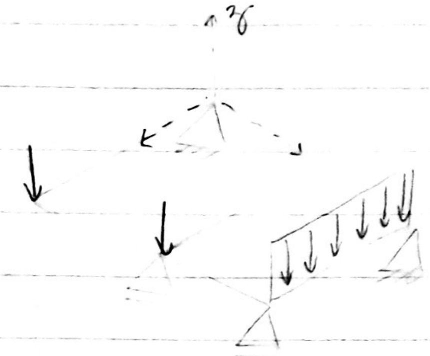
PEF - P2 - Lucas Novaki

Estruturas Tridimensionais

- ISOSTÁTICAS $\left[\begin{array}{l}\text{grelhas} \\ \text{pórticos espaciais}\end{array}\right.$

- EQUILÍBRIO $\left\{\begin{array}{l}\vec{R} = \vec{0} \\ \vec{M}_o = \vec{0}\end{array}\right.$

Grelhas: estrutura no plano e carregamento $\perp$ ao plano

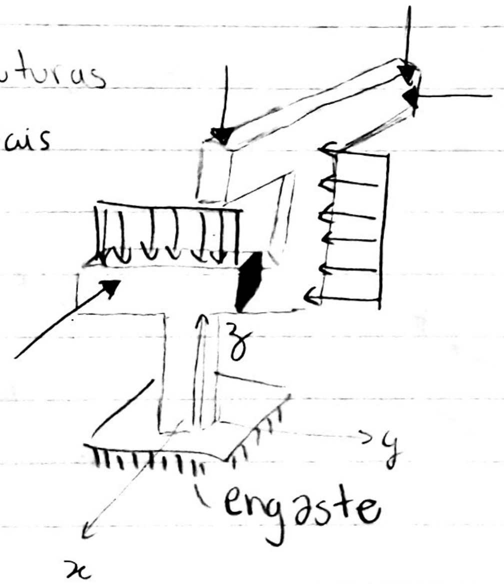


$R_x = 0$

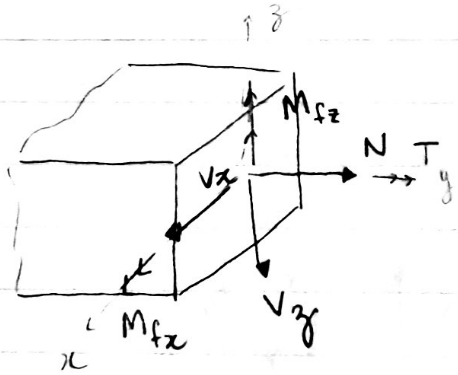
$R_y = 0$

$M_z = 0$

Estruturas Poligonais



No corte:

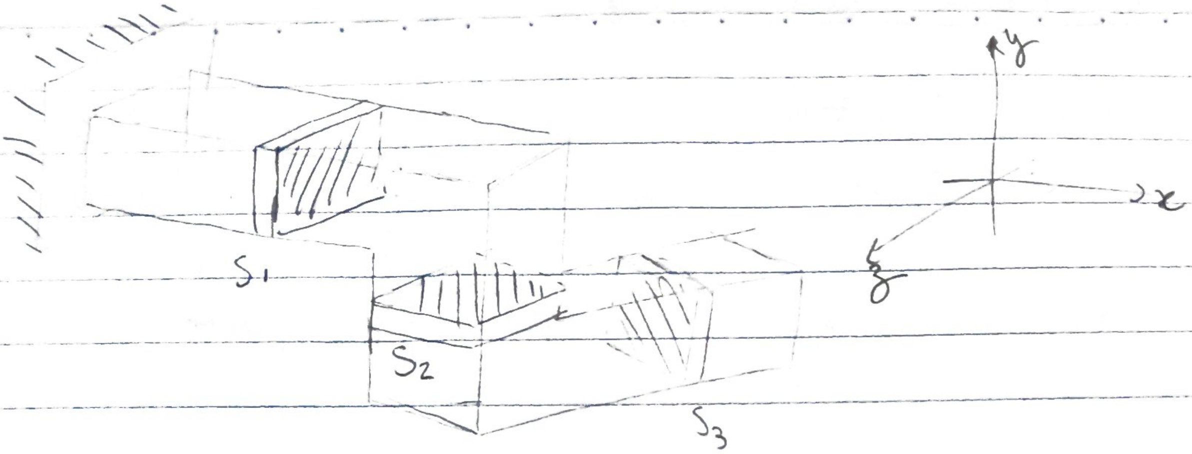


Convenção de sinais:

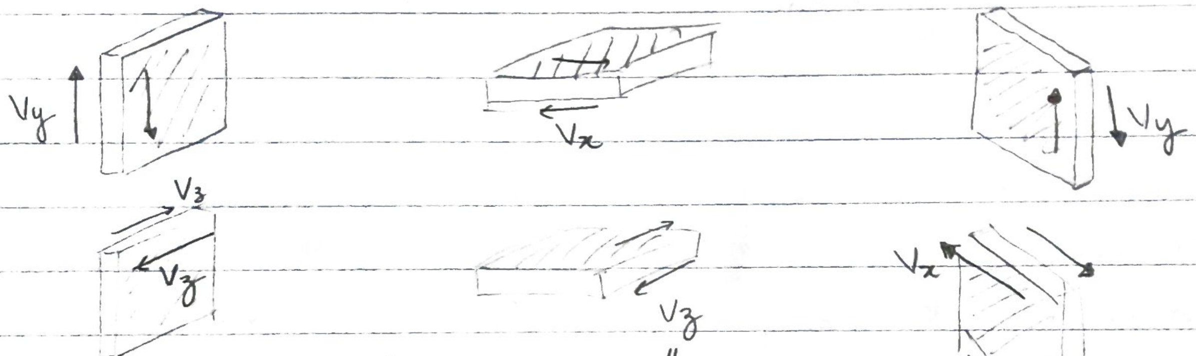
$N > 0$: saindo da seção

$T > 0$: saindo da seção

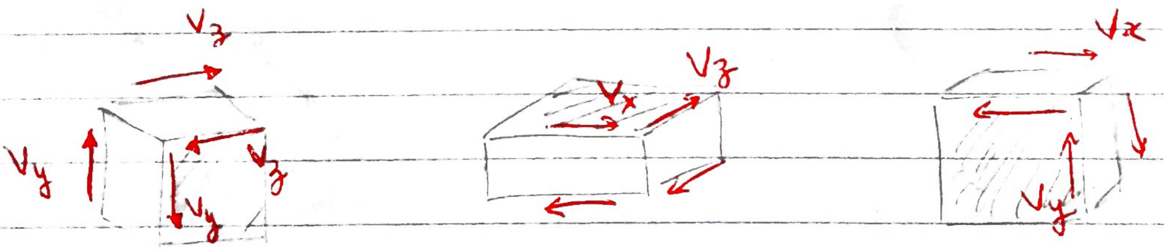
M_{fx}, M_{fx}, M_{fz} desenhar do lado tracionado.

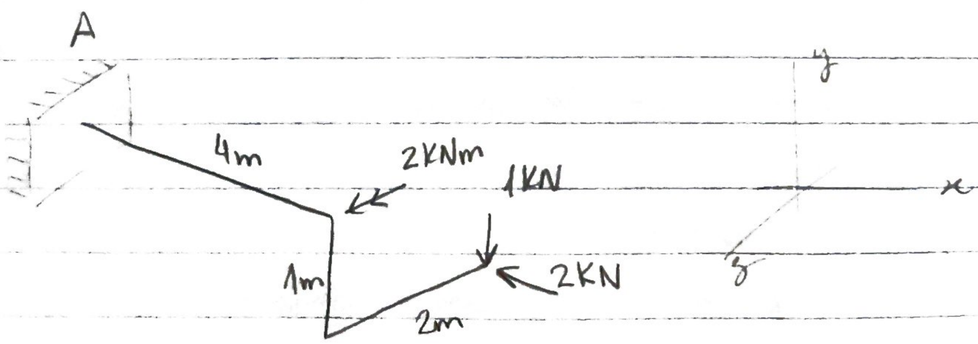


SEÇÃO S1 SEÇÃO S2 SEÇÃO S3

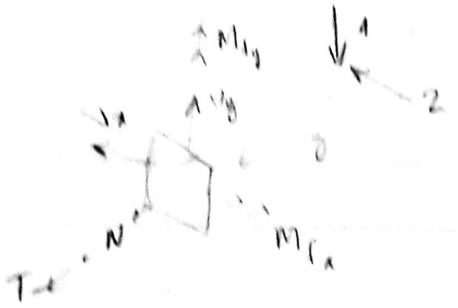


"contra o eixo"





Barra CD $(0 < y < 2m)$



$\Sigma F_x = 0 \rightarrow V_x = -2$

$\Sigma F_y = 0 \rightarrow V_y = 1$

$\Sigma F_z = 0 \rightarrow N = 0$

$\Sigma M_{s,x} = 0 \rightarrow M_{fx} - 1 \cdot z = 0$

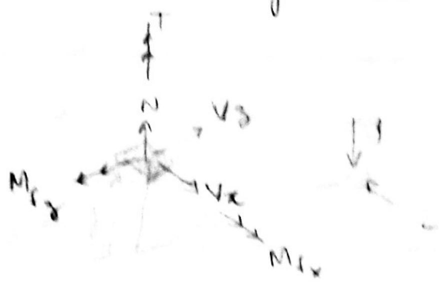
$M_{fx} = z$ ⊕ em cima

$\Sigma M_{s,y} = 0 \rightarrow M_{fy} + 2z = 0$

$M_{fy} = -2z$

$\Sigma M_{s,z} = 0 \rightarrow T = 0$

Barra BC $(0 < y < 4m)$:



