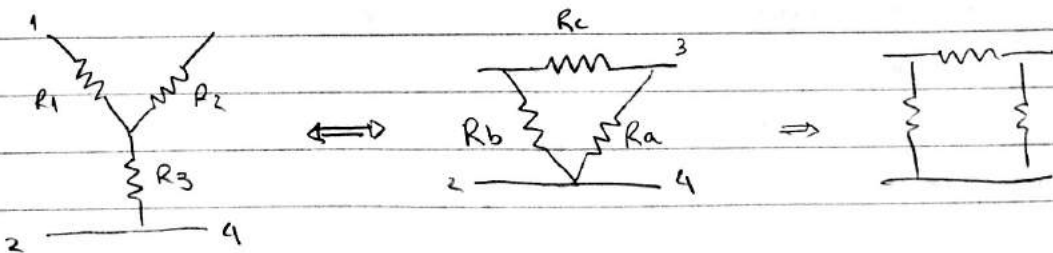
SISDIN 2 - Parte II

- Circuitos Elétricos
- Impedâncias Complexas
- Função de Transferência
- Amplificadores Operacionais
- Ponte Y-Δ
- Kirchhoff

Convenção



Transformação Y-Δ ou T-Π



Terminais 1 e 2

$$\begin{aligned} Y &\rightarrow R_{eq} = R_1 + R_3 \\ \nabla &\rightarrow R_{eq} = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \end{aligned} \quad (1)$$

Terminais 1 e 3

$$Y \rightarrow R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \quad (2)$$

Terminais 3 e 4

$$R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \quad (3)$$

De (1), (2) e (3):

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

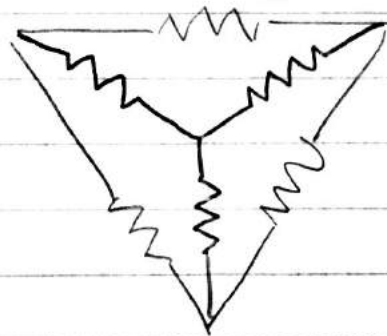
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

ou

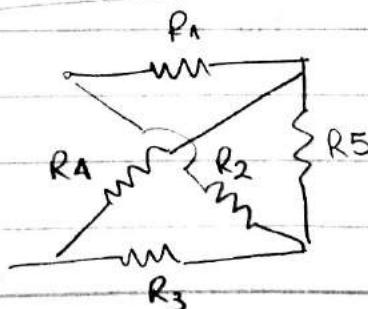
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

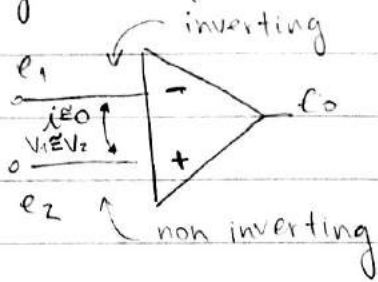
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$



OUTRO CIRCUITO



Modelling of Operational Amplifiers



$$e_0 = K(e_2 - e_1)$$

↳ ganho diferencial $10^5 - 10^6$

impedância $10^5 - 10^{13} \Omega$

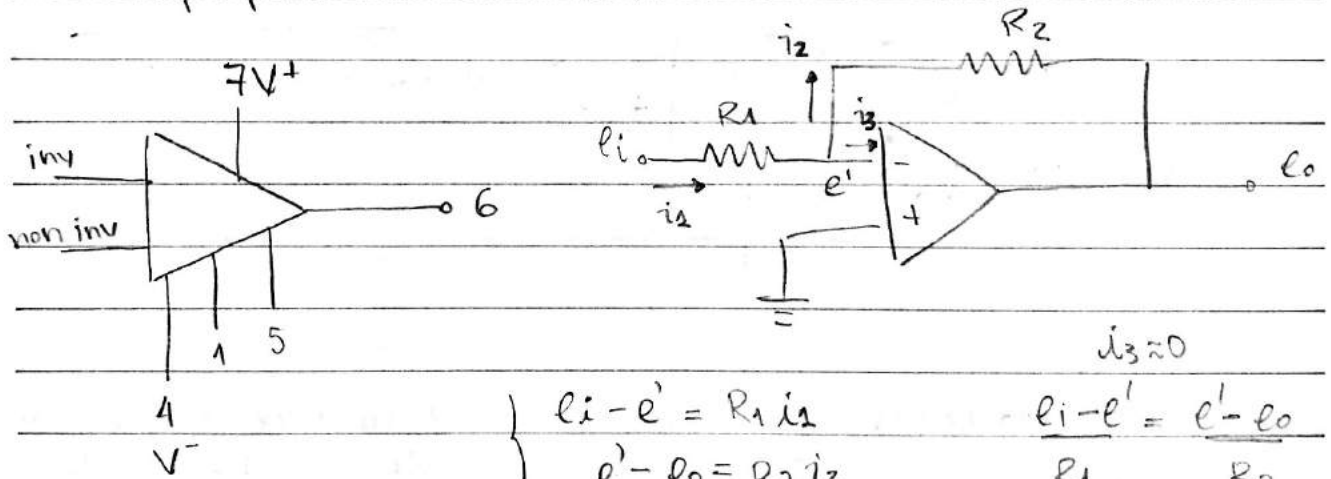
no current flow

zwischen e_1 und e_2

output impedance = zero

balance	1	8	none
inv input	2	7	V^+
non inv input	3	6	Output
V^-	4	5	Balance

Amp Op 741



$$\left. \begin{aligned} e_i - e' &= R_1 i_1 \\ e' - e_o &= R_2 i_2 \end{aligned} \right\}$$

$$\left. \begin{aligned} \frac{e_i - e'}{R_1} &= i_1 \\ \frac{e' - e_o}{R_2} &= i_2 \end{aligned} \right\}$$

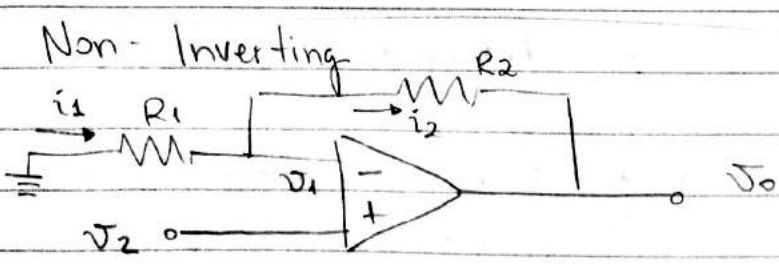
$i_3 \approx 0$

$$\frac{e_i - e'}{R_1} = \frac{e' - e_o}{R_2}$$

$e' = 0V$

$$\frac{e_i}{R_1} = \frac{e'}{R_2}$$

$$e_o = -\frac{R_2}{R_1} e_i$$



$$i_1 = i_2$$

$$0 - U_1 = \frac{U_1 - U_0}{R_2}$$

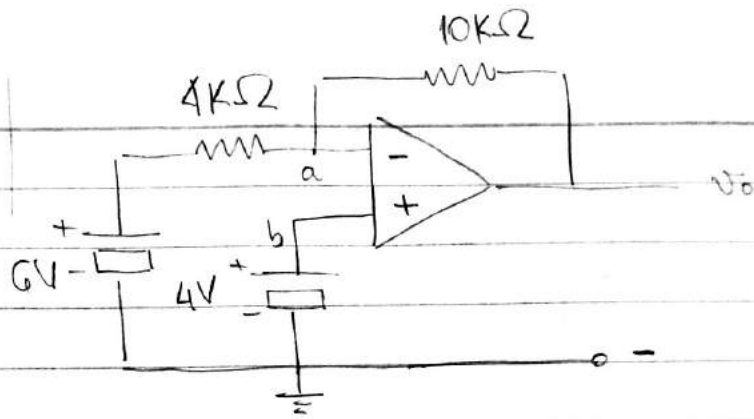
$$U_0 = \left(1 + \frac{R_2}{R_1}\right) \cdot U_1$$



/ /

Para $R_2 = R_1$:

$$V_o = 2V_2$$

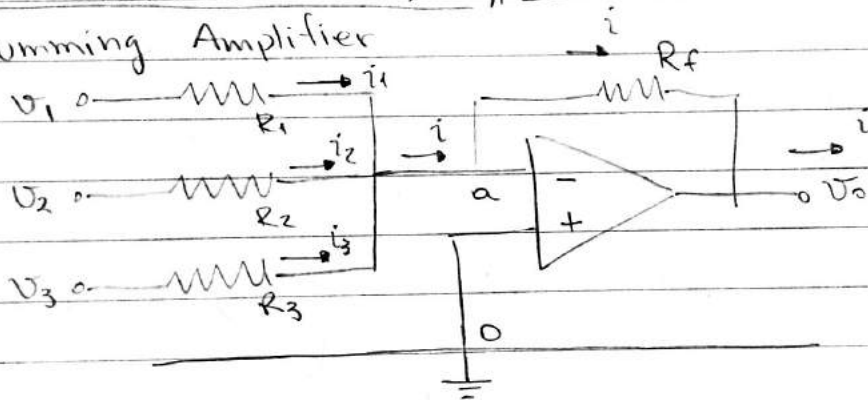


$$\frac{6 - V_a}{4K} = \frac{V_a - V_o}{10K}$$

Como $V_a = V_b = 4V$:

$$V_o = \left(\frac{6 - 4 \cdot 10K}{4K} \right) \cdot (-1) + V_a = -1V \checkmark$$

Summing Amplifier



$$i = i_1 + i_2 + i_3$$

$$i = \frac{V_a - V_o}{R_f}$$

$$\frac{V_a - V_o}{R_f} = \frac{V_1 - V_a}{R_1} + \frac{V_2 - V_a}{R_2} + \frac{V_3 - V_a}{R_3}$$

Como $V_a = 0$

$$V_o = - \left(\frac{R_f V_1}{R_1} + \frac{R_f V_2}{R_2} + \frac{R_f V_3}{R_3} \right)$$

Seja $R_f = 10K\Omega$

$R_1 = 10K\Omega$

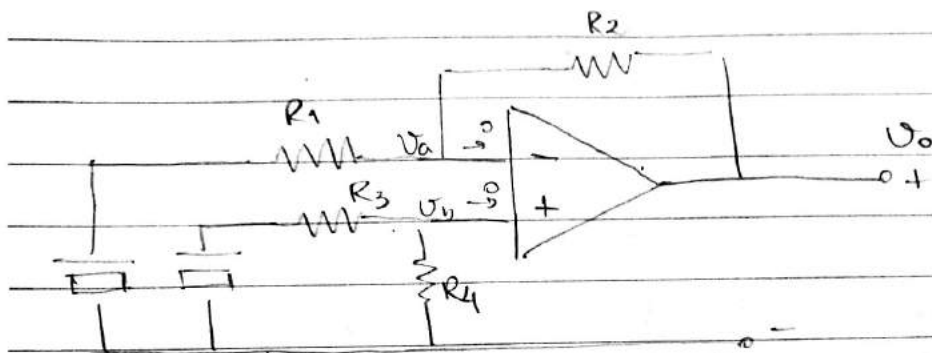
$R_2 = 20K\Omega$

$R_3 = 40K\Omega$

$$-V_o = V_1 + 0,5V_2 + 0,25V_3$$

INPUT			OUTPUT
$[V_1$	V_2	$V_3]$	V_o
$[0$	0	$0]$	0
$[0$	0	$1]$	$0,25$
$[0$	1	$0]$	$0,5$

converter D/A

Difference Amplifier

$$\text{nó A: } \frac{V_1 - V_a}{R_1} = \frac{V_a - V_0}{R_2}$$

$$V_0 = \left(\frac{R_2 + 1}{R_1} \right) V_a - \frac{R_2}{R_1} V_1$$

$$\text{nó B: } \frac{V_2 - V_b}{R_3} = \frac{V_b - 0}{R_4}$$

$$V_b = \frac{R_4}{R_3 + R_4} V_2$$

$$V_0 = \frac{R_2 \left(1 + \frac{R_1}{R_2} \right) \cdot V_2}{R_1 \left(1 + \frac{R_3}{R_4} \right)} - \frac{R_2}{R_1} V_1$$

Usando $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

difference

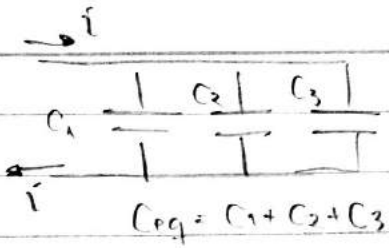
$$\rightarrow V_0 = \frac{R_2}{R_1} (V_2 - V_1)$$