$\Sigma F_x = 0 \quad V_x = 2$

$\Sigma F_y = 0 \quad N = 1$

$\Sigma F_z = 0 \quad V_z = 0$

$\Sigma M_{s,x} = 0 \quad M_{fx} = 2$ ⊕ atrás

$\Sigma M_{s,y} = 0 \quad T = -4$

$\Sigma M_{s,z} = 0 \quad M_{fz} = 2y$ ⊕ direita

Barra AB $(0 < \eta < 4m)$

$N = -2 kN$

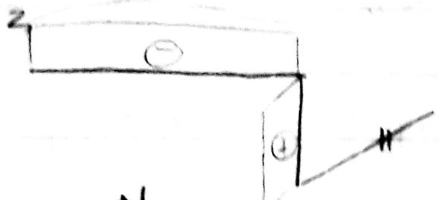
$V_y = 1 kN$

$V_z = 0$

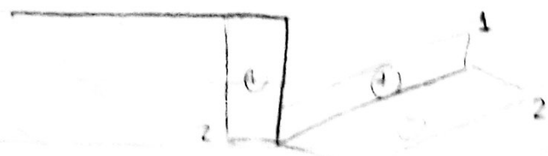
$T = -2 kNm$

$M_{fy} = -4 kNm$ ⊕ atrás

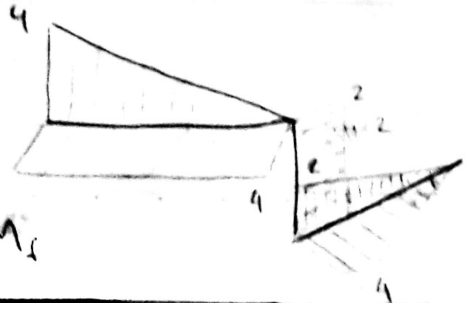
$M_{fz} = \eta$ ⊕ em cima



N

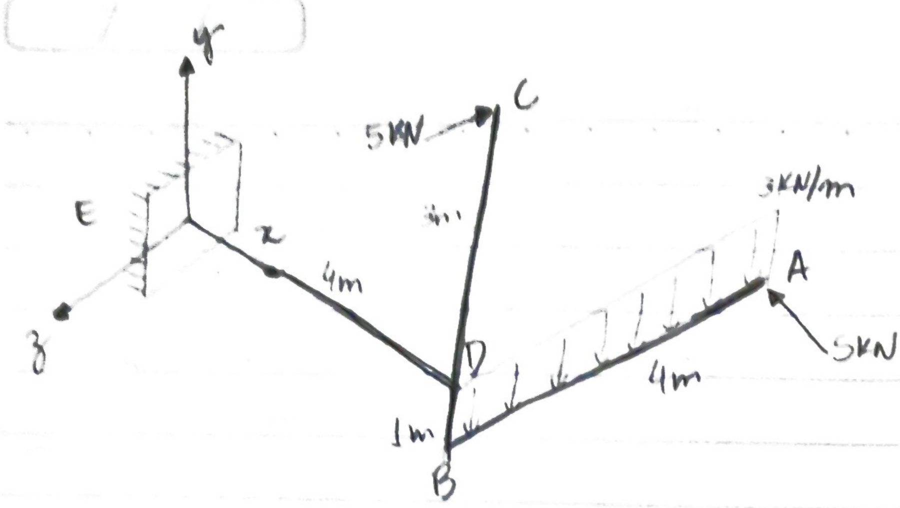


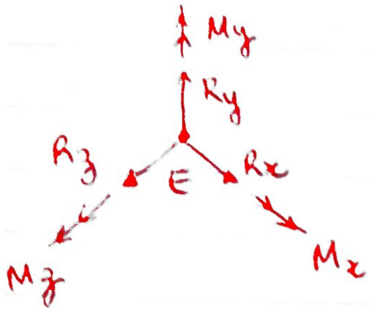
V



M_f

T





Reações de Apoio

$\Sigma F_x = 0: \quad R_x - 5 = 0 \Rightarrow R_x = 5KN$

$\Sigma F_y = 0: \quad R_y - 34 = 0 \Rightarrow R_y = 12KN$

$\Sigma F_z = 0: \quad R_z - 5 = 0 \Rightarrow R_z = 5KN$

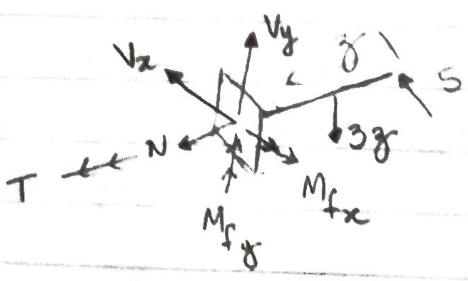
$\Sigma M_{E,x} = 0: \quad M_x - 5\,3 - 3.4.2 = 0$

$M_x = 39KNm$

$\Sigma M_{E,y} = 0: \quad M_y = -40KNm$

$\Sigma M_{E,z} = 0: \quad M_z = 55KNm$

Barra AB $(0 < z < 4m)$



$\Sigma F_x = 0 \Rightarrow V_x = -5KN$

$\Sigma F_y = 0 \Rightarrow V_y = 3z$

$\Sigma F_z = 0 \Rightarrow N = 0$

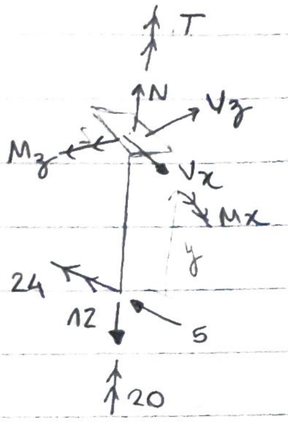
$\Sigma M_{S,x} = 0: \quad M_{fx} - 3z\,\frac{z}{2} = 0$

$M_{fx} = \frac{3z^2}{2}$ + em cima

$\Sigma M_{S,y} = 0 \quad M_{fy} = -5z$ + esquerda

$\Sigma M_{S,z} = 0 \Rightarrow T = 0$

Barra BD $(0 < y < 1\,m)$

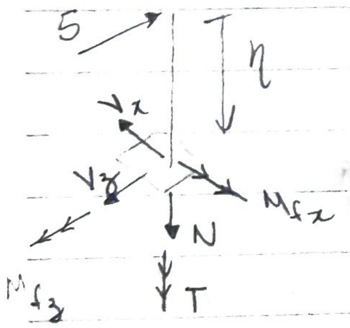


$V_x = 5KN$ $\quad$ $T = -20\,KNm$

$V_z = 0$ $\quad$ $M_{fz} = 5y$ ⊕ direita

$N = 12KN$ $\quad$ $M_{fx} = 24KNm$ ⊕ atrás

Barra CD $(0 < \eta < 3m)$



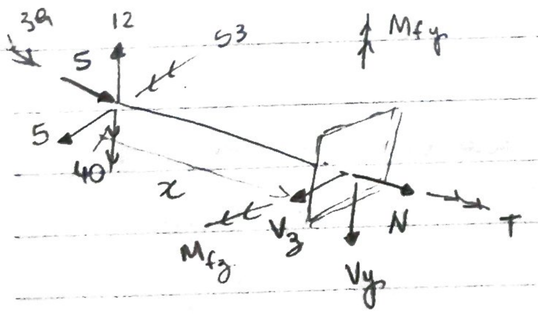
$T = 0$ $\quad$ $V_x = 0$

$N = 0$

$V_z = 5$

$M_{fz} = 0$ $\quad$ $M_{fx} = 5\eta$ ⊕ frente

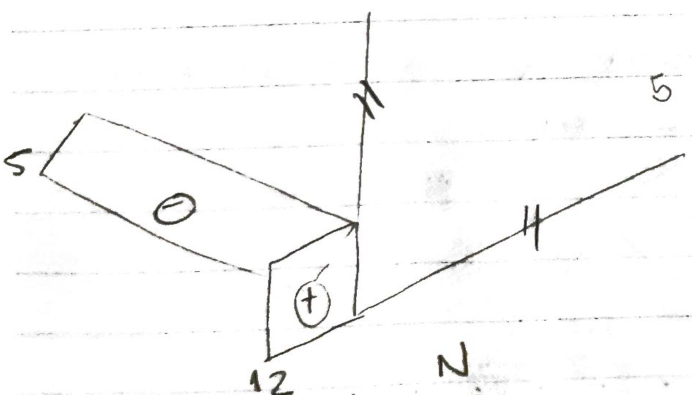
Barra ED $(0 < x < 4m)$



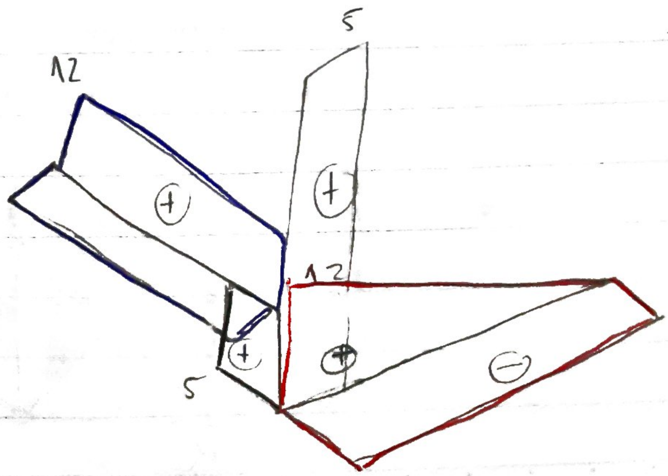
$N = -5KN$ $\quad$ $T = -39KNm$

$V_y = 12KN$ $\quad$ $M_{fz} = -53 + 12x$ ⊕ embaixo

$V_z = -5KN$ $\quad$ $M_{fy} = 40 - 5x$ ⊕ frente

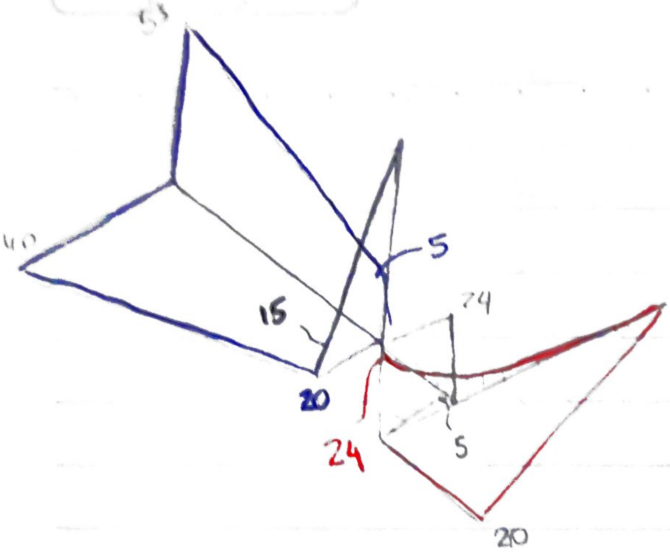


N [KN]