CAPACITORES

$$C = \frac{\epsilon A}{D} \quad q = C \sigma \quad i = \frac{dq}{dt} = C \frac{dv}{dt}$$

$$V = \frac{1}{C} \int i dt + v(0)$$

Potência $\rightarrow P = Vi = V C \frac{dv}{dt}$



$$i = C_{eq} \frac{du}{dt}$$

INDUTORES

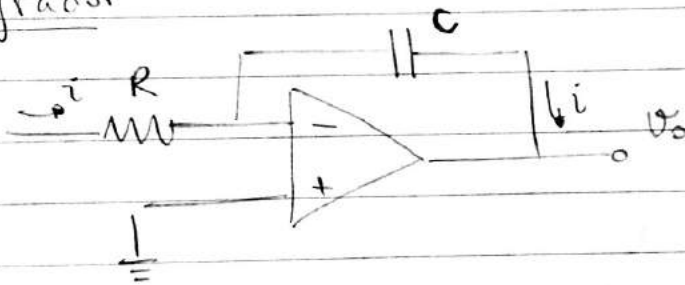
$$v = L \frac{di}{dt}$$

$$i = \frac{1}{L} \int v dt + i(0)$$

Em série: $L_{eq} = L_1 + L_2 + \dots$

Em paralelo: $\frac{1}{L_{eq}} = \frac{1}{L_1} + \dots + \frac{1}{L_n}$

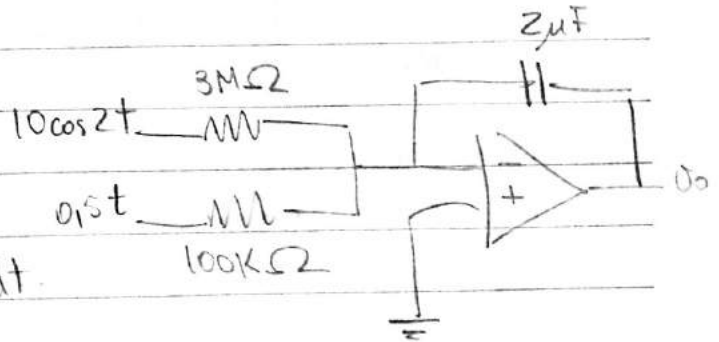
Integrador



$$i_R = i_C$$

$$\frac{v_1}{R_1} = -C \frac{dv_0}{dt}$$

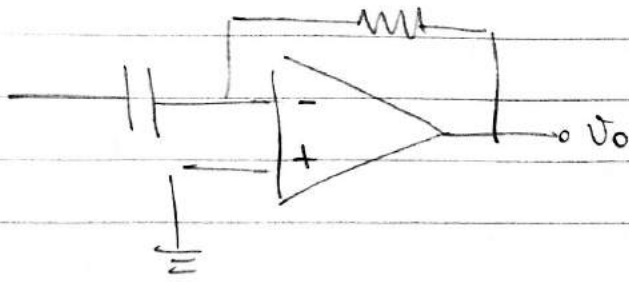
$$v_0 = -\frac{1}{RC} \int v_1 dt$$



$$v_0 = -\frac{1}{RC} \int v_1 dt - \frac{1}{RC} \int v_2 dt$$

$$v_0 = -0,183 \sin 2t - 1,25 t^2 \text{ [mV]}$$

Diferencial

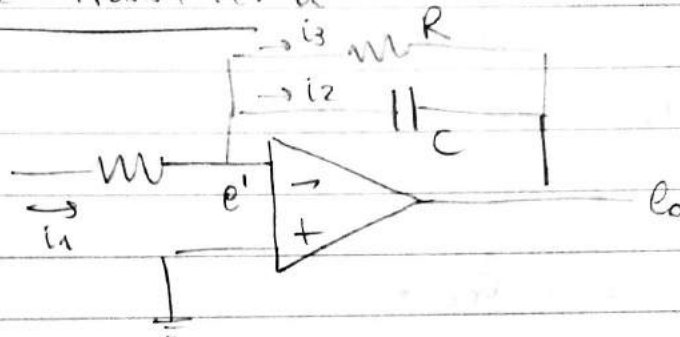


$$i_R = i_C$$

$$\frac{v_o}{R} = -C \frac{dv_o}{dt}$$

$$v_o = -RC \frac{dv_o}{dt}$$

Função de Transferência



$$G(s) = ?$$

$$i_1 = \frac{e_i - e'}{R_1}$$

$$i_3 = \frac{e' - e_o}{R_2}$$

$$i_2 = \frac{d}{dt} (e' - e_o) C$$

$$e' = 0$$

$$\therefore \frac{e_i}{R_1} = -C \frac{de_o}{dt} - \frac{e_o}{R_2}$$

$$\mathcal{L} \left[\frac{e_i}{R_1} \right] = \mathcal{L} \left[-C \frac{de_o}{dt} \right] - \mathcal{L} \left[\frac{e_o}{R_2} \right]$$

$$\frac{E_i(s)}{R_1} = -C s E_o(s) - \frac{E_o(s)}{R_2}$$

$G(s) = \frac{E_o(s)}{E_i(s)} \Rightarrow$	$G(s) = \frac{-R_2}{R_1} \cdot \frac{1}{(R_2 C s + 1)}$
--------------------------------------------	---------------------------------------------------------

$$V(s) = Z(s) I(s)$$

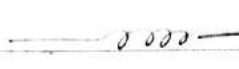
Método de Impedâncias Complexas

- R
- Ls
- $\frac{1}{Cs}$



$$V_R = RI$$

$$V_R(s) = R I(s)$$



$$V_L = L \frac{dI}{dt}$$

$$V_L(s) = Ls I(s)$$

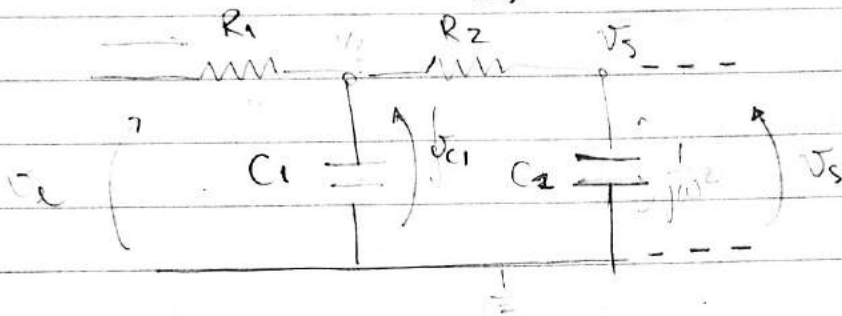


$$C \cdot \dot{V}_C = I$$

$$V_C(s) = \frac{1}{Cs} I(s)$$

$$V_C = \frac{1}{C} \int Idt$$

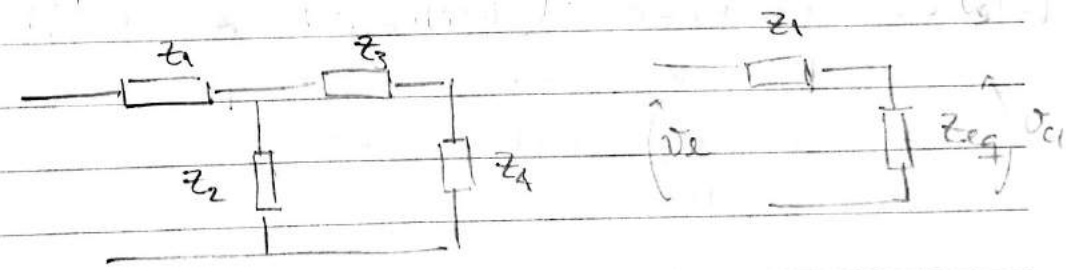
$$Cs$$



$$V_{C1} =$$

$$\frac{V_s - V_{C1}}{R} = C \dot{V}_{C1} + \frac{(V_{C1} - V_s)}{R_2} + C_2 \dot{V}_s$$

$V_{C1}(s) =$



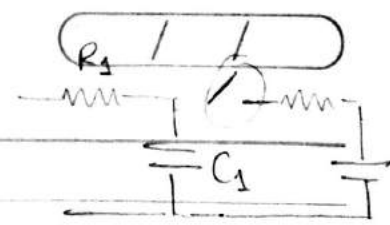
$$Z_{eq} = \frac{Z_2(Z_3 + Z_4)}{Z_2 + Z_3 + Z_4} = \frac{\frac{1}{Cs} (R_2 + \frac{1}{C_2s})}{\frac{1}{Cs} + R_2 + \frac{1}{C_2s}} = \frac{C_2 R_2 s + 1}{R_2 C_1 C_2 s^2 + (C_1 + C_2)s}$$

$$V_{C1}(s) = \frac{V_s(s) Z_{eq}(s)}{Z_1(s) + Z_{eq}(s)} = \frac{V_s(s) \cdot \frac{C_2 R_2 s + 1}{R_2 C_1 C_2 s^2 + (C_1 + C_2)s}}{R_2 C_1 C_2 s^2 + (C_1 + C_2)s + 1}$$

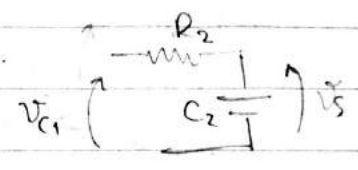
La Place:

$$\frac{1}{R} (V_s(s) - Z(s) V_{C1}(s)) = C_1 s Z(s) V_{C1}(s) + \frac{1}{R_2} (Z(s) V_{C1}(s) - V_s(s)) + C_2 s V_{C1}(s)$$

04 $\frac{V_{C1}}{V_i} = \frac{1}{R_1 C_1 s + 1} = G_1(s) \rightarrow$



$\frac{V_s}{V_{C1}} = \frac{1}{R_2 C_2 s + 1} = G_2(s)$

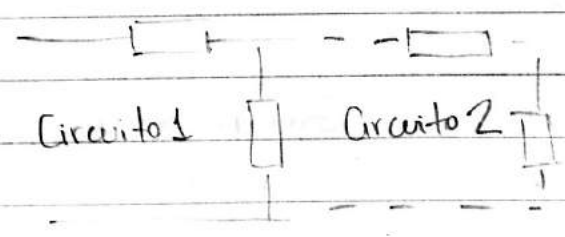


$\frac{V_s}{V_i} = \frac{1}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_1 C_2 s} \neq G_1(s) \cdot G_2(s)$

↳ termo de acoplamento
SIST. EM PARALELO

Quanto menor a impedância do circuito 2, maior a corrente drenada do circuito 1 ($I_{C1} \downarrow$, $V_{C1} \downarrow$)

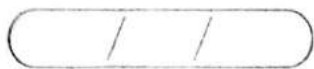
Por outro lado, quanto maior a impedância do circuito 2 face àquele do circuito 1, mais próximos estiverem de tensão de saída do capacitor 1 igual à tensão do circuito $R_1 \cdot C_1$ isolado.



Impedância do Circuito 2 : $Z(s) = R_2 + \frac{1}{C_2 s}$

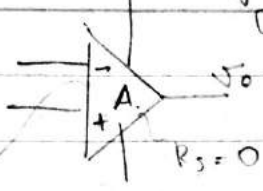
1) K_1 cte, $C_2 \uparrow$
 $R_2 \gg R_1 \Rightarrow$ acoplamento pequeno

$R_2 \downarrow \Rightarrow$ acoplamento aumenta



elemento ativo

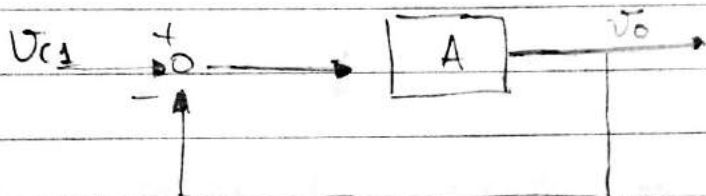
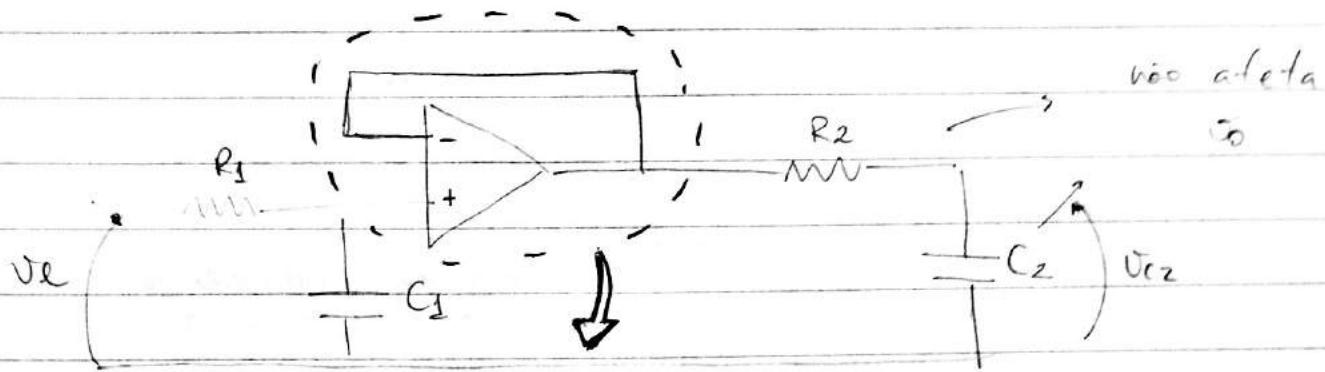
V_{cc} injeta energia no circuito



$$V_o = A(V_i - V_-)$$

A mto grande

$R_i = \infty$



$$\frac{V_o(s)}{V_{c1}(s)} = \frac{A}{A+1}$$

$$A \gg 1$$

Portanto, tensão de saída do AmpOp não depende de elementos adicionais como R_2 e C_2 .

$$\frac{V_o}{V_{c1}} \cong 1$$

$$V_{c1}(s) = V_e(s) \cdot \frac{1}{R_1 C_1 s + 1} = G_1(s) \cdot V_e(s)$$

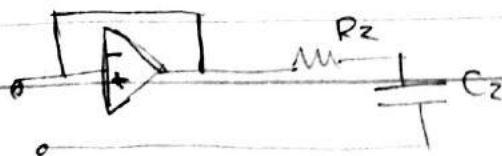
$$V_s(s) = V_{c2}(s) = V_o(s) \cdot \frac{1}{R_2 C_2 s + 1} = G_2(s) V_o(s) = G_2(s) \cdot V_{c1}(s)$$

$$\Rightarrow V_s(s) = V_e(s) G_1(s) \cdot G_2(s)$$

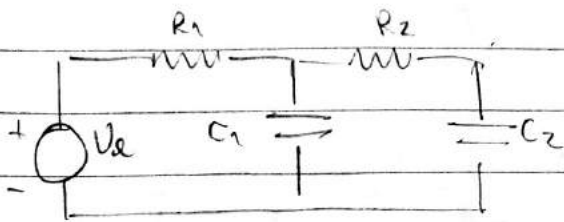
(*) Toda corrente vai para C_1 , circuito 2 não drena corrente. Assim, chegamos em relação s/ termo de acoplamento