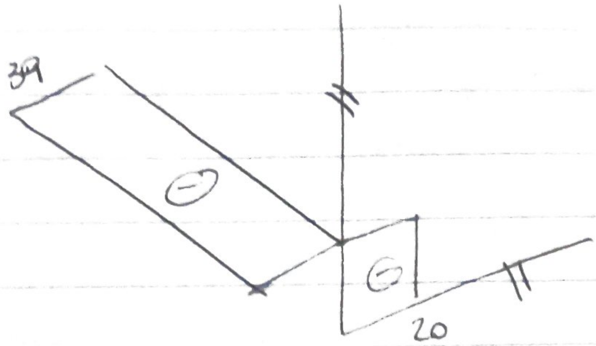


V [KN]

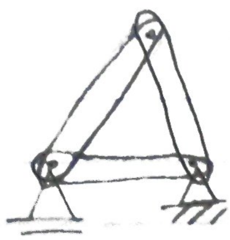
M_f [KNm]



T [KNm]

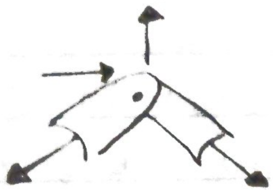


TRELIÇA - Barras ligadas por articulações
- Carregamentos exclusivamente nodais



CONVENÇÃO DE SINAIS

Treliças Planas

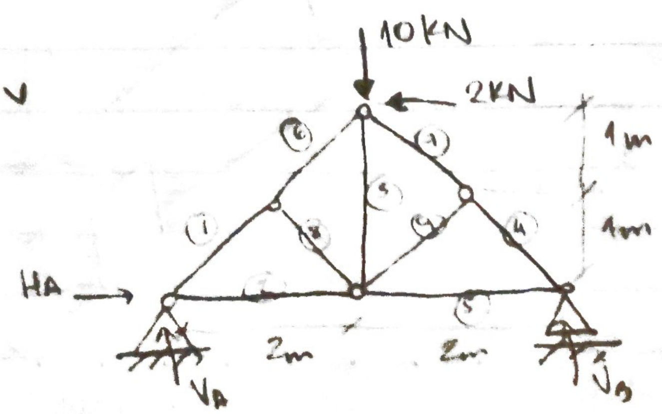


2 EQS DE EQUILÍBRIO

Para n nós. 2n eqs de equilíbrio

incógnitas: b forças normais
v reações vinculares

$2n = b + v$



H_A	2
V_A	6
V_B	4
N_1	$-6\sqrt{2}$
N_2	4
N_3	0
N_4	$-4\sqrt{2}$
N_5	4
N_6	$-6\sqrt{2}$
N_7	$-4\sqrt{2}$
N_8	0
N_9	0

Reações de Apoio: $\Sigma F_H = 0: \quad H_A = 2kN$

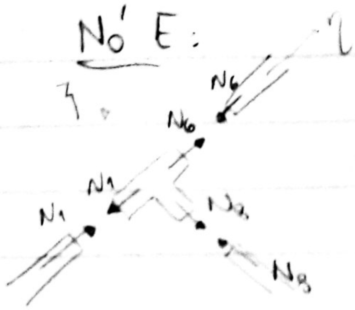
$\Sigma F_V = 0: \quad V_A + V_B = 10$

$\Sigma M_{(A)} = 0: \quad {}^{2}\cancel{4}\, V_B + 2.\cancel{2} - 10.\cancel{2} = 0$

$V_B = \frac{8}{2} = 4KN$

$V_A = 6KN$

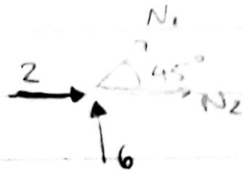
Nó E:



$\Sigma F_x = 0: \quad N_1 = N_6$

$\Sigma F_y = 0: \quad N_8 = 0$

Nó A:

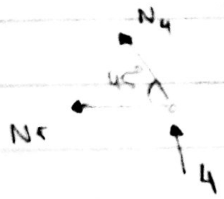


$\Sigma F_H = 0: \quad 2 + N_2 + \frac{N_1}{\sqrt{2}} = 0$

$\Sigma F_V = 0: \quad 6 + \frac{N_1}{\sqrt{2}} = 0 \Rightarrow N_1 = -6\sqrt{2}$

$N_2 = 4$

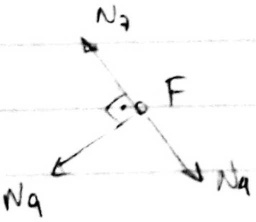
Nó B



$\frac{N_4}{\sqrt{2}} + 4 = 0 \rightarrow N_4 = -4\sqrt{2}$

$N_5 + \frac{N_4}{\sqrt{2}} = 0 \rightarrow N_5 = 4$

Nó F



$N_7 = N_4 = -4\sqrt{2}$

$N_9 = 0$

Nó D

Como $N_8 = N_9 = 0$,

$N_3 = 0$

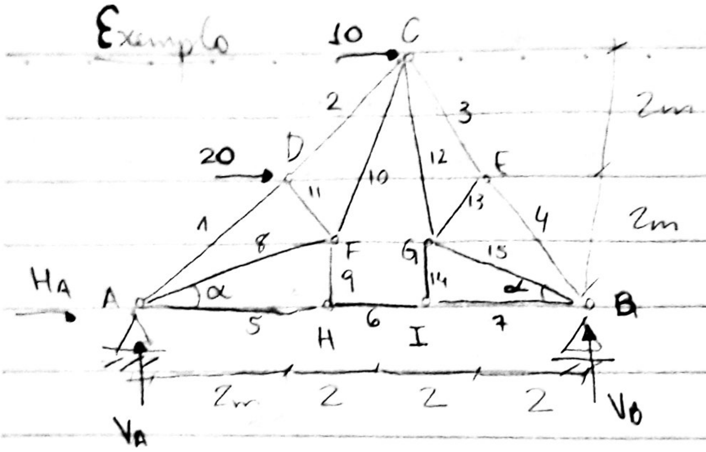
$N_2 = N_5$

Corte de Ritter

- corta-se a treliça em 2 partes
- verifica-se o equilíbrio em uma das partes

$\alpha = 30°$

Exemplo



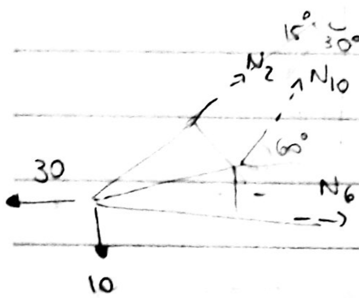
Reações

$V_A = -10$

$V_B = 10$

$H_A = -30$

Corte em 2, 10, 6:



$N_6 = 10$

$N_2 = -19{,}32$

$N_{10} = 27{,}32$

Nó H

$N_5 = N_6$; $N_9 = 0$

$N_5 = 10$

Nó I: $N_{14} = 0$

$N_7 = 10$

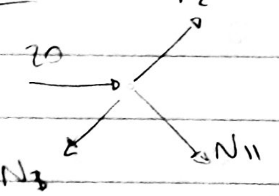
Nó E:

$N_3 = N_4$

$N_{13} = 0$

Nó D:

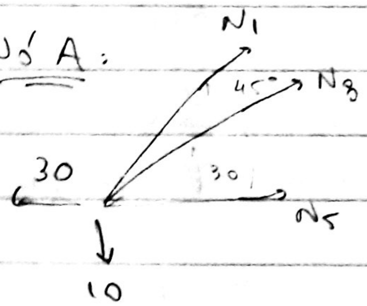
$N_2 = -19{,}32$



$N_{11} = -10\sqrt{2}$

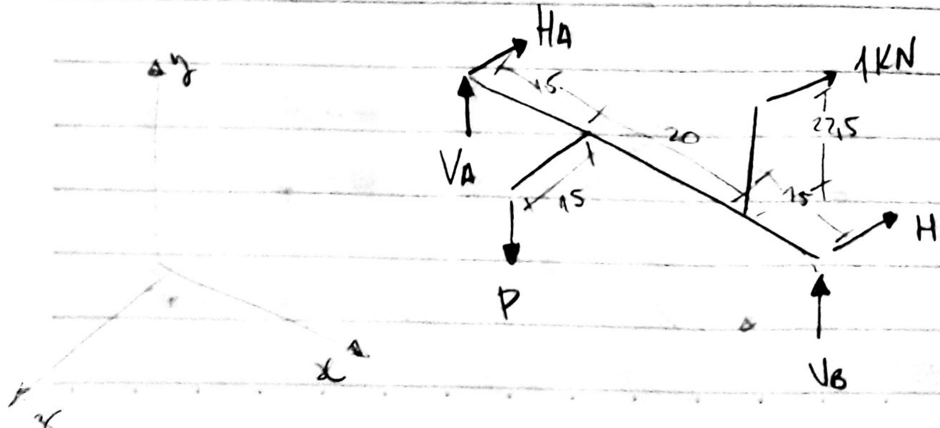
$N_1 = -5{,}118$

Nó A:



$N_8 = 27{,}32$

Lista: Ex 11



$V_A + V_B = P$

$H_A + H_B = -1$

$\sum M_z^{(A)} = 0$:

$-15P + 50V_B = 0$

$-3P + 10V_B = 0$

$\sum M_y^{(A)} = 0$:

$50H_B + 35 = 0$

$H_A = -0{,}3KN$ $H_B = -0{,}7KN$

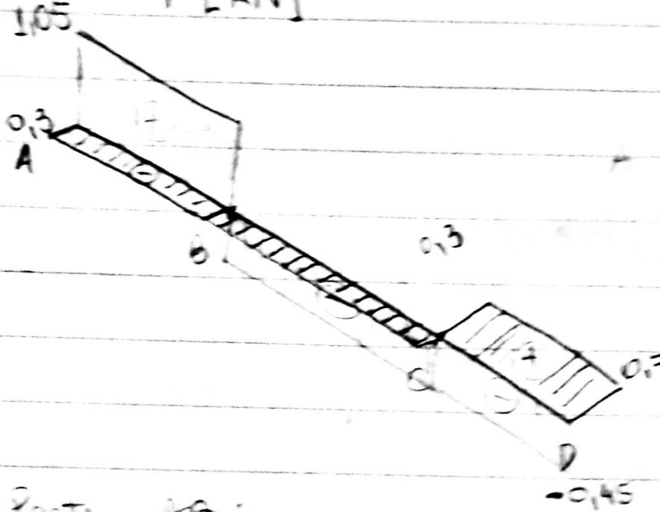
$\sum M_x^{(A)} = 0: \quad 15P - 22{,}5 = 0$

$$P = \frac{22{,}5}{15} = \frac{2{,}5 \cdot 9}{2{,}5 \cdot 6} = \underline{1{,}5\,KN}$$

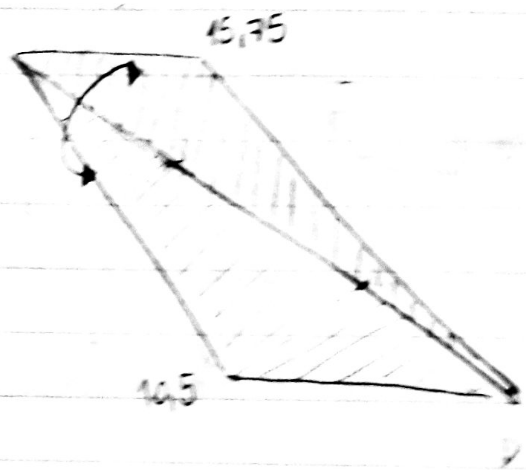
$-3 \cdot 1{,}5 + 10\,V_B = 0$

$$V_B = \frac{4{,}5}{10} = \underline{0{,}45\,KN} \Rightarrow \underline{V_A = 1{,}05\,KN}$$

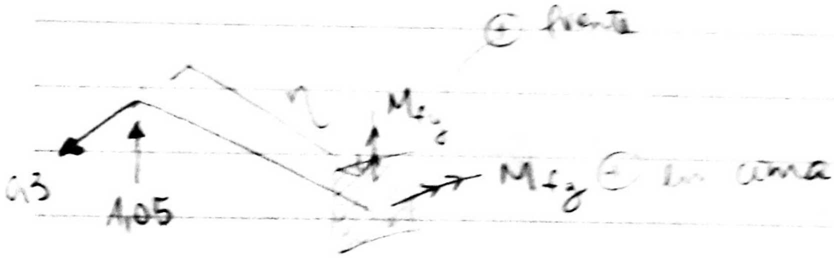
V [KN]



M_f [KNm]



Parte AB:



$M_{f_y} = 0{,}3\,\eta \quad$ para $0 \leq \eta \leq 15$

$M_{f_z} = 1{,}05\,\eta$

Parte BD

3,5
3,3
10,5

$M_{f_z} = 1{,}05\,\eta - 1{,}5\,\eta + 15{,}75$

$= 15{,}75 - 0{,}45\,\eta$

$M(\eta = 35) = 0$

Parte CD:

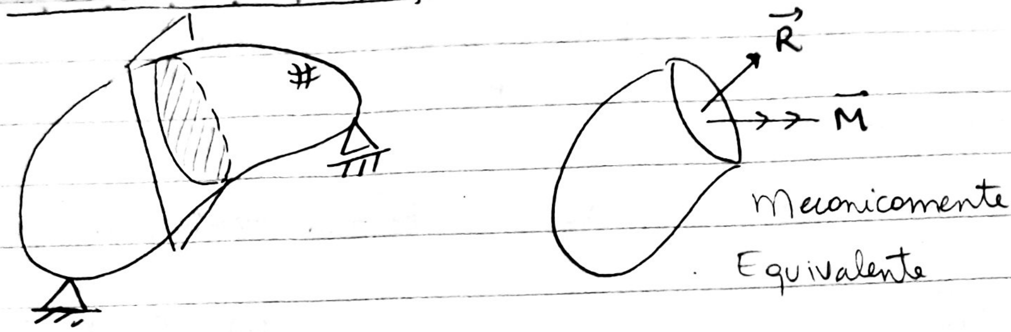
$M_{f_y} = 10{,}5 - z + 0{,}3z$

$0 \leq z \leq 15$

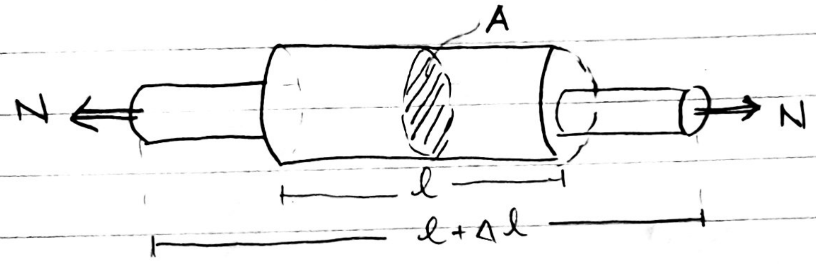
$= 10{,}5 - 0{,}7z$

$M_{f_y}(z = 15) = 0$

Tensões e Deformações



Lei de Hooke



Definimos tensão normal como:

$$\sigma_N = \frac{N}{A}$$

Deformação longitudinal: $\varepsilon_N = \frac{\Delta \ell}{\ell}$

módulo de elasticidade

$$\sigma_N = E\,\varepsilon_N$$

Material	E [GPa]	ν
Aço	210	0,3
Alumínio	70	0,25
Concreto	$\simeq 25$	0,15
Madeira	$\simeq 10$	?
Nylon	$\simeq 2$	0,42

ε_T : deformação transversal

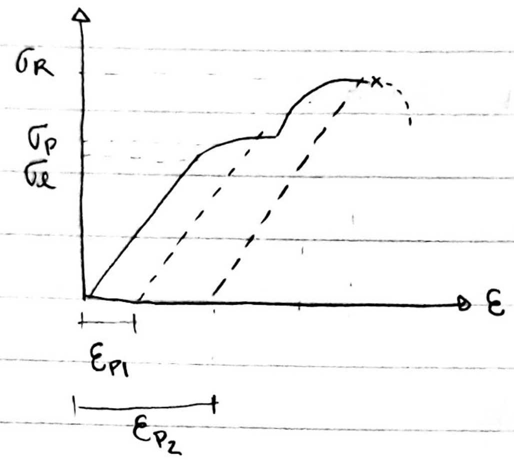
$$\varepsilon_T = \frac{\Delta\phi}{\phi} < 0 \quad \text{(estricção)}$$

50 0,12

$$\varepsilon_T = -\nu\,\varepsilon_N \Rightarrow \boxed{\nu = \frac{-\varepsilon_T}{\varepsilon_N}}$$

Curva tensão-deformação

MATERIAL DÚCTIL

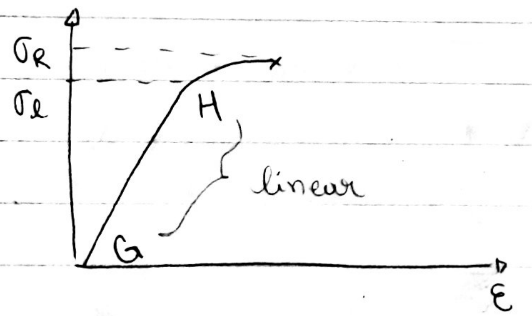


σ_ℓ: tensão limite de proporcionalidade

σ_p: tensão de plastificação / escoamento

σ_R: tensão de ruptura

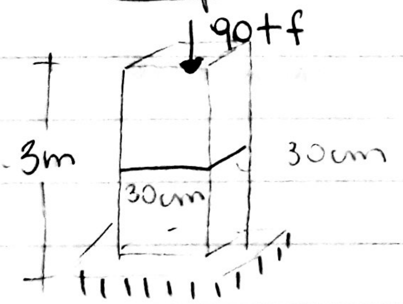
MATERIAL FRÁGIL



$$\frac{\Delta l}{l} = \frac{\sigma}{E}$$

$$\boxed{\Delta l = \frac{l\,N}{E\,A}}$$

Exemplo: Calcular o encurtamento do pilar de concreto

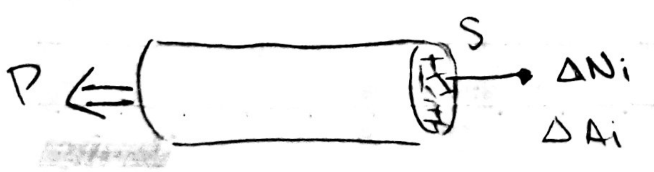


$E_c = 25\,GPa$

$$\Delta l = \frac{3m \cdot 90 \cdot 10^4\,N}{25 \cdot 10^9\,Pa \cdot 900 \cdot 10^{-4}}$$

$$\Delta l = \frac{3}{2500} = 0{,}12 \cdot 10^{-2} = -1{,}2\,mm$$

A tensão σ_N é uma média!