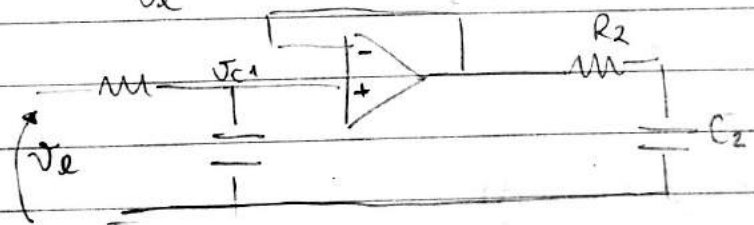
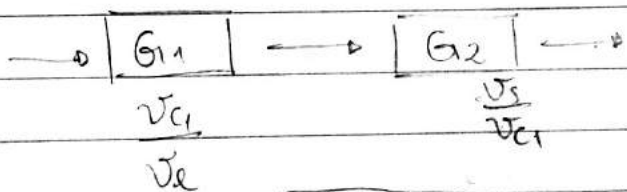
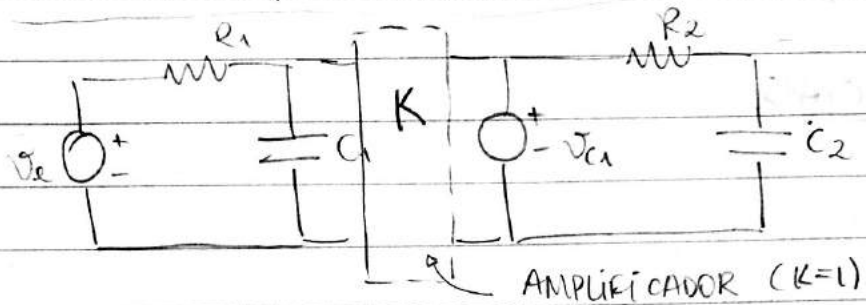
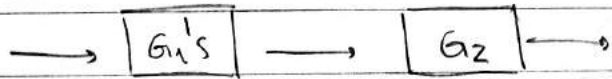
SENSOR
DE MEDIÇÃO
DE TENSÃO V_{c1}



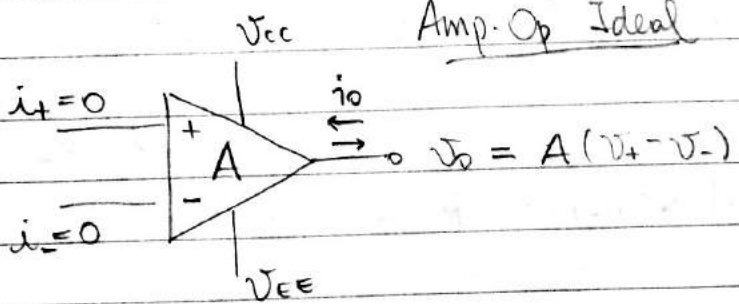
Revisão Aula Passada



$$G_1(s) = \frac{R_2 C_2 s + 1}{R_1 R_2 C_1 C_2 s^2 + R_1 (C_1 + C_2) s + R_2 C_2 s + 1}$$



Amp. Op Ideal



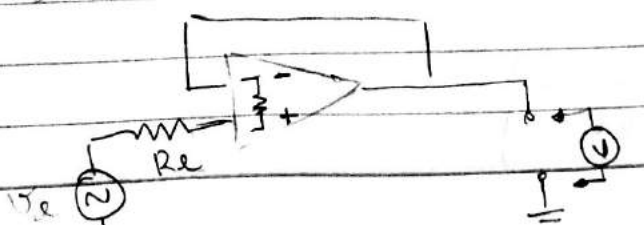
1. $V_o = A(V_+ - V_-)$
2. $i_+ = i_- = 0, R_i = \infty$
3. $R_o = 0 \Rightarrow V_o = A(V_+ - V_-)$

independente da corrente
dissipada na saída

4. $A = \infty \gg 1$

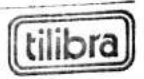
INFLUENCIA DAS RESISTÊNCIAS DE ENTRADA E SAÍDA

1) $R_i \neq 0$



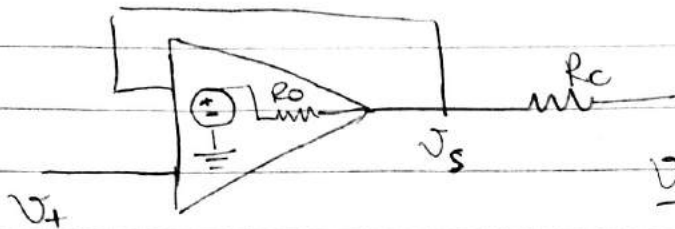
$$V_+ = V_e - R_e i$$

$\neq 0$ maior $R_i \Rightarrow i \Rightarrow V_+ \rightarrow V_e$

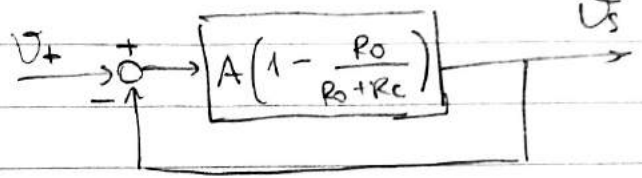


///

2) $R_0 \neq 0$



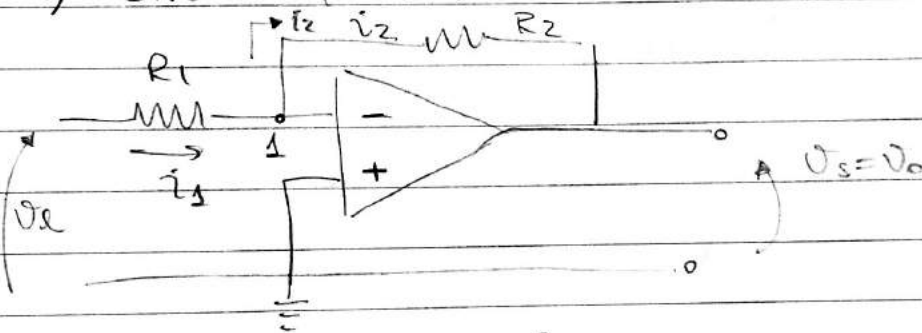
$$A(V_+ - V_-) - R_0 i = R_c i$$



$$R_0 \downarrow \Rightarrow V_s \rightarrow A(V_+ - V_-)$$

CASOS ESPECIAIS

1) Circuito Inversor



Nó 1: $i_1 = i_2 + i_-$

$$i_1 = \frac{V_e - V_-}{R_1}$$

$$i_2 = \frac{V_- - V_0}{R_2}$$

$$\frac{V_e - V_-}{R_1} = \frac{V_- - V_0}{R_2} \quad (1)$$

$$\Rightarrow V_e = V_- \left(1 + \frac{R_1}{R_2}\right) - \frac{R_1}{R_2} V_0 \quad (2)$$

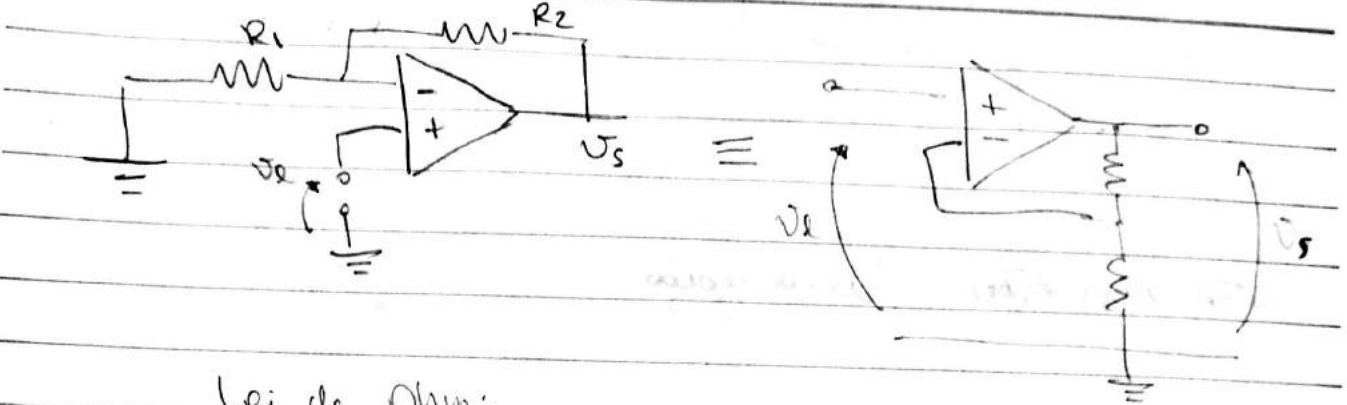
Além disso, $V_0 = A(V_+ - V_-) = -A V_- \quad (3)$

(3) \rightarrow (2)

$$V_e = -\frac{V_0}{A} \left(1 + \frac{R_1}{R_2}\right) - \frac{R_1}{R_2} V_0 \approx -\frac{V_0 R_1}{R_2} \quad |A| \gg 1$$

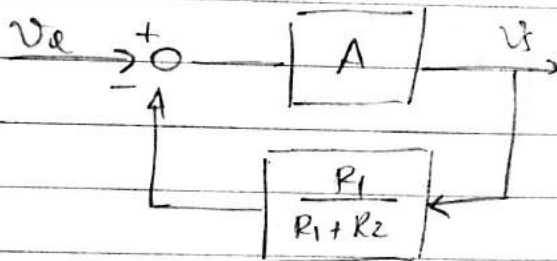
$$V_0 = -\frac{R_2}{R_1} V_e$$

2) Circuito Não-Inversor



Lei de Ohm:

$$\frac{V_-}{R_1} = \frac{V_s}{R_1 + R_2} \Rightarrow V_- = \frac{R_1}{R_1 + R_2} V_s$$



$$\frac{V_s}{V_e} = 1 + \frac{R_2}{R_1}$$

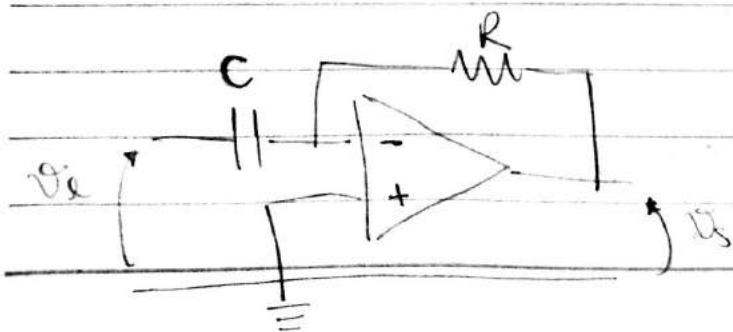
$$\frac{V_s}{V_e} = \frac{A}{1 + \frac{A R_1}{R_1 + R_2}} = \frac{1}{\frac{1}{A} + \frac{R_1}{R_1 + R_2}} = 1 + \frac{R_2}{R_1}$$

Combinando-se estes resultados com o conceito de impedâncias complexas, podemos construir outras operações com circuitos baseados no amp-op.

— R — $Z(s) = R$

— L — $Z(s) = Ls$

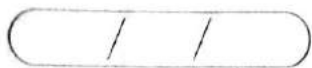
— C — $Z(s) = \frac{1}{Cs}$



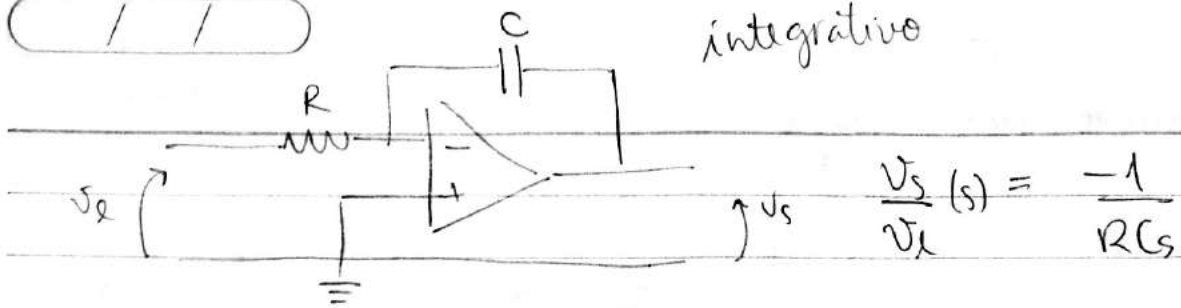
$$V_s(s) = \frac{-R}{1/Cs} V_e(s)$$

$$\frac{V_s(s)}{V_e} = -RCs$$

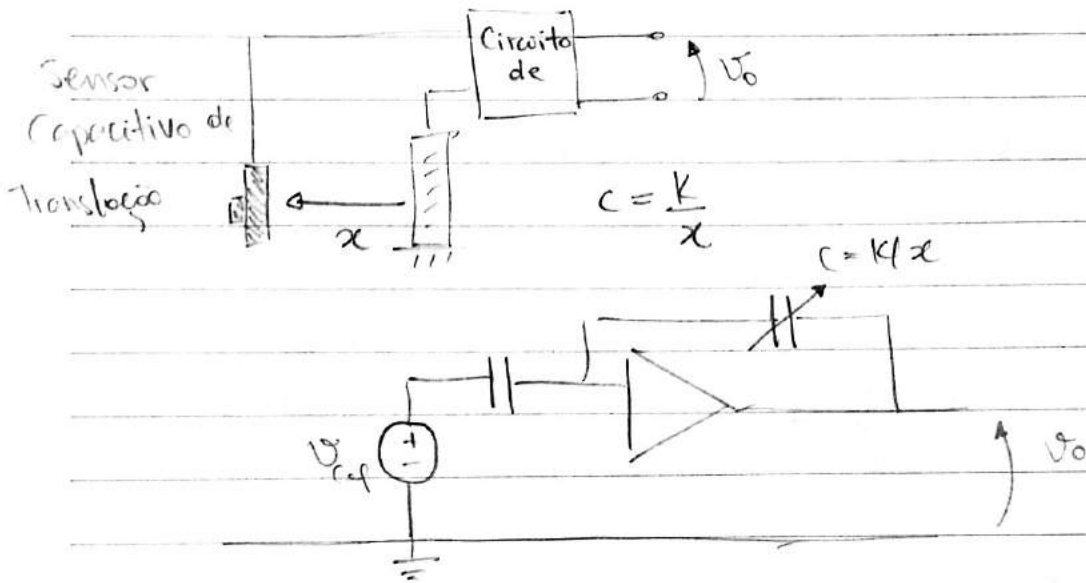
derivativo



integrativo



OUTRA APLICAÇÃO: Linearização



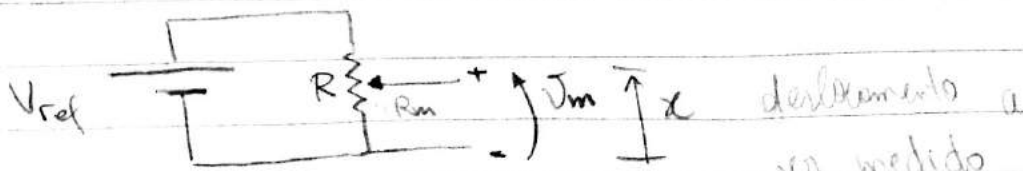
$$V_s(s) = \frac{-1/Cs \cdot V_{ref}(s)}{1/Ces}$$

p/ $V_{ref}(s) \equiv$ diagrama de amplitude

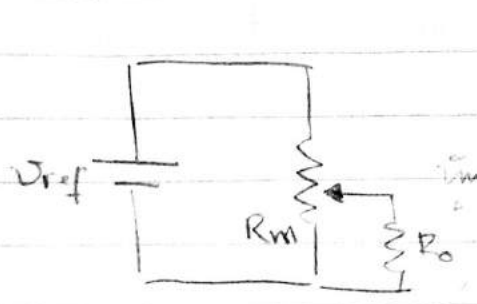
$$V_s(s) = -\frac{V_{ref} C_{ref}}{K} x(s)$$

Sensores de Deslocamento Linear e Angular

1. Potenciômetro Linear



Na prática, temos o efeito da carga = (o circuito de medida de tensão, acoplado ao potenciômetro possui impedância finita).



$$R \text{ --- } L$$

$$R_m \text{ --- } \alpha$$

$$R_m = \frac{\alpha R}{L}$$

$$R_{eq} = \frac{R_m P_o}{R_m + P_o} = \frac{R \frac{\alpha}{L} \cdot P_o}{R \frac{\alpha}{L} + P_o}$$

Lei de Ohm:

$$\frac{U_m}{R_{eq}} = \frac{U_{ref}}{R(1 - \frac{\alpha}{L}) + R_{eq}}$$

$$\therefore U_m = U_{ref} \left(\frac{R_{eq}}{R(1 - \frac{\alpha}{L}) + R_{eq}} \right) = U_{ref} \cdot \frac{1}{\frac{R(1 - \frac{\alpha}{L})}{R_{eq}} + 1} \quad (1)$$

Temos:

$$\frac{R(1 - \frac{\alpha}{L})}{R_{eq}} = \frac{R(1 - \frac{\alpha}{L})}{\frac{R \frac{\alpha}{L} \cdot P_o}{R \frac{\alpha}{L} + P_o}} = \frac{(R/L)R + P_o(1 - \frac{\alpha}{L})}{\frac{\alpha}{L} \cdot P_o}$$

$$= \left(\frac{R}{P_o} + \frac{L}{\alpha} \right) \left(1 - \frac{\alpha}{L} \right) \quad (2)$$

Portanto: $U_m = U_{ref} \frac{1}{\left(\frac{R}{P_o} + \frac{L}{\alpha} \right) \left(1 - \frac{\alpha}{L} \right) + 1} = U_{ref} \frac{1}{\frac{R - R/L + L}{P_o \alpha} + 1}$

$$U_m = U_{ref} \frac{\alpha}{L} \left[\frac{R}{P_o} \left(\frac{\alpha}{L} \right) \left(1 - \frac{\alpha}{L} \right) + 1 \right]^{-1}$$

RELAÇÃO NÃO LINEAR EM α

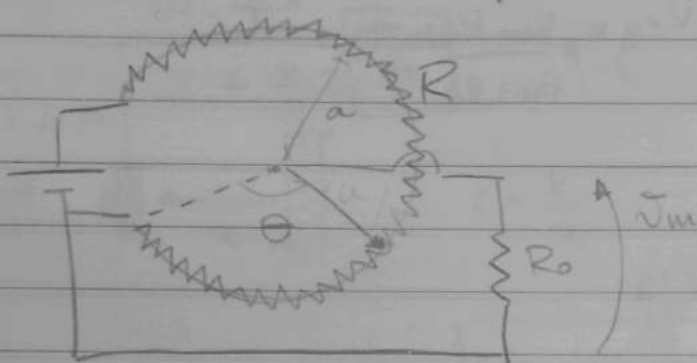
GOSTARÍAMOS DE LINEARIDADE

$$U_m = K \alpha$$

$$p/ \frac{R}{R_0} \rightarrow 0 \Rightarrow \left| \bar{V}_m = \bar{V}_{ref} \cdot \frac{x}{L} \right| = Kx$$

Portanto, é importante utilizarmos um circuito de medida de alta impedância face à impedância do sensor

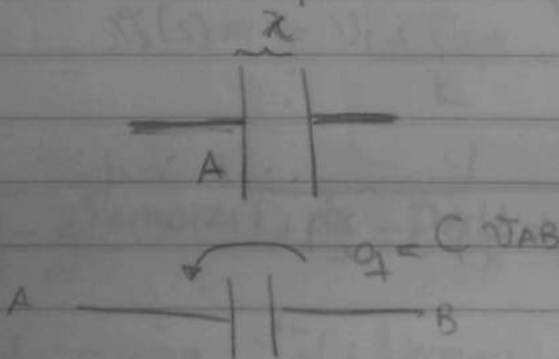
2. Potenciômetro Angular



$$\bar{V}_m = \bar{V}_{ref} \frac{\theta}{\theta_{max}} \left[\frac{1 + \frac{R}{R_0} \left(\frac{\theta}{\theta_{max}} \right) \left(1 - \frac{\theta}{\theta_{max}} \right)}{\theta_{max}} \right]^{-1}$$

$$p/ \frac{R}{R_0} \rightarrow 0 : \left| \bar{V}_m = \theta \frac{\bar{V}_{ref}}{\theta_{max}} \right|$$

3. Sensores Capacitivos

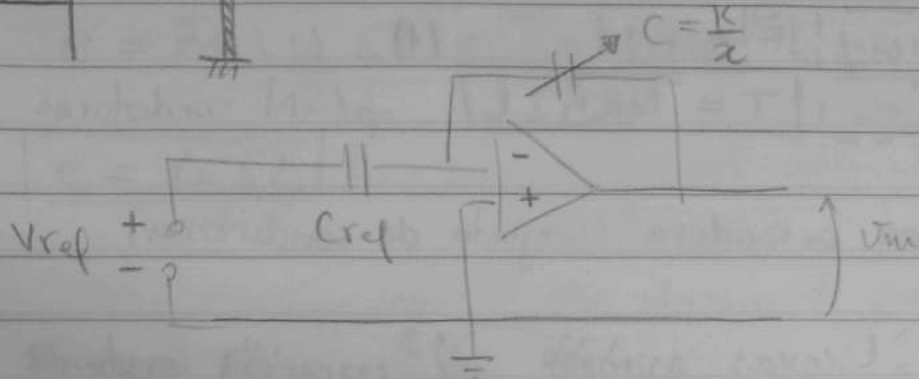
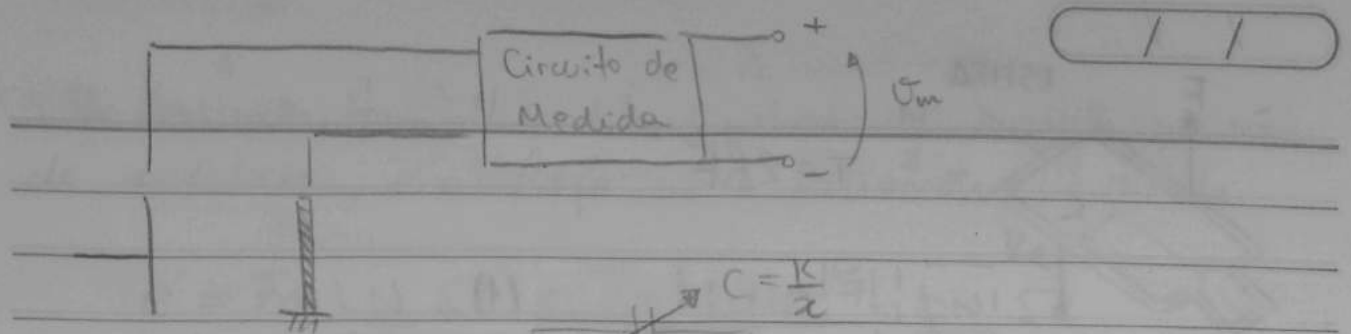


$$C = \frac{\epsilon A}{x}$$

ϵ = constante dielétrica
 área comum, sobreposta entre chapas
 x = espessura do dielétrico

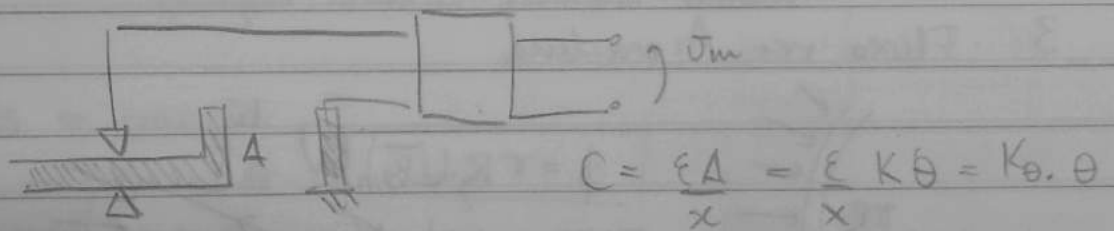
p/ V_{AB} de x e C variável

$$i = V_{AB} \frac{dC}{dt}$$



$$V_m = - \frac{C_{ref} \cdot V_{ref}}{K} \cdot s = K' \cdot s$$

4. Capacitivo para deslocamento angular



Podemos medir vel. angular

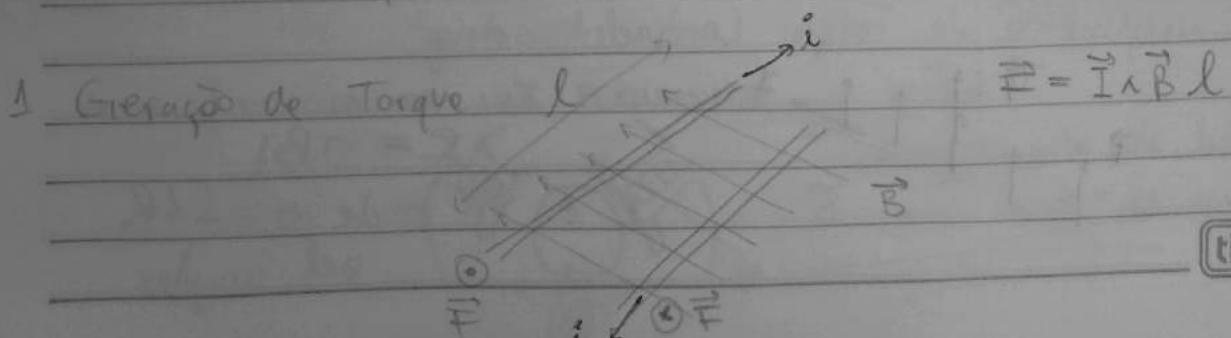
$$\therefore i = V_{ref} \cdot \frac{dC}{dt} = V_{ref} \cdot K_\theta \frac{d\theta}{dt} \Rightarrow \left| \frac{d\theta}{dt} = \frac{1}{K_\theta} \cdot i \right|$$

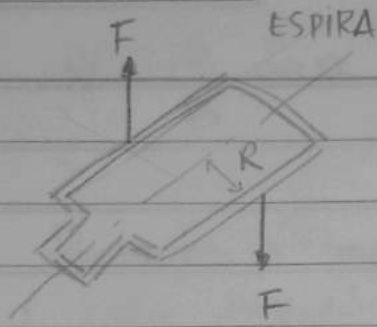
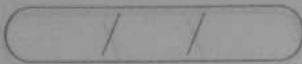
Sistemas Eletromecânicos

Motor CC de Escovas de Ímã Permanente

LIVRO:

Kenjo



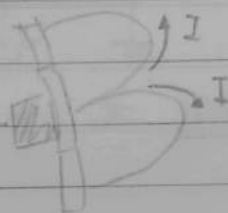
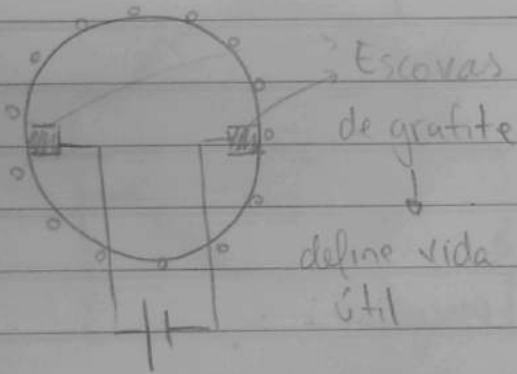


$$T = 2RF$$

$$|F| = BIL \quad (1)$$

$$T = NRBI L \quad \text{p/ } N \text{ condutores}$$

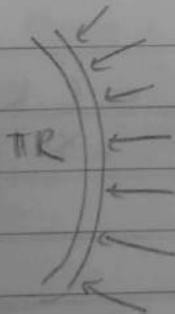
2. Corrente na armadura (conjunto de condutores)



Por construção:

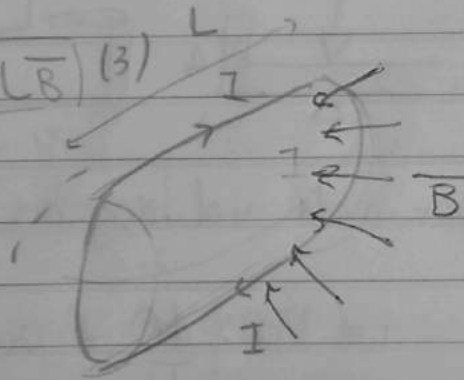
$$I_a = 2I \quad (2)$$

3. Fluxo na Armadura



$$\phi = \pi R L B \quad (3)$$

FLUXO

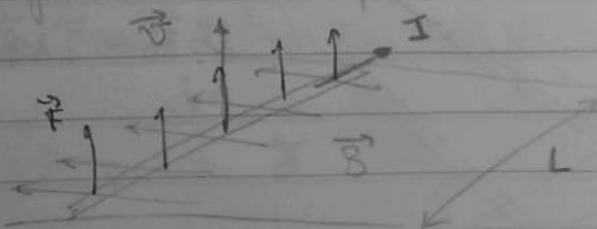


(2), (3) em (1):

$$T = \frac{N\phi}{2\pi} I_a = K_T I_a$$

$$T = K_T I_a$$

4. Surgimento de Força Contraeletromotriz



$$e = \sigma BL$$

$$\text{onde } \sigma = \omega R$$

vel. angular

Por construção, para N condutores, 2 polos N arranjos em série de condutores, e a força contra eletromotriz resulta

$$e = \frac{B R L N}{2} \Omega = \frac{B \pi R L N^2}{2\pi} \cdot \frac{2\pi}{N} \Omega = K_e \Omega$$