$$A = \lim_{n \to \infty} \sum_{i=1}^{N} \Delta A_i = \int_S dA$$

$$N = \int_S dN$$

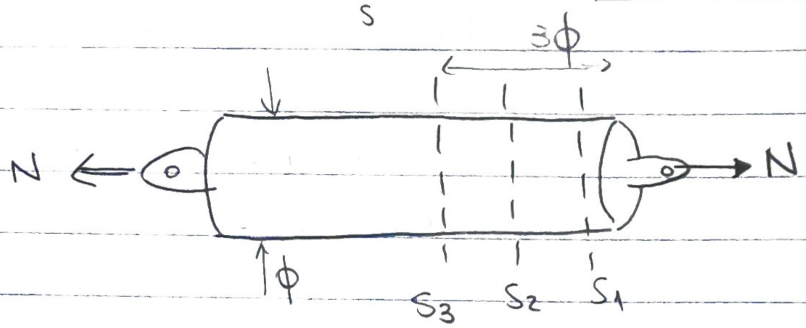
Para uma área infinitesimal dA:

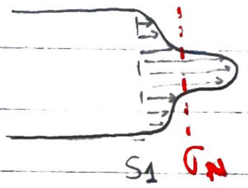
$$\sigma_x = \frac{dN}{dA} \Rightarrow dN = \sigma_x \, dA$$

Logo:

$$N = \int_S \sigma_x \, dA \quad . \quad \text{Se } \sigma_x = \sigma_N, \ \forall P \in S:$$

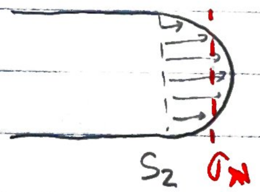
$$N = \sigma_N \int_S dA \Rightarrow \boxed{N = \sigma_N A}$$



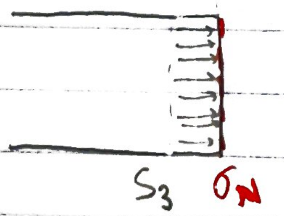


$\sigma_x >> \sigma_N$

σ_N: tensão média

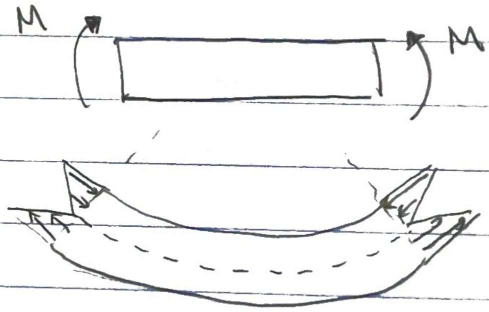


$\sigma_x > \sigma_N$



$\sigma_x \simeq \sigma_N$

Tensões na Flexão



- cortante
- torçor

} CISALHAMENTO

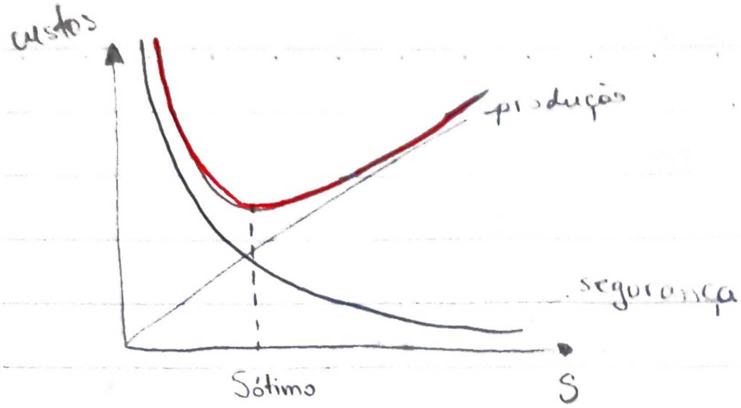
Coeficiente de Segurança

$$\sigma_{lim} = \begin{cases} \sigma_e \text{ (ducteis)} \\ \sigma_R \text{ (frágeis)} \end{cases}$$

$$\bar{\sigma} = \frac{\sigma_{lim}}{S}$$

$\bar{\sigma}$: admissível

S: Fator de segurança



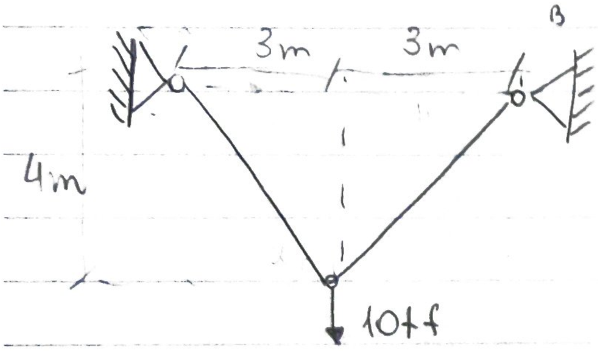
Alguns coeficientes utilizados:

$S = 1{,}6$ (aço)

$S = 2\text{-}3$ (concreto)

$S = 3\text{-}4$ (madeira)

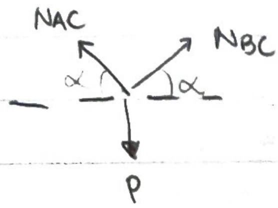
Exemplo: Determinar o diâmetro do fio para que o coeficiente de segurança seja 2. Considere $\sigma_c = 600\,MPa$ e $E = 210\,GPa$.



Tensão Admissível:

$$\bar{\sigma} = \frac{\sigma_{lim}}{S} = \frac{600}{2} = 300\,MPa$$

Nó C:



$\Sigma F_H = 0: N_{AC} = N_{BC}$

$\Sigma F_V = 0: 2 N_{AC} \operatorname{sen}\alpha = P$

$$N_{AC} = \frac{P}{2\operatorname{sen}\alpha} = \underline{62{,}5\,kN}$$

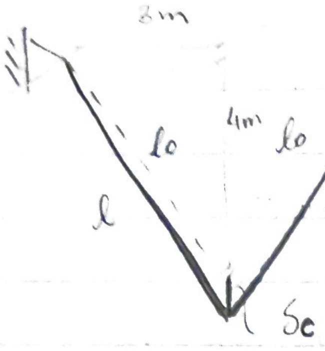
Cálculo da Área:

$$\sigma_N \leq \bar{\sigma} \Rightarrow \frac{N}{A} \leq \bar{\sigma} \Rightarrow A \geqslant \frac{N}{\bar{\sigma}}$$

$$\frac{\pi \phi^2}{4} \geqslant \frac{N}{\bar{\sigma}} \Rightarrow \phi \geqslant \sqrt{\frac{4N}{\pi \bar{\sigma}}} = 0{,}0163\,m$$

$$\boxed{\phi_{min} = 1{,}63\,cm}$$

Calculando deslocamento do ponto C:



$l = l_0 + \Delta l$

$l = l_0 + l_0 \varepsilon$

$l = l_0(1+\varepsilon)$

Calculando ε: $\quad \varepsilon = \dfrac{\sigma}{E} = \dfrac{300 \cdot 10^6}{210 \cdot 10^9}$

$\varepsilon = 1{,}43 \cdot 10^{-3}\, m/m$

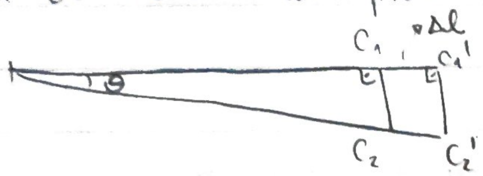
Descobrindo δ_c: $\quad l = 5(1 + 1{,}43 \cdot 10^{-3})$

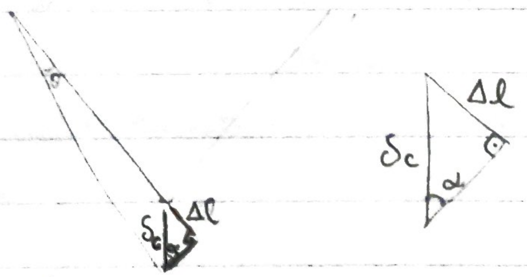
$= 5{,}0071$

$3^2 + (4+\delta_c)^2 = l^2$

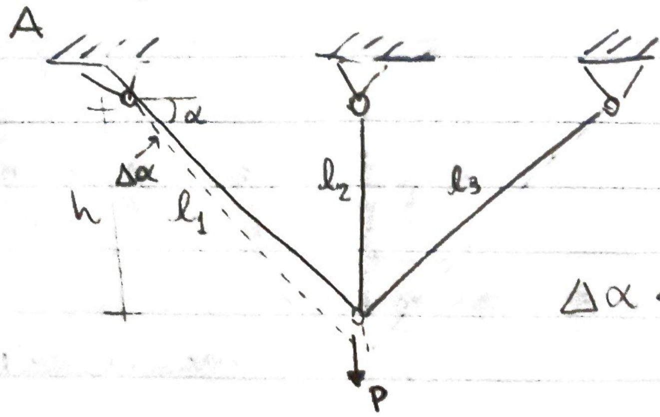
$\therefore \quad \delta_c = 0{,}0089\, m$

Calcular δ_c de maneira aproximada:





$$\boxed{\delta_c = \frac{\Delta l}{\operatorname{sen}\alpha}}$$

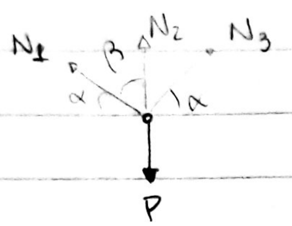


Estrutura Hiperestática

$\Delta\alpha << 1$

- supor que a geometria permanece inalterada no equilíbrio
- permite a linearização do problema

Equilíbrio no Nó C:



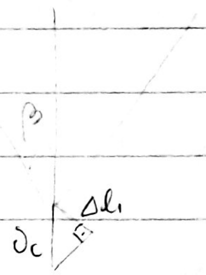
$\Sigma F_H = 0: \; -N_1 \,\text{sen}\,\beta + N_3 \,\text{sen}\,\beta = 0 \Rightarrow N_1 = N_3$

$\Sigma F_V = 0: \; N_1 \cos\beta + N_3 \cos\beta + N_2 = P$

$\Rightarrow 2 N_1 \cos\beta + N_2 = P$

- COMPATIBILIDADE DE DESLOCAMENTO: $l_1 = l_3 = \dfrac{h}{\cos\beta}$, $l_2 = h$

diagrama de Williot



Logo: $\Delta l_1 = \delta_c \cos\beta$

$\Delta l_2 = \delta_c$

$\Delta l_3 = \delta_c \cos\beta$

Sabemos que: $\dfrac{N}{A} = \dfrac{E\Delta l}{l} \rightarrow N = \dfrac{EA}{l}\Delta l$

Assim: $N_1 = \dfrac{EA\Delta l_1}{l_1} = \dfrac{EA\cos^2\beta\,\delta_c}{h} = N_3$

$N_2 = \dfrac{EA}{l_2}\Delta l_2 = \dfrac{EA}{h}\delta_c$

Substituindo no equilíbrio: $2\cos\beta\left(\dfrac{EA}{h}\cos^2\beta\,\delta_c\right) + \left(\dfrac{EA}{h}\delta_c\right) = P$

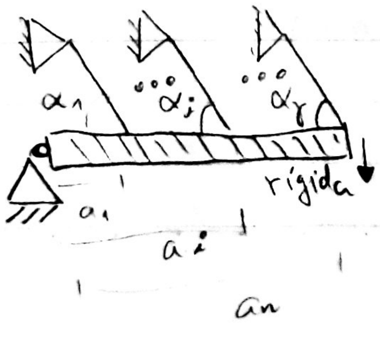
$\dfrac{EA}{h}\delta_c\,(2\cos^3\beta + 1) = P$

$\hookrightarrow \delta_c = \dfrac{Ph}{EA(2\cos^3\beta - 1)}$

$N_1 = N_3 = \dfrac{P\cos^2\beta}{2\cos^3\beta + 1}$

$N_2 = \dfrac{P}{2\cos^3\beta + 1}$

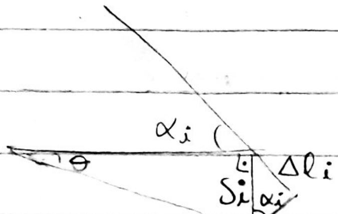
Estrutura Estaiada



(n-1) vezes hiperestático

Fazendo aproximação:

$$(I)\quad tg\theta = \frac{\delta_1}{a_1} = \frac{\delta_i}{a_i} = \frac{\delta_n}{a_n} = cte$$



$$sen\,\alpha_i = \frac{\Delta l_i}{\delta_i} \Rightarrow \Delta l_i = sen\alpha_i . \delta_i$$

Como $N_i = \frac{E_i A_i}{l_i} \Delta l_i$

$$N_i = \frac{E_i A_i \delta_i sen\alpha_i}{l_i}$$

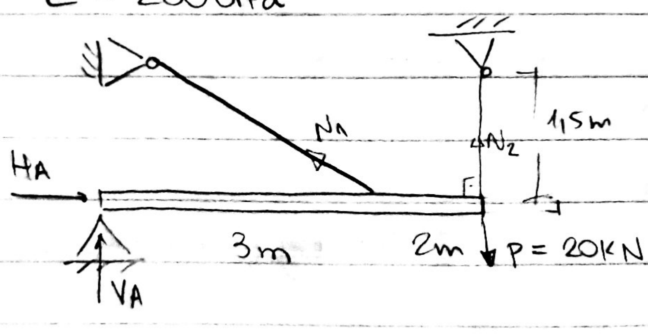
$$\delta_i = \frac{N_i l_i}{E_i A_i sen\alpha_i} \quad (II)$$

Em (I):

$$\frac{N_i l_i}{E_i A_i a_i sen\alpha_i} = tg\theta$$ (n-1) eqs de compatibilidade

Exemplo: calcular o diâmetro mínimo dos estais, considerando:

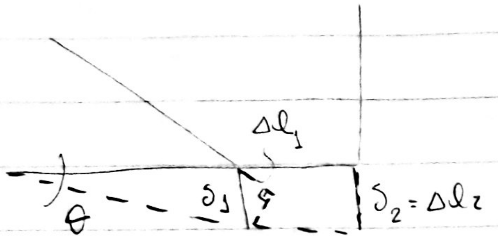
$\bar{\delta} = 1\,cm$
$\bar{\sigma} = 60\,MPa$
$E = 200\,GPa$



$\sum F_H = 0 \quad H_A - N_1 \cos\alpha_1 = 0$

$\sum F_V = 0: \quad V_A + N_1 \operatorname{sen}\alpha_1 + N_2 = P$

$\sum M_{(A)} = 0: \quad N_1 \operatorname{sen}\alpha_1 \cdot 3 + 5N_2 = 5P$



$\operatorname{sen}\alpha_1 = \dfrac{\Delta l_1}{\delta_1} \; ; \; \delta_1 = \dfrac{\Delta l_1}{\operatorname{sen}\alpha_1}$

$tg\theta = \dfrac{\delta_1}{3} = \dfrac{\delta_2}{5}$

$$\frac{\Delta l_1}{3 \operatorname{sen}\alpha_1} = \frac{\Delta l_2}{5}$$

$$\Delta l_i = \frac{N_i . l_i}{EA} \; , \quad \frac{N_1 l_1}{3 \operatorname{sen}\alpha_1} = \frac{N_2 l_2}{5}$$

$\boxed{N_1 = 0{,}12 N_2}$

$\begin{cases} N_1 = 232{,}5 \text{ Kgf} \\ N_2 = 1937{,}6 \text{ Kgf} \end{cases}$

$H_A = 207{,}9 \text{ Kgf}$

$V_A = -41{,}6 \text{ Kgf}$

Critério de Tensão máxima:

$\sigma \leq \bar{\sigma} \qquad \sigma = N/A$

$\sigma_2 \leq \bar{\sigma}$

$A \geqslant \dfrac{N_2}{\bar{\sigma}} \Rightarrow A \geqslant 3{,}23 \text{ cm}^2$

Critério de Deslocamento Máximo:

$\delta \leq \bar{\delta}_c$

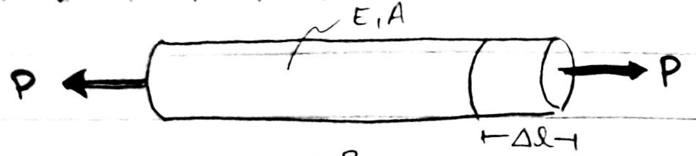
$\delta_2 \leq \bar{\delta}_c$

$\delta_2 = \Delta l_2 = \dfrac{N_2 l_2}{EA} \Rightarrow A \geqslant \dfrac{N_2 l_2}{E \bar{\delta}_c} = 0{,}14 \text{ cm}^2$

Logo: $A_{min} = 3{,}23 \text{ cm}^2$

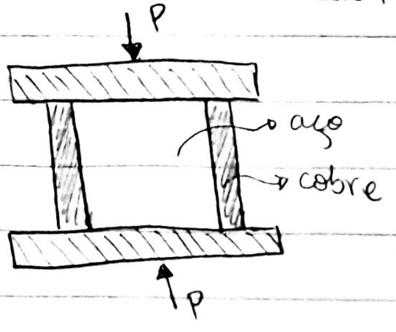
$\dfrac{\pi \phi^2}{4} = A_{min} \Rightarrow \boxed{\phi_{min} = 2{,}03 \text{ cm}}$

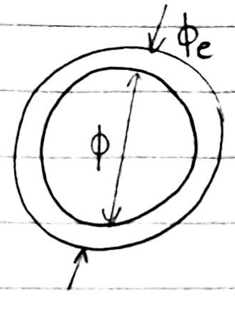
Forças Normais em estruturas hiperestáticas



$$\Delta l = \frac{P.L}{EA}$$

Exemplo:





Pede-se ΔL ($\sigma_{aço} = \sigma_1$, $\sigma_{cu} = \sigma_2$)

$$A_a = \frac{\pi \phi_a^2}{4} \qquad A_c = \frac{\pi \phi_c^2}{4} - A_a$$

Condição de compatibilidade:

$$\varepsilon_a = \varepsilon_c = \frac{\Delta l}{l} = \varepsilon$$

Podemos escrever: $\sigma_a = E_a \cdot \varepsilon$

$\sigma_c = E_c \cdot \varepsilon$

Por equilíbrio: $\sigma_a \cdot A_a + \sigma_c A_c = P$

Substituindo: $E_a \varepsilon A_a + E_c \varepsilon A_c = P$

$$(E_a A_a + E_c A_c) \frac{\Delta l}{l} = P$$

$$\Delta l = \frac{P l}{E_a A_a + E_c A_c}$$

$$\Delta l = \frac{P l}{(EA)_{equivalente}}$$

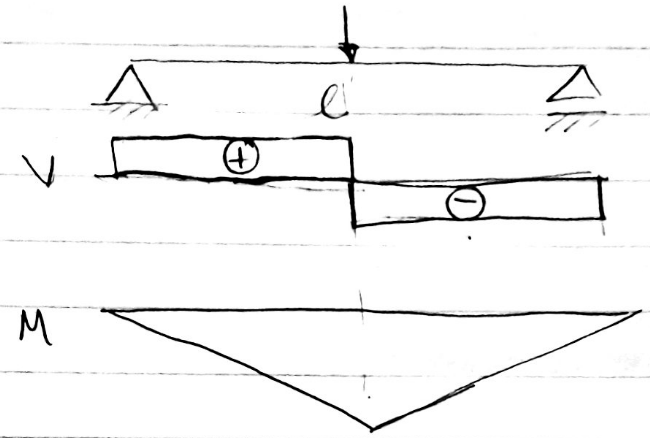
$$\sigma_a = \frac{P E_a}{E_a A_a + E_c A_c} \qquad \sigma_c = \frac{P E_c}{E_a A_a + E_c A_c}$$