MOTORES ELÉTRICOS em ROBÓTICA

Desvantagem: rotação ↑ torque ↓

Vantagem: alta exatidão e movimento contínuo, relações lineares facilitam controle

FATORES DE SELEÇÃO

- Fonte de Alimentação
- Condições ambientais
- Consumo e Manutenção
- Controlabilidade

- Partes do Motor
- ROTOR
 - ESTATOR
 - SISTEMA DE CAMPO : produção de fluxo magnético
 - ARMADURA : produção de corrente

* Comutador em contato com escovas

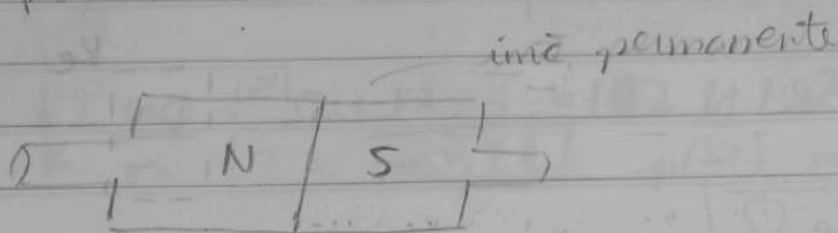
MOTOR S/ ESCOVA ⇒ IMÃ PERMANENTE NO ROTOR



chaveamento de bobinas pelo sist. de controle

MOTOR DE PASSO: vantagem \rightarrow conecta direto em sist. digital
 \hookrightarrow pode trabalhar em malha aberta

motor híbrido



Motor CC Imã Permanente de Escovas

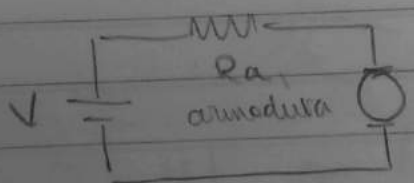
RELAÇÕES FUNDAMENTAIS

$T = K_t I_a$ ϕ unidades coerentes:
 $E = K_e \Omega$ $K_t = K_e$

Análise Estática

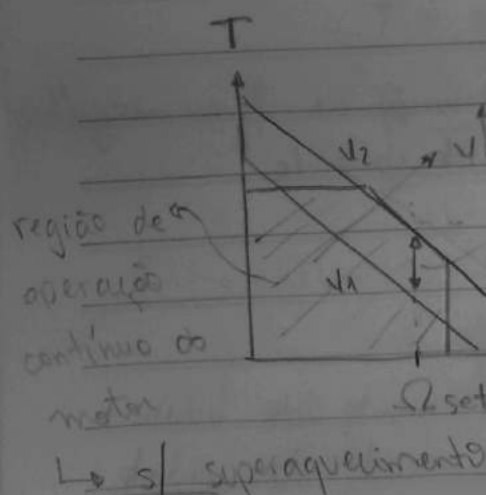
$V = cte$, $\Omega = cte$
 circuitos Equivalentes

$V = R_a I_a + K_e \Omega$
 $I_a = \frac{V - K_e \Omega}{R_a}$



Portanto, o torque pode ser expresso por:

$T = K_t I_a = \frac{K_t V}{R_a} - \frac{K_t K_e \Omega}{R_a} = K - K_m \Omega$



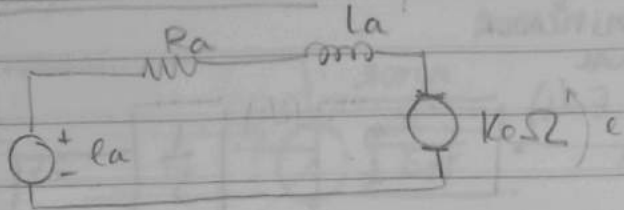
circuitos do controle acerta V para manter $\Omega = cte$.

LIGA/DESLIGA

1 / 1

Análise Dinâmica

efeito da indutância da armadura

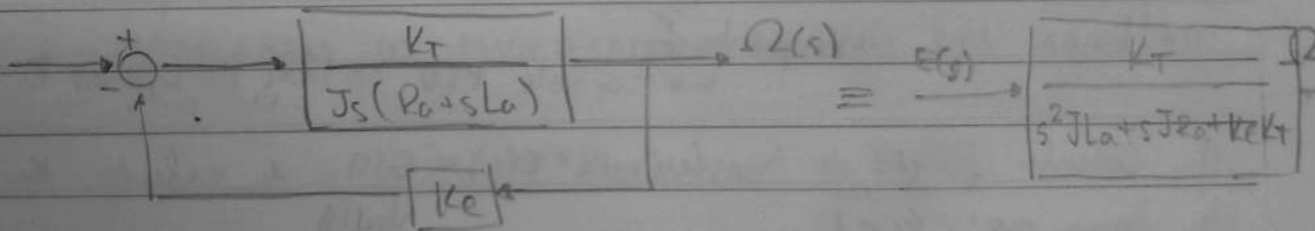
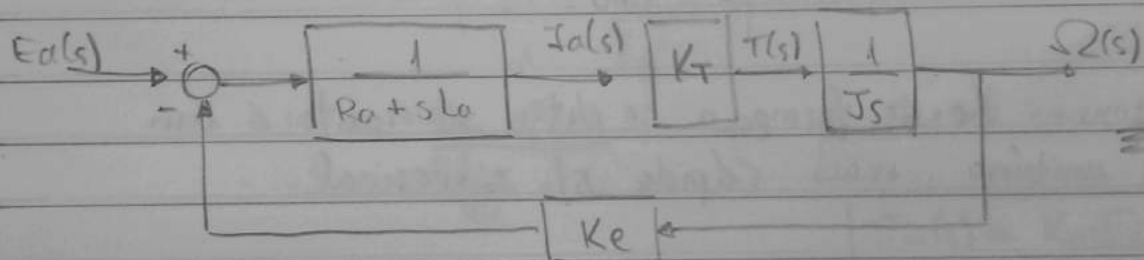


$$E_a(s) = R_a I_a(s) + L_a s I_a(s) + K_e \Omega(s)$$

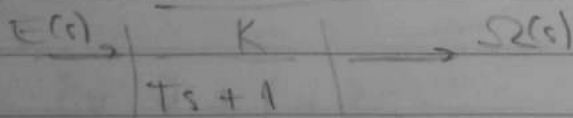
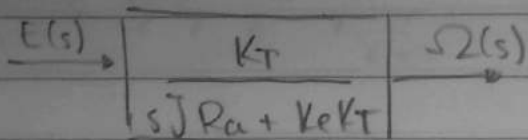
$$E(s) = K_e \Omega(s)$$

$$T(s) = K_T I_a(s) = J s \Omega(s)$$

$J \equiv$ momento de inércia de carga em relação ao eixo do motor

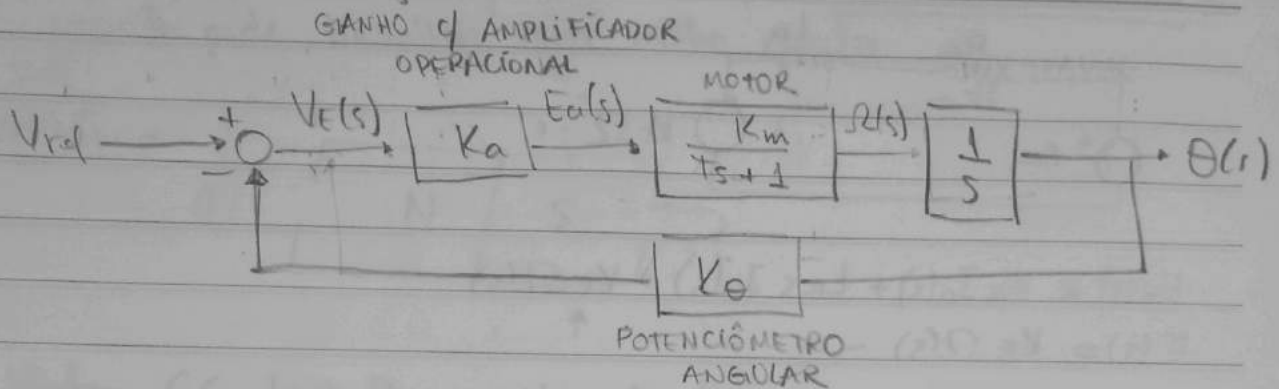


pl $L_a \approx 0$:



$$\left. \begin{aligned} K &= 1/K_e \\ T &= \frac{J R_a}{K_T K_e} \end{aligned} \right\}$$

Exemplo de Aplicação: Servo Mecanismo



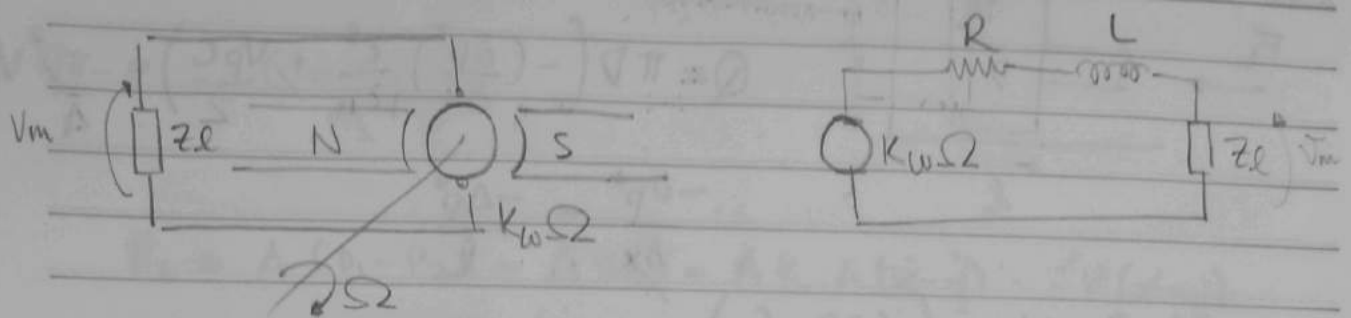
Suponha o diagrama de blocos anterior representando o sistema de controle de posição da pena plottum, onde: plotter

$$K_m = 0.25 \quad T_m = 0.5 \quad K_e = 1$$

- 1) Determine K_a de forma a se obter a resposta a um degrau unitário, mais rápida e sobressinal.
- 2) Calcule a cte de tempo do sistema. Como você pode melhorar esse desempenho?
↳ controle PID, ao invés de apenas P.

1) Calcule função de transferência $F(s) = \frac{\Theta(s)}{V_{ref}(s)}$ e calcule K_a de forma que $\zeta = 1$.

Sensor de Velocidade: Talogenerador



$$e = K_w \Omega(t) = R i(t) + L \frac{di(t)}{dt} + Z_e i$$

$$K_w \Omega(s) = R I(s) + L s I(s) + Z_e I(s)$$

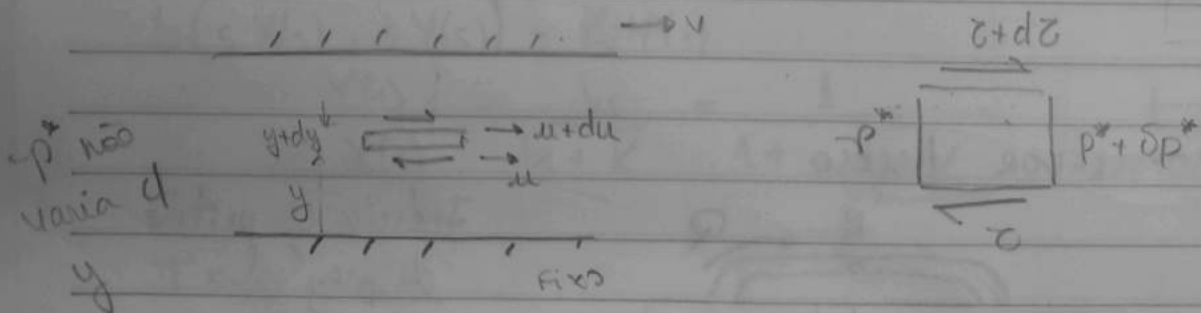
$$= \left(\frac{R}{Z_e} + \frac{Ls}{Z_e} + 1 \right) \tilde{V}_m(s)$$

impedância do sensor

$$\tilde{V}_m(s) = \frac{K_w \cdot \Omega(s)}{\frac{R+Ls}{Z_e} + 1} \Rightarrow \text{pt } Z_e \gg R+Ls :$$

$$\tilde{V}_m(s) = K_w \Omega(s)$$

MODELAGEM DO AMORTECEDOR VISCOZO



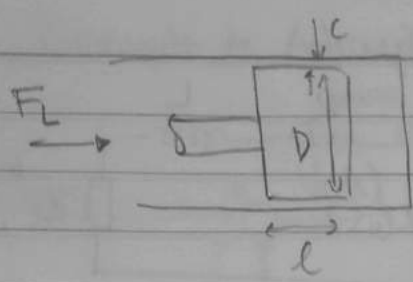
p^* não varia d

$$-\delta p^* dy dz + d\tau \cdot dz dx = 0$$

$$\frac{\delta p}{\delta x} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right)$$

$$\left| \frac{\delta p^*}{\delta x} = \frac{\partial \tau}{\partial y} \right|$$

$$\frac{\delta p^*}{\delta x} \frac{y^2}{2} = \mu u + C_1 y + C_2$$



Admitindo $b = \pi D$

$$Q = \pi D \left(- \left(\frac{\partial p^*}{\partial x} \right) \frac{c^3}{12\mu} + \frac{V_p c}{2} \right) = \frac{-\pi D^2 V_p}{4}$$

$$\frac{-\delta p^*}{\delta x} = \frac{-\Delta p^*}{l}$$

$$\Delta p^* = V_p \frac{\mu l}{c^3} (3D - 6c)$$

$$\Delta p^* = \frac{Q \mu l \cdot 12}{\pi D} \left(1 - \frac{2c}{D} \right) \quad R$$

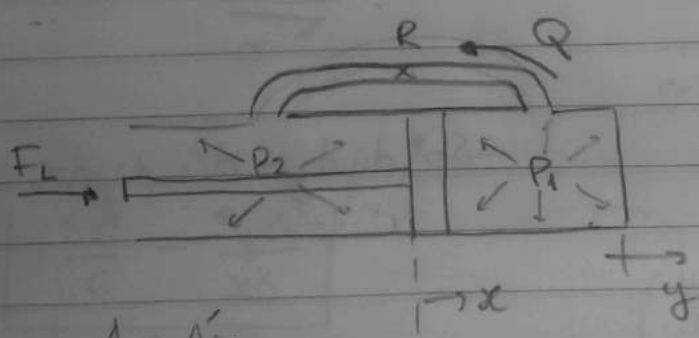
Resistência ao Fluxo nos Conais: $R = \frac{\mu l 12}{\pi D} \left(1 - \frac{2c}{D} \right)$

Desprezando cisalhamento e inércia do pistão:

$$\Delta p^* A_p = F_L$$

* Mechanics of Fluids (B.S. Massey, John W. Smith)

AMORTECEDOR VISCOSO



Inércia do pistão desprezível

$$\textcircled{1} F_L = A(P_1 - P_2)$$

Eq. da Continuidade

A = Área Transversal do Pistão

$$\textcircled{2} \rho Q dt = A \rho (dx - dy)$$

$$Q = A(\dot{x} - \dot{y})$$

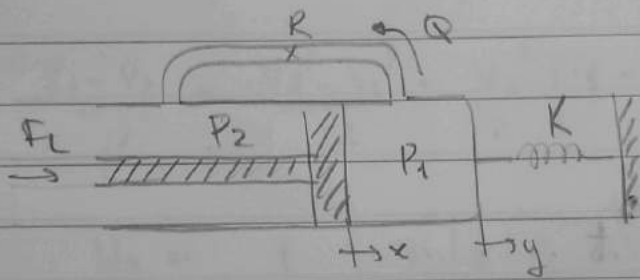
③ Escoamento laminar no conduto

$$Q = \frac{P_1 - P_2}{R = cts}$$

$$F_L = A(P_1 - P_2) = ARQ = AR A(\dot{x} - \dot{y}) = A^2 R(\dot{x} - \dot{y})$$

$F_L = b(\dot{x} - \dot{y})$ FORÇA DO AMORTECEDOR (Reação)
 $f_a = -b(\dot{x} - \dot{y})$

$A^2 R$



Desprezando inércia do conjunto:

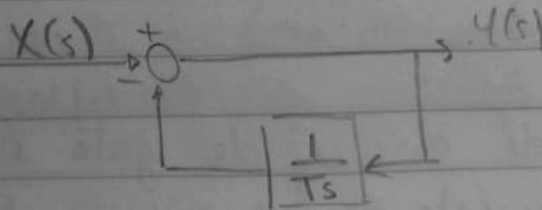
$$F_L - K y = 0 \Rightarrow b(\dot{x} - \dot{y}) - K y = 0$$

Admitindo C.T. nulas, aplicamos transf. de Laplace:

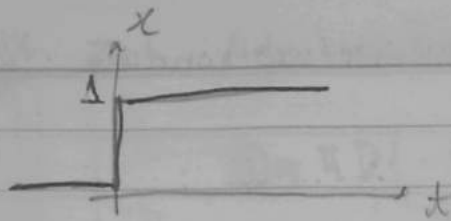
$$b(s X(s) - s Y(s)) = K Y(s)$$

$$\frac{Y(s)}{X(s)} = \frac{b s}{b s + K} = \frac{1}{1 + \frac{1}{\frac{b s}{K}}} = \frac{1}{1 + \frac{1}{T s}}$$

$$T = \frac{b}{K} = \frac{A^2 R}{K}$$



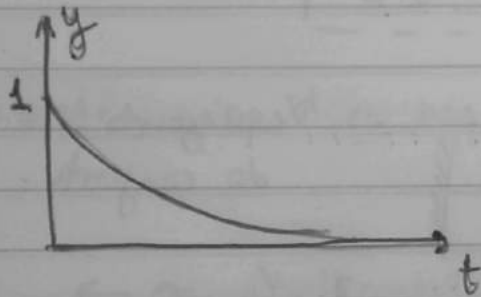
P/ $X(s) = \frac{1}{s}$



$$Y(s) = \frac{1}{s} \frac{1}{1 + \frac{1}{T_s}} = \frac{1}{s} \frac{T_s}{T_s + 1}$$

$$Y(s) = sT \left(\frac{1}{s} \frac{1}{T_s + 1} \right) \Rightarrow y(t) = \frac{dT}{dt} (1 - e^{-t/T})$$

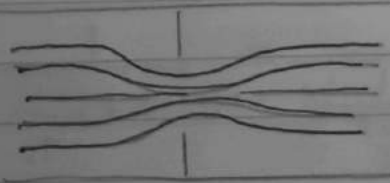
$$y(t) = e^{-t/T}$$



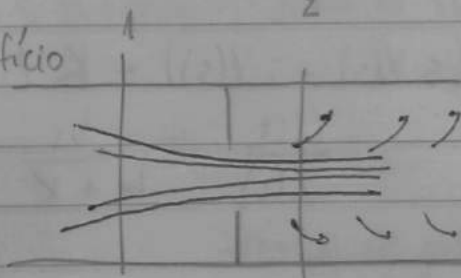
Servo Válvula

1. Introdução

Escoamento em orifício



Queda de pressão é devido a tensões de cisalhamento



Queda de pressão é devido à aceleração do fluido.

A situação onde a área de jato é mínima (seção 2) é chamado de "vena contracta".



Entre 1 e 2, o escoamento pode ser admitido

potencial, sempre segundo linhas de corrente

Vale a relação de Bernoulli:

$$\frac{\rho}{2} v_1^2 + P_1 + \rho g h_1 = \frac{\rho}{2} v_2^2 + P_2 + \rho g h_2$$

$$\frac{\rho v_1^2}{2} + P_1 = P_2 + \frac{\rho v_2^2}{2} \quad (1)$$

Além disso, vale eq. da Continuidade:

$$A_1 v_1 = A_2 v_2 \quad (2)$$

$$\Rightarrow \frac{P_1 - P_2}{\rho} = \frac{v_2^2 - v_1^2}{2} = \frac{v_2^2}{2} \left(1 - \left(\frac{A_2}{A_1} \right)^2 \right)$$

$$v_2 = \frac{1}{\sqrt{1 - \left(\frac{A_2}{A_1} \right)^2}} \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

$$Q_2 = A_2 v_2 = \frac{A_2}{\sqrt{1 - \left(\frac{A_2}{A_1} \right)^2}} \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

Coef. de Ajustes

1) A velocidade do jato é um pouco menor devido a efeitos viscosos (aparece o coef. de velocidade: C_v)

2) A_2 é diferente da área do orifício A_0 .
Aparece o coeficiente de contração: C_c .

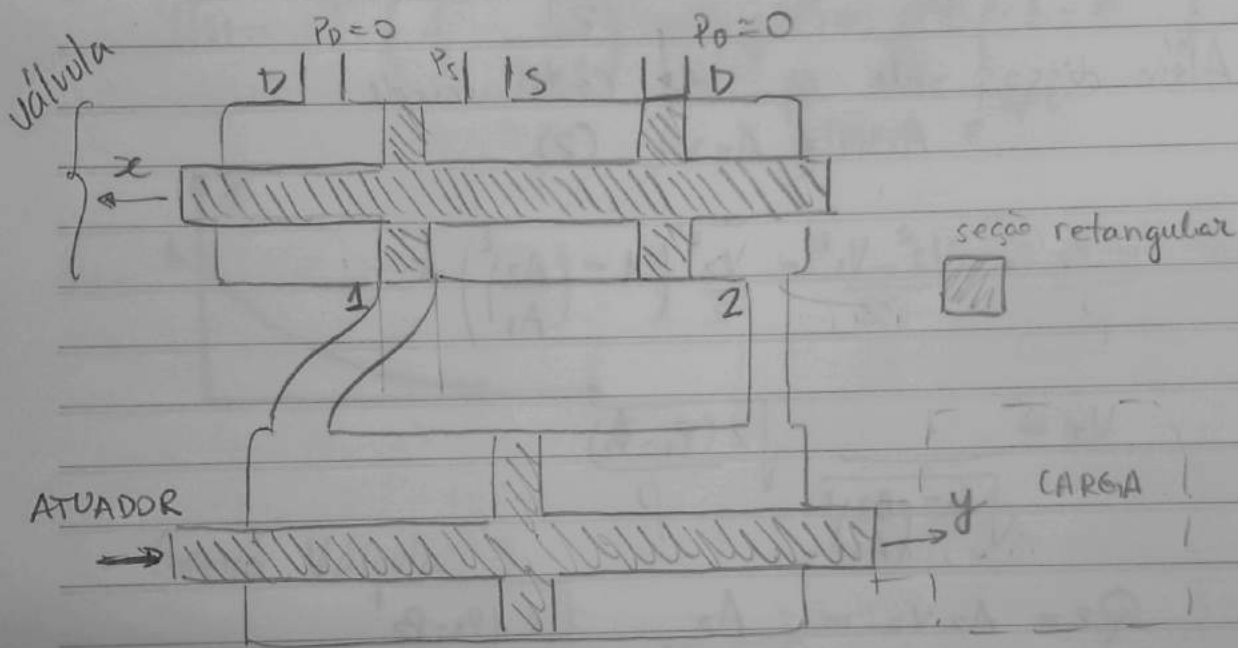
$$\therefore Q_2 = C_d A_0 \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

$$\text{Então } C_d = \frac{C_c C_v}{\sqrt{1 - C_c^2 \left(\frac{A_0}{A_1} \right)^2}} \quad [A_2 = C_c A_0] \quad \text{coef. de descarga}$$

Coef. de Descarga para orifícios pontiagudos,
regime permanente e $A_0 \ll A_r =$

Vamos admitir $C_d = 0,6$

SERVO VÁLVULA



Escoamento em Reg. Permanente

Trecho 1: V_c entre 5 e 1

$$P_s + \frac{\rho}{2} V_s^2 = P_1 + \frac{\rho}{2} V_1^2$$

$$Q_1 = C_d A_r \sqrt{\frac{2}{\rho} (P_s - P_1)}$$

$$A_r = L x$$

$$Q_1 = C_d L x \sqrt{\frac{2}{\rho} (P_s - P_1)}$$

$$= C_v x \sqrt{P_s - P_1}$$

com $C_v = C_d L \sqrt{\frac{2}{\rho}}$

Trecho 2: V_c entre 2 e D

$$Q_D = C_d L x \sqrt{\frac{2(P_2 - P_D)}{\rho}} = C_v x \sqrt{P_2}$$

$$A_D = Lx$$

A princípio admite-se escoamento incompressível, sem vazamentos, onde:

$$Q_1 = Q_D = Q$$

$$P_L = P_1 - P_2$$

$$P_1 = P_s - \left(\frac{Q}{C_v x}\right)^2 \quad P_2 = \left(\frac{Q}{C_v x}\right)^2$$

$$\Rightarrow Q = C_v x \sqrt{\frac{1}{2}(P_s - P_L)} \quad (1)$$

Atuador

$$Q = A_p \dot{y} \quad (2)$$

Força da

carga $F_L = P_1 A_p - P_2 A_D = A_p(P_1 - P_2) = A_p P_L$

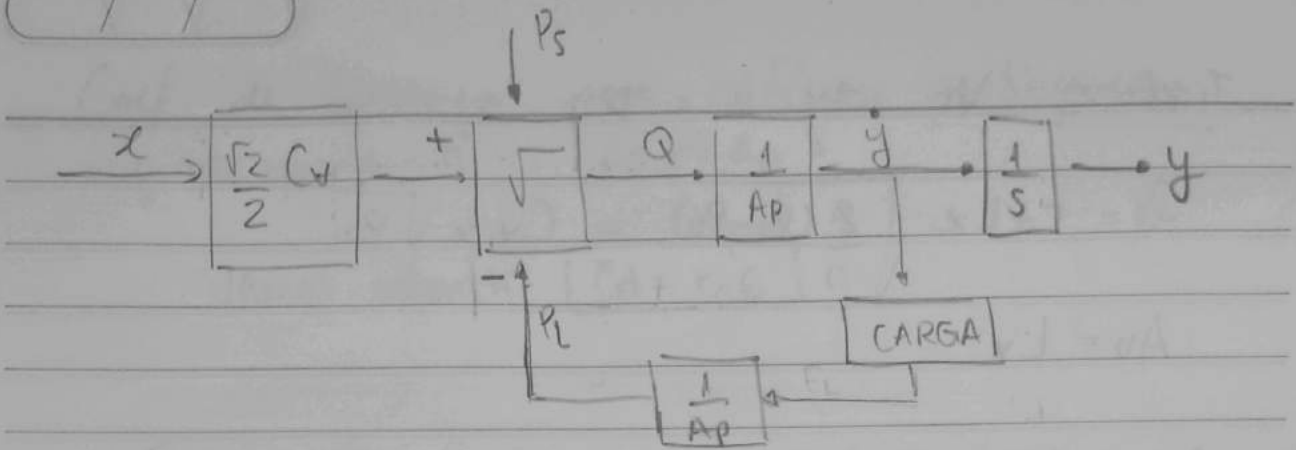
$$P_L = \frac{F_L}{A_p} \quad (3)$$

Inércia do Êmbolo Desprezível

Aplicando (3), (2) em (1):

$$A_p \dot{y} = C_v x \sqrt{\frac{1}{2} \left(P_s - \frac{F_L}{A_p} \right)}$$

$$\dot{y} = \frac{C_v}{A_p} \sqrt{\frac{1}{2} \left(P_s - \frac{F_L}{A_p} \right)} \cdot x \quad (4)$$