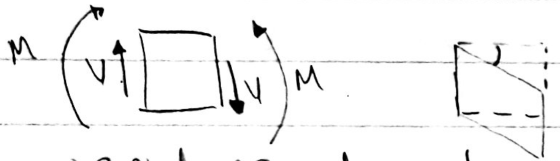
Sabemos que: $A_{TOTAL} = A_a + A_c$

$$\boxed{E_{eq} = \frac{E_a A_a + E_c A_c}{A_{TOT}}}$$

TORÇÃO

- Deformação / tensão: cisalhamento





distorção [rad]

$\gamma \simeq tg\,\gamma = \frac{dy}{dx} = cte$

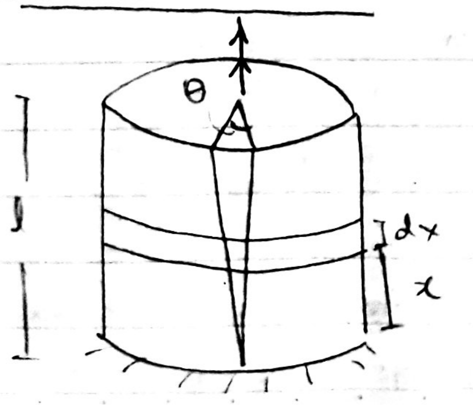
Tensão média de cisalhamento $\quad \tau = \frac{V}{A}$

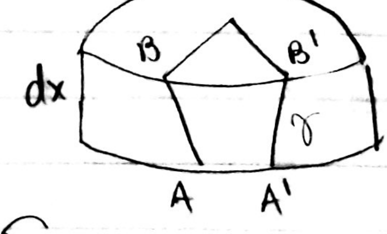
Lei de Hooke: $\boxed{\tau = G\gamma}$

└ módulo de elasticidade transversal

Material elástico e isotrópico: $G = \frac{E}{2(1+\nu)}$

TENSÃO DE EIXOS

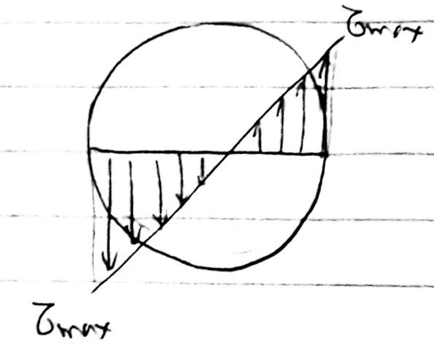




$\left.\begin{array}{l} \overset{\frown}{B'B} = r\,d\theta \\ \overset{\frown}{B'B} = \gamma\,dx \end{array}\right\} \quad r\,d\theta = \gamma\,dx$

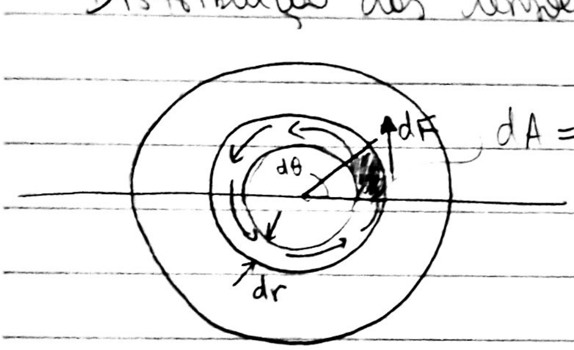
$$\gamma = r\frac{d\theta}{dx} = \varphi\, r \quad \text{se} \quad \frac{d\theta}{dx} = cte$$

Lei de Hooke: $\tau = G\gamma \Rightarrow \tau = G\varphi r$



$$\tau_{max} = G\varphi R$$

Distribuição das tensões de cisalhamento:



$dA = r\,dr\,d\theta$

$$dF = \tau\, dA$$
$$dT = r\, dF$$
$$dT = \tau r\, dA = \tau r^2 dr\, d\theta$$
$$dT(r) = \int_0^{2\pi} dT = \int_0^{2\pi} \tau r^2 dr\, d\theta$$

$$dT(r) = 2\pi \tau r^2 dr$$
$$T = \int_0^R dT(r) = \int_0^R 2\pi G\varphi r^3 dr$$

$$T = \frac{\pi G \varphi R^4}{2} \Rightarrow \boxed{\varphi = \frac{2T}{\pi G R^4}}$$

Substituindo na Lei de Hooke: $\tau = G\varphi r = G\left(\frac{2T}{\pi G R^4}\right) r$

$$\boxed{\tau = \frac{T}{J} r}$$

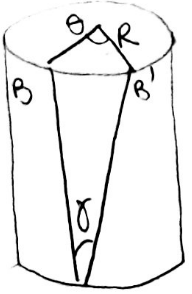
onde $J = \frac{\pi R^4}{2} = \frac{\pi D^4}{32}$

$J \equiv$ momento polar de inércia de um círculo cheio de raio externo R

$$\gamma = \varphi r$$

$$\varphi = \frac{T}{GJ}$$

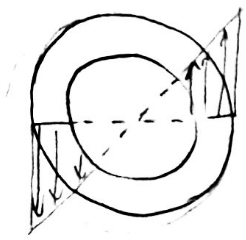
GJ: módulo de rigidez à torção



$$\widehat{BB'} = R\theta = \gamma L$$

$$\gamma = \frac{TR}{GJ} = \frac{R\theta}{L} \Rightarrow \boxed{\theta = \frac{TL}{GJ}}$$

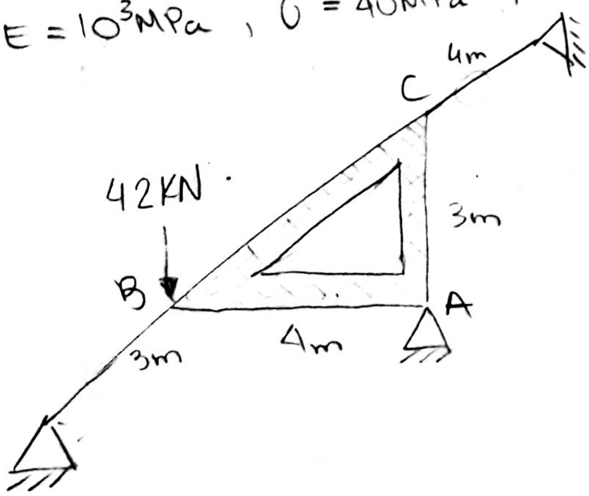
Eixo vazado



$$\tau_{max} = \frac{T}{J} Re \quad , \quad J = \frac{\pi}{2}\left(Re^4 - Ri^4\right)$$

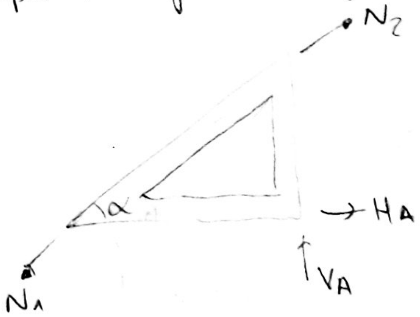
Ex 14): mesma S, T e A = ? chapa triangular rígida

$E = 10^3 MPa$, $\bar{\sigma} = 40 MPa$, $V_B \leq 0{,}1\,m$



$\operatorname{sen}\alpha = 3/5$

$\cos\alpha = 4/5$



Equilíbrio: $\sum F_H = 0$:

$$-N_1\cos\alpha + H_A + N_2\cos\alpha = 0$$

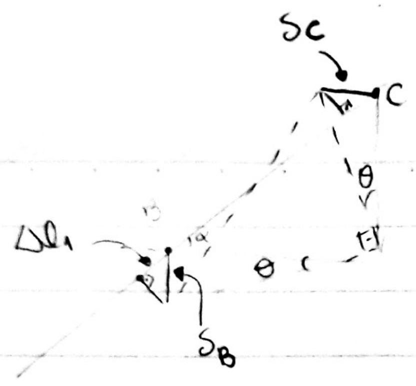
$$\sum F_V = 0 .$$

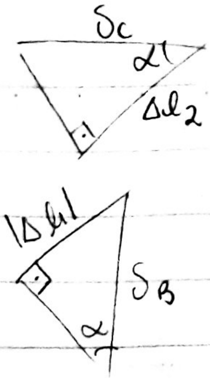
$$-N_1\operatorname{sen}\alpha + V_A + N_2\operatorname{sen}\alpha = 0$$

$$\sum M_B = 0 \Rightarrow V_A = 0$$

$$H_A = 48KN$$

$$N_2 - N_1 = 70$$





$$\cos\alpha = \frac{\Delta l_2}{\delta_c}$$

$$\text{sen}\,\alpha = \frac{|\Delta l_1|}{\delta_B}$$

$$tg\,\theta = \frac{\delta_c}{3} = \frac{\delta_B}{4}$$

$$\frac{1}{3}\frac{\Delta l_2}{\cos\alpha} = \frac{1}{4}\frac{|\Delta l_1|}{\text{sen}\,\alpha}$$

$$\frac{\Delta l_2}{3}\cdot\frac{5}{4} = \frac{|\Delta l_1|}{4}\cdot\frac{5}{3}$$

$$\Delta l_2 = |\Delta l_1|$$

considerando Williot: $\Delta l_2 = -\Delta l_1$

Lembrando: $N_i = EA\frac{\Delta l_i}{l_i} \Rightarrow \Delta l_i = \frac{N_i \cdot l_i}{EA}$

$$\frac{N_2 l_2}{EA} = -\frac{N_1 l_1}{EA} \Rightarrow N_1 = -\frac{4}{3}N_2$$

$$N_2 + \frac{4}{3}N_2 = 70 \quad \begin{cases} N_2 = 30KN \\ N_1 = -40KN \end{cases}$$