Linearização

$$Q = Q_0 + \left. \frac{\partial Q}{\partial x} \right|_{x=x_0} (x-x_0) + \left. \frac{\partial Q}{\partial P_L} \right|_{P_L=P_{L0}} (P_L-P_{L0})$$

$$Q = C_v \sqrt{\frac{\Delta(P_S - P_L)}{2}} \cdot x$$

$$\Rightarrow Q \approx \bar{Q} + \left. C_v \sqrt{\frac{1}{2}(P_S - P_L)} \right|_{P_L=P_{L0}} (x-x_0) + C_v \cdot x_0 \left. \frac{-1/2}{\sqrt{1/2(P_S - P_L)}} \right|_{P_L=P_{L0}} (P_L - P_{L0})$$

$$\Rightarrow q = K_1 x + K_2 \Delta P_L$$

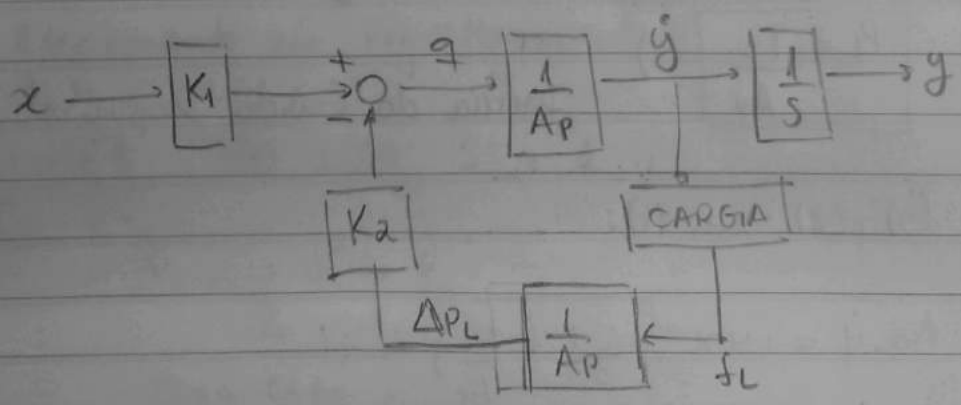
$$q = Q - \bar{Q} = A_p \dot{y}$$

$$x = X - X_0$$

$$\Delta P_L = P_L - P_{L0}$$

$$K_1 = C_v \left[\frac{1}{2}(P_S - P_{L0}) \right]^{1/2}$$

$$K_2 = \frac{C_v \cdot x_0}{2} \left[\frac{-1/2}{\sqrt{1/2(P_S - P_{L0})}} \right]$$

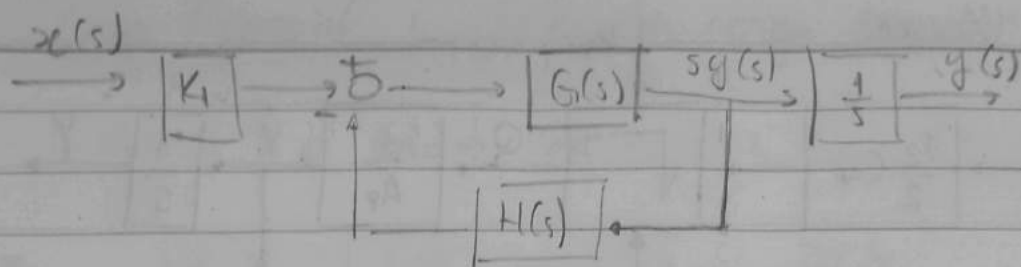


CARGA : Consideremos sist. massa-amortecados

$$m \ddot{y} + b \dot{y} = 0$$

$$m (\ddot{y} + \ddot{y}_0) + b (\dot{y} + \dot{y}_0) =$$

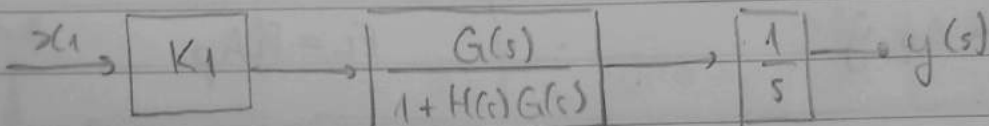
$$= (ms^2 + bs) Y(s) = (ms + b) s Y(s)$$



$$G(s) = 1/A_p$$

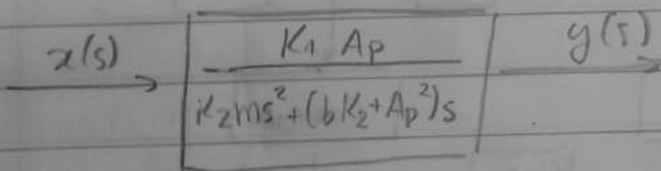
$$H(s) = (ms+b) \cdot \frac{1}{A_p} \cdot K_2$$

O diagrama de blocos pode ser reduzido a:



$$\frac{A_p}{\Delta_p^2 + (ms+b)K_2}$$

* Analytics Robotics and Mechanisms: Stadler



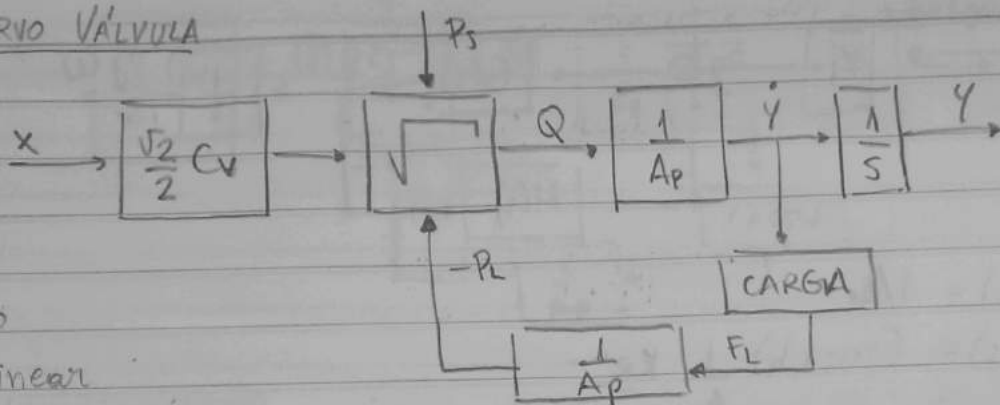
$$\frac{1}{s} \cdot \frac{\frac{K_1 A_p}{bk_2 + A_p^2}}{\left(\frac{mk_2}{bk_2 + A_p^2} s + 1\right)} = \frac{1}{s} \cdot \frac{K}{T s + 1}$$

$$K = \frac{K_1 A_p}{bk_2 + A_p^2} ; \quad T = \frac{mk_2}{bk_2 + A_p^2}$$

pl $\frac{mk_2}{bk_2 + A_p^2} \ll 1$, simplifico-se pl

$$\frac{Y(s)}{X(s)} = \frac{K}{s}$$

SERVO VÁLVULA



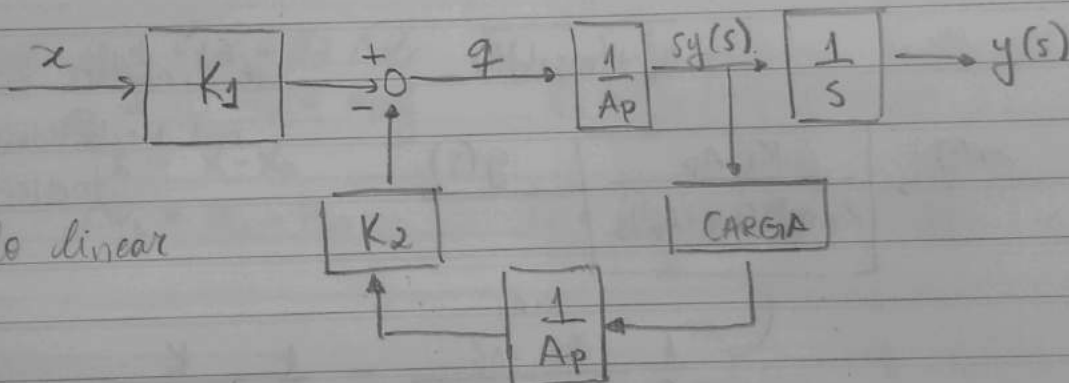
Modelo não-linear

$$Q = C_v x \sqrt{\frac{1}{2} (P_s - P_L)}$$

$$P_L = P_1 - P_2$$

$$Q = A_p \cdot \dot{y}$$

$$F_L = P_L A_p$$



Modelo linear

$$x = x - x_0$$

$$y = y - y_0$$

$$q = Q - Q_0$$

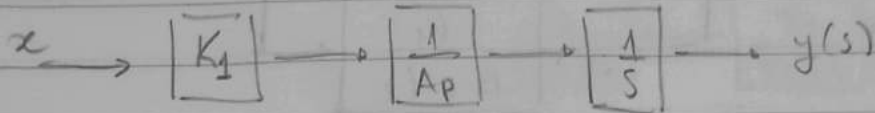
$$\Delta P = P_L - P_0$$

$$q = K_1 x - K_2 \Delta P \quad \text{com} \quad K_1 = C_v \left[\frac{1}{2} (P_s - P_L) \right]^{1/2}$$

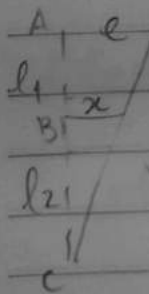
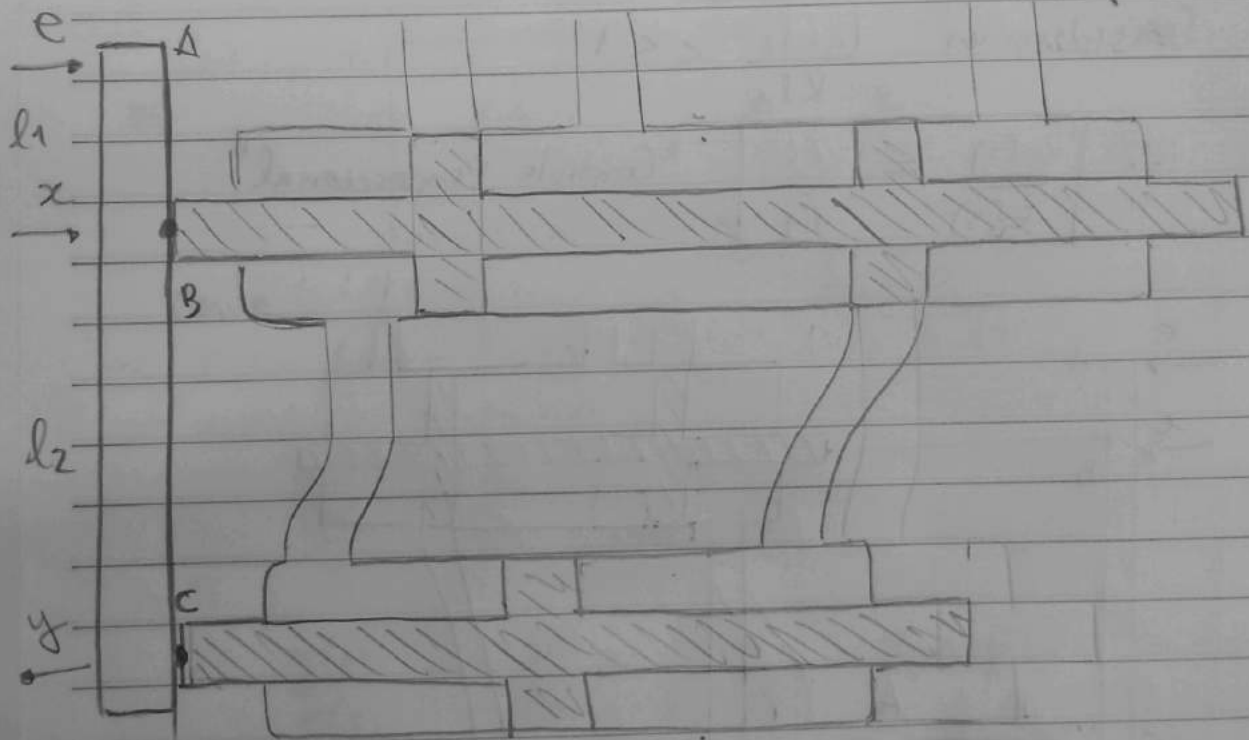
$$K_2 = C_v x_0 \left[\frac{1}{2} (P_s - P_L) \right]^{-1/2}$$

Caso Especial : efeito da carga pequena.

$x_0 = 0$ e vamos negligenciar $k_2 \neq (k_2 \approx 0)$



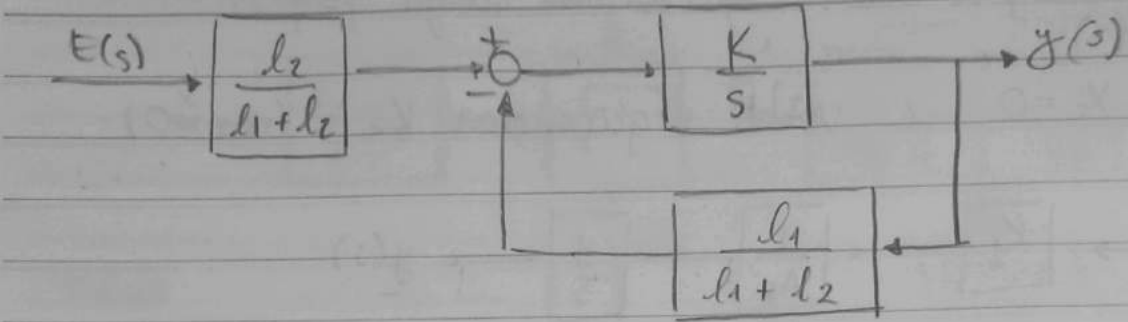
$$\frac{y(s)}{x(s)} = \frac{K_1/A_p}{s} = \frac{K}{s} \quad (\text{integrador})$$



Pequenos deslocamentos

$$e = x \Rightarrow x = \frac{l_2}{l_1+l_2} e \quad (1)$$

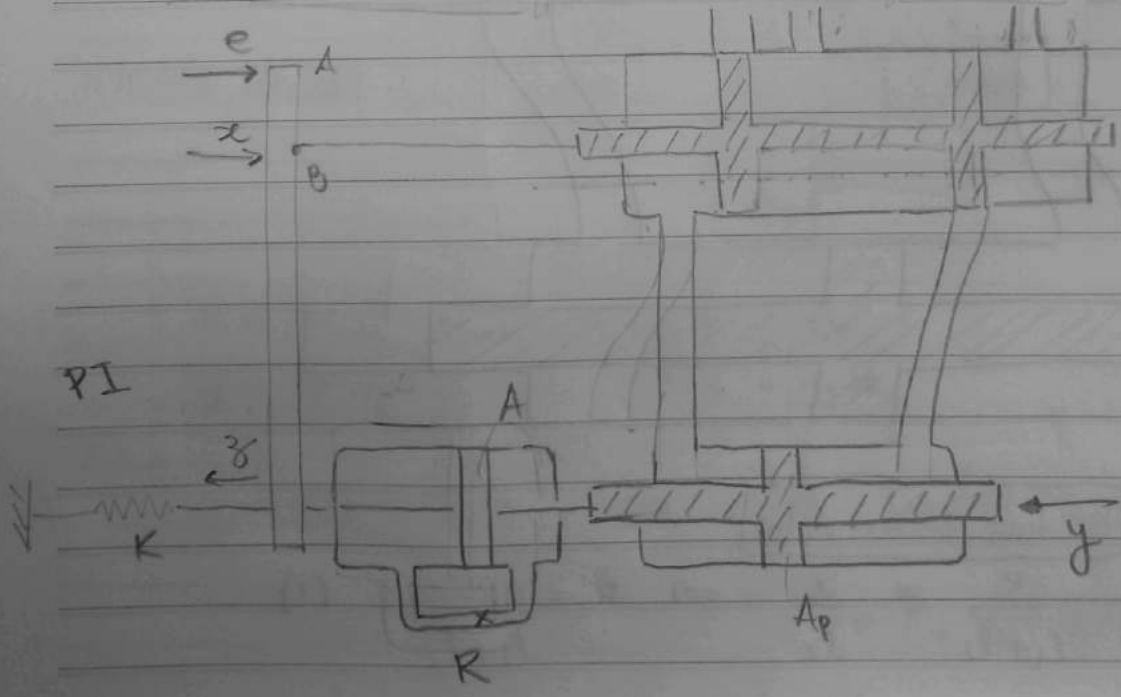
$$x = y \Rightarrow |x| = \frac{l_1}{l_1+l_2} y \quad (2)$$



$$\frac{Y(s)}{E(s)} = \frac{K l_2}{s(l_1+l_2)+K l_1} = \frac{l_2/l_1}{s(l_1+l_2)/K l_1 + 1}$$

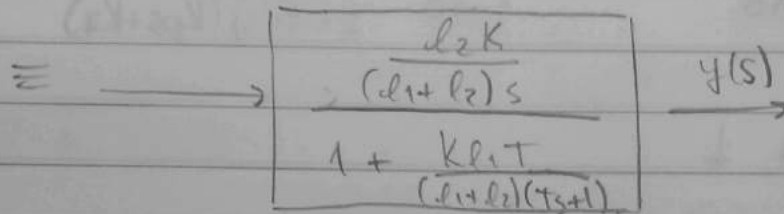
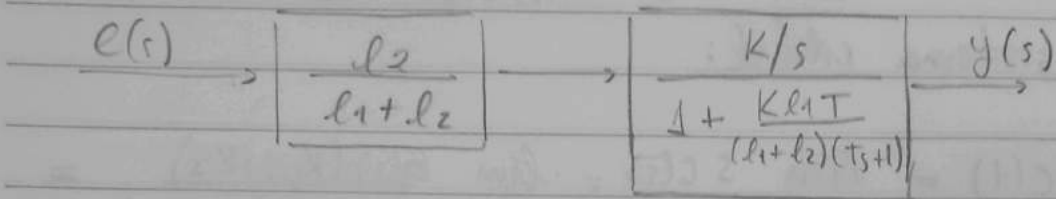
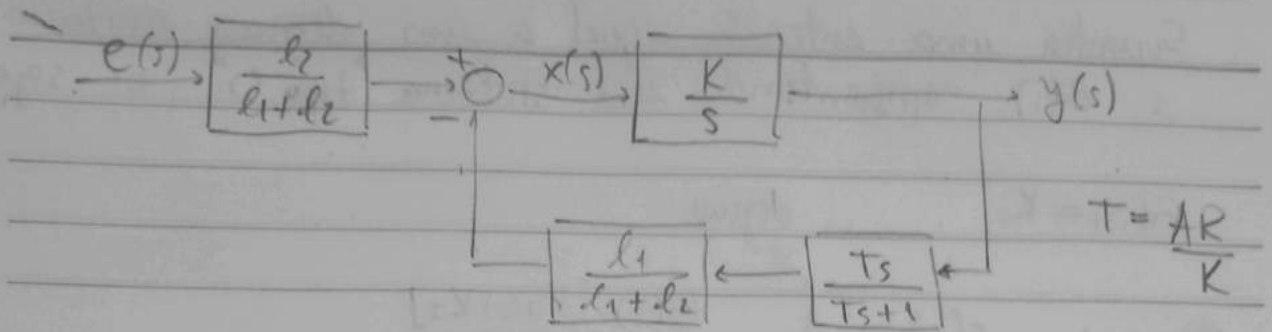
Consideramos $\frac{l_1+l_2}{K l_1} \ll 1$

$$\therefore \frac{Y(s)}{E(s)} = \frac{l_2}{l_1} \quad \text{"Control Proporcional"}$$

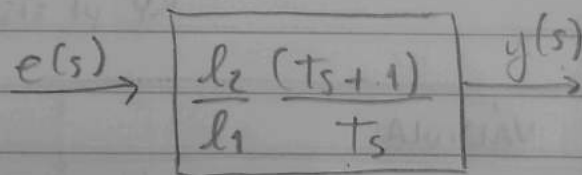


Pequeños Desplazamientos $x = \frac{l_2}{l_1+l_2} e$

$$|x| = \frac{l_1}{l_1+l_2} y$$



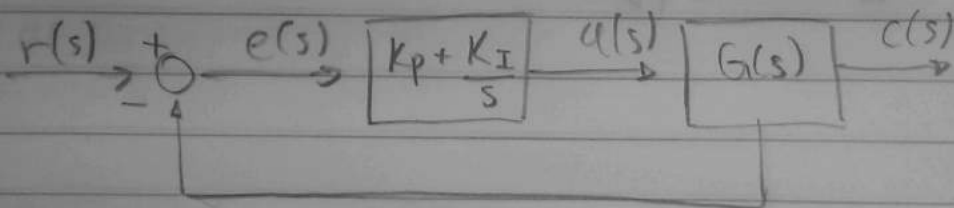
Como $\frac{K \cdot l_1 \cdot T}{(l_1 + l_2)(T_s + 1)} \gg 1$, simplificamos o diagrama:



$$\therefore \frac{y(s)}{e(s)} = \frac{l_2}{l_1} + \frac{l_2}{l_1} \frac{1}{T_s} = K_p + \frac{K_I}{s}$$

"Controle Proporcional Integral"

Efeito deste tipo de controle



$$\frac{c(s)}{r(s)} = \frac{G(s) [K_p + K_I/s]}{1 + G(s) [K_p + K_I/s]} = \frac{G(s) (K_p s + K_I)}{s + G(s) (K_p s + K_I)}$$

Suponha uma entrada igual a um degrau unitário e $G(s)$ representando um sistema "tipo 0", ou seja,

$$G(0) = K \quad \text{degrau}$$

$$\text{Tema: } c(s) = \frac{1}{s} \frac{G(s) \cdot [K_p s + K_I]}{s + G(s) [K_p s + K_I]}$$

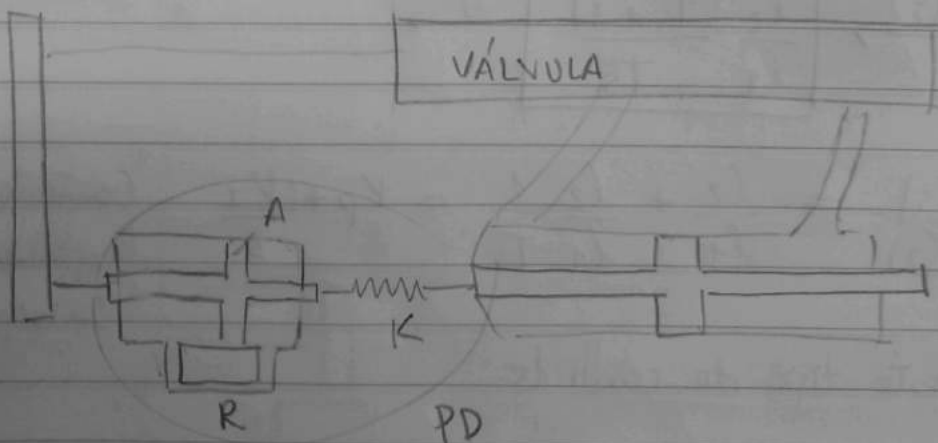
Supondo sistema estável:

$$\lim_{t \rightarrow \infty} c(t) = \lim_{s \rightarrow 0} s c(s) = \lim_{s \rightarrow 0} \frac{G(s) (K_p s + K_I)}{s + G(s) (K_p s + K_I)} =$$

$$= \frac{G(0) K_I}{G(0) K_I} = 1$$

$$\lim_{t \rightarrow \infty} \text{Erro}(t) = r(t) - c(t) = 0$$

Integrativo
retira erro em
RP pl sistema "tipo 0"



Efeito de Compressibilidade e Vazamento

$$\bar{q} = 0, \quad x_0 \neq 0, \quad p_1 = p_0, \quad y_0 = 0$$

Partindo do modelo linear:

$$q = K_1 x - K_2 \Delta p = \underbrace{A_p y}_{\text{variação de volume}} + q_L + q_c$$

Γ vazamento
 \hookrightarrow compressibilidade

$$q_L = L \Delta p \quad \Delta p = p_1 - p_0 \quad \text{no cilindro}$$

$$q_c = \frac{\rho V}{K} \dot{\Delta p} \quad \text{em qual ponto ótimo no metade do volume do cilindro}$$

$$q_v = A_p \cdot \dot{y}$$

APLICANDO LAPLACE

$$K_1 x(s) - K_2 \Delta P(s) = A_p s y(s) + L \Delta P(s) + K_c s \Delta P(s) \quad (1)$$

$$(ms^2 + bs + K) y(s) = A_p \Delta P(s) \quad (2)$$

vamos escolher carga como sendo sist. massa-mola-amortecedor

$$y(s) = \frac{A_p}{ms^2 + bs + K} \Delta P(s)$$

$$\text{Em (1), vem: } K_1 x(s) = \left[(K_2 + L) + \frac{A_p^2 s}{ms^2 + bs + K} + K_c s \right] \Delta P(s)$$

$$\Rightarrow G_n(s) = \frac{\Delta P(s)}{x(s)} = \frac{K_1 [ms^2 + bs + K]}{(ms^2 + bs + K)(K_c s + (K_2 + L)) + A_p^2 s}$$

$$G_1(s) = \frac{[ms^2 + bs + K]/A_p}{x(s)} y(s)$$

$$G_2(s) = \frac{y(s)}{x(s)} = \frac{A_p K_1}{(ms^2 + bs + k)(K_c s + (K_2 + L)) + A_p^2 s}$$

• $\bar{Q} \neq 0$, $x_0 \neq 0$, $P_L = P_0$, $\dot{y}_0 = 0$

$$Q = C_v x_0 \sqrt{\frac{1}{2}(P_s - P_{L0})} = A_p \dot{y}_0 + K P_{L0} + K_c \dot{P}_{L0} \rightarrow 0 \text{ (RP)}$$

$\neq 0$ $\neq 0$ \downarrow pode ser 0

No caso incompressível e sem vazamentos, teríamos:

$$\bar{Q} = C_v x_0 \sqrt{\frac{1}{2}(P_s - P_{L0})} = A_p \dot{y}_0$$

carga 1: $m \ddot{y} = F_L$

No equilíbrio $\rightarrow \ddot{y}_0 = 0$, $\dot{y}_0 = \text{cte} = \frac{\bar{Q}}{A_p}$

$$\Rightarrow P_{L0} = \frac{F_{L0}}{A_p} = 0$$

$$\bar{Q} = C_v x_0 \sqrt{\frac{P_s}{2}}$$

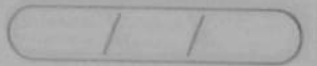
carga 2: $m \ddot{y} + b \dot{y} = F_L$

No equilíbrio $\rightarrow \ddot{y}_0 = 0$, $\dot{y}_0 = \text{cte}$

$$\Rightarrow F_{L0} = b \dot{y}_0 \rightarrow P_{L0} = \frac{b \dot{y}_0}{A_p}$$

p/ $x_0 \neq 0$ e $P_{L0} \neq P_s$, temos $\bar{Q} \neq 0$

vazamento já fica mais problemático



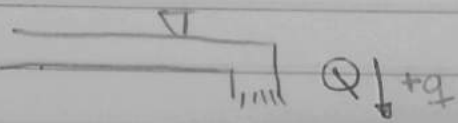
Carga 3: $m\ddot{y} + b\dot{y} + Ky$

No equilíbrio $\rightarrow \ddot{y}_0 = 0$

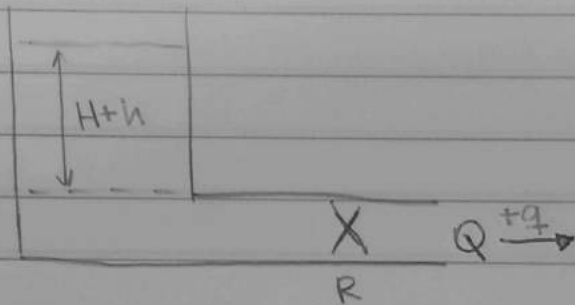
$$b\dot{y}_0 + Ky_0 = F_{10} = \text{cte}$$

$$\bar{Q} = A_p \dot{y}_0 = 0, \quad p_1 x_0 \neq 0$$

$$\Rightarrow \bar{Q} = 0 = C_v x \sqrt{\frac{1}{2}(\rho_2 - \rho_1)} \Rightarrow P_2 = P_1 = \frac{K y_0}{A_p}$$



Condição de Equilíbrio



$$H = \bar{H}$$

$$Q = \bar{Q}$$

$$Q = K\sqrt{H}$$

$$\bar{Q}^2 = K\bar{H}$$

$$c \frac{dH}{dt} = Q_i - Q_o$$

f(geometria)

Em relação a condição de equilíbrio:

$$q_i \rightarrow h$$

ENERGIA
CARETA

$$h - Rq_o = 0$$

CONTINUIDADE

$$ch = q_i - q_o$$

$$h(s) = Rq_o(s) \quad (1)$$

$$csh(s) = q_i(s) - q_o(s) \quad (2)$$

(2) em (1):

$$csh(s) = q_i(s) - \frac{h(s)}{R}$$

$$\rightarrow (RCs + 1)h(s) = Rq_i(s)$$

$$\Rightarrow \frac{h(s)}{q_i(s)} = \frac{R}{RCs + 1}$$