· Critérios:

(1) $\sigma \leq \bar{\sigma}$

$|\sigma_1| > |\sigma_2|$

$$\frac{|N_1|}{A} \leq \bar{\sigma}$$

$$A \geq \frac{|N_1|}{\bar{\sigma}} \qquad \underline{A \geq 10^{-3}m^2}$$

(2) $\delta_B \leq V_B$

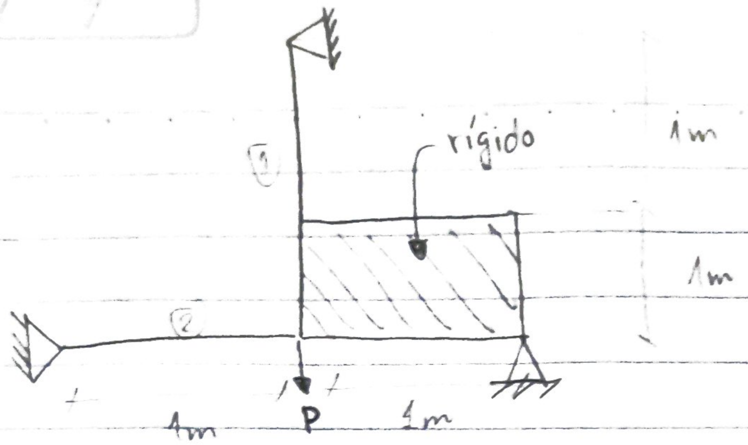
$$\frac{|\Delta l_1|}{\text{sen}\,\alpha} \leq V_B$$

$$\frac{|N_1|\, l_1}{EA}\cdot\frac{1}{\text{sen}\,\alpha} \leq V_B$$

$$A \geq 2 \cdot 10^{-3}m^2$$

De (1) e (2):

$A_{min} = 2 \cdot 10^{-3} m^2$



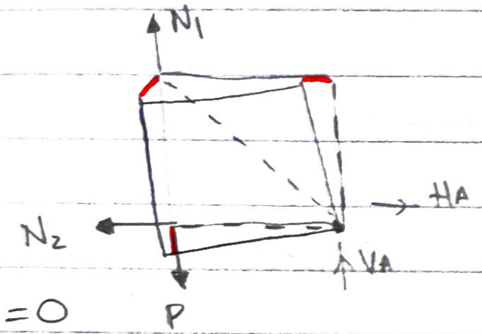
$\sigma_e = 400MPa$

$E = 200GPa$

$S = 2$

$\delta_{A,max} = 0,8mm$

$P = 1000kN$



$\Sigma F_H = 0: \quad H_A = N_2$

$\Sigma F_V = 0: \quad N_1 + V_A = P \Rightarrow N_1 = P$

$\Sigma M_{(A)} = 0: \quad V_A = 0$

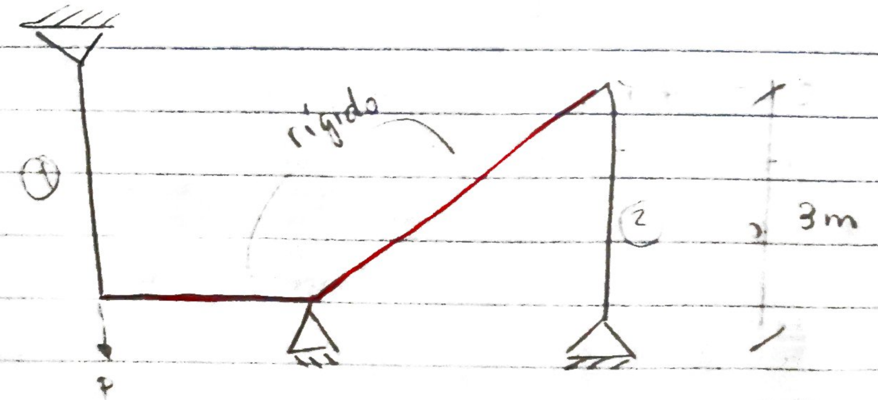
$\Delta l_2 = 0$

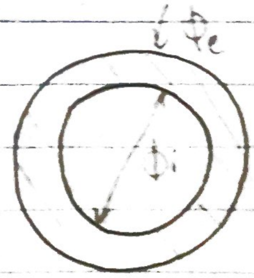
$N2 = 0$, pois a barra não "precisa" ter alongamento para atingir novo ponto.

$\dfrac{N_1}{A} \leq \bar{\sigma} \Rightarrow \dfrac{P}{A} \leq \bar{\sigma}$ $\quad \boxed{A \geq 5.10^{-3} m^2}$

$\dfrac{N_1 l_1}{EA} = \Delta l_1 \leq \delta_{A\,max}$

$\boxed{A \geq 6,25.10^{-3} m^2}$ $\quad \boxed{\phi_{min} = 89,2mm}$





$E_a = 21000 kN/cm^2$

$E_F = 2100 kN/cm^2$

$\phi_i = 18cm$

$\phi_e = 20cm$

Determinar P_{max}

$\delta_c \leq \delta = 1cm$

$\sigma_G \leq \overline{\sigma_G} = 15 KN/cm^2$

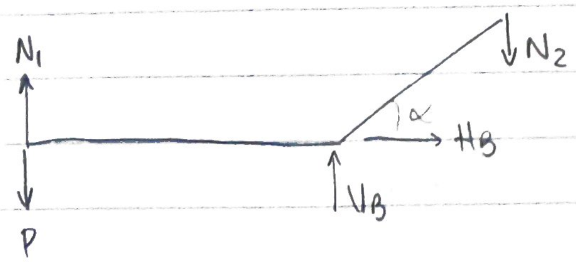
$\sigma_F \leq \overline{\sigma_F} = 5 KN/m^2$

EA_{eq}

$A_F = 81\pi cm^2 \qquad A_G = 19\pi cm^2$

$EA_{eq} = E_F A_F + E_G A_G$

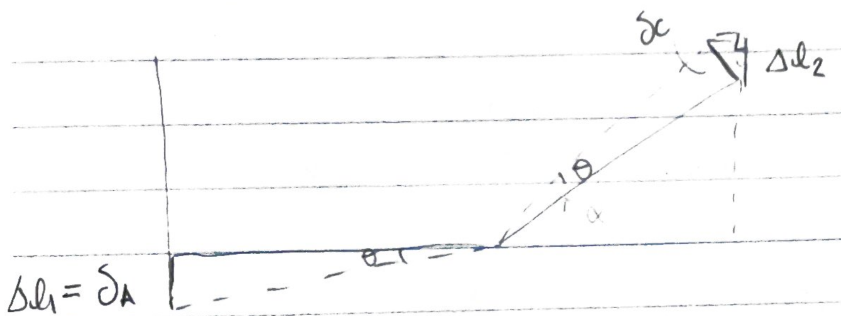
$EA_{eq} = 1\,780\,880 KN$



$\sum F_H = 0: H_B = 0$

$\sum F_V = 0: N_1 - P + V_B - N_2 = 0$

$\sum M_{(B)} = 0: N_1 + \frac{4}{3} N_2 = P$



$\Delta l_1 = \frac{3}{5}\delta_c$

$\Delta l_2 = \frac{4}{5}\delta_c$

Williot

$\frac{\delta_A}{3} = \frac{\delta_c}{5}$, $\delta_A = \Delta l_1$

$\delta_c = \frac{\Delta l_2}{\cos\alpha}$

$\varepsilon_1 = \frac{\Delta l_1}{l_1} = \frac{1}{500}\delta_c$

$\varepsilon_2 = \frac{\Delta l_2}{l_2} = \frac{4}{1500}\delta_c$

Condições de Deformação máxima:

$\sigma_F = E_F \varepsilon_F \leq \overline{\sigma_F} \Rightarrow \varepsilon_F \leq \overline{\sigma_F}/E_F$

$\sigma_G = E_G \varepsilon_G \leq \overline{\sigma_G} \Rightarrow \varepsilon_G \leq \overline{\sigma_G}/E_G$

Substituindo: $\varepsilon_G \leq 7{,}143 \cdot 10^{-4} cm/cm$

$\varepsilon_F \leq 2{,}38 \cdot 10^{-3} cm/cm$

Usarei como limite $\overline{\varepsilon} = 7{,}143 \cdot 10^{-4}$

$\max(\varepsilon_1, \varepsilon_2) \leq \overline{\varepsilon}$

$\varepsilon_2 \leq \overline{\varepsilon}$

$\frac{4\delta_c}{1500} \leq \overline{\varepsilon} \Rightarrow \boxed{\delta_c \leq 0{,}27 cm}$

$\varepsilon = \frac{N}{EA}$ $\varepsilon_1 = \frac{N_1}{EA_{eq}}$ $\varepsilon_2 = \frac{N_2}{EA_{eq}}$

como $\frac{4}{3}\varepsilon_1 = \varepsilon_2$, então:

$\boxed{\frac{4N_1}{3} = N_2}$ Assim $N_2 > N_1$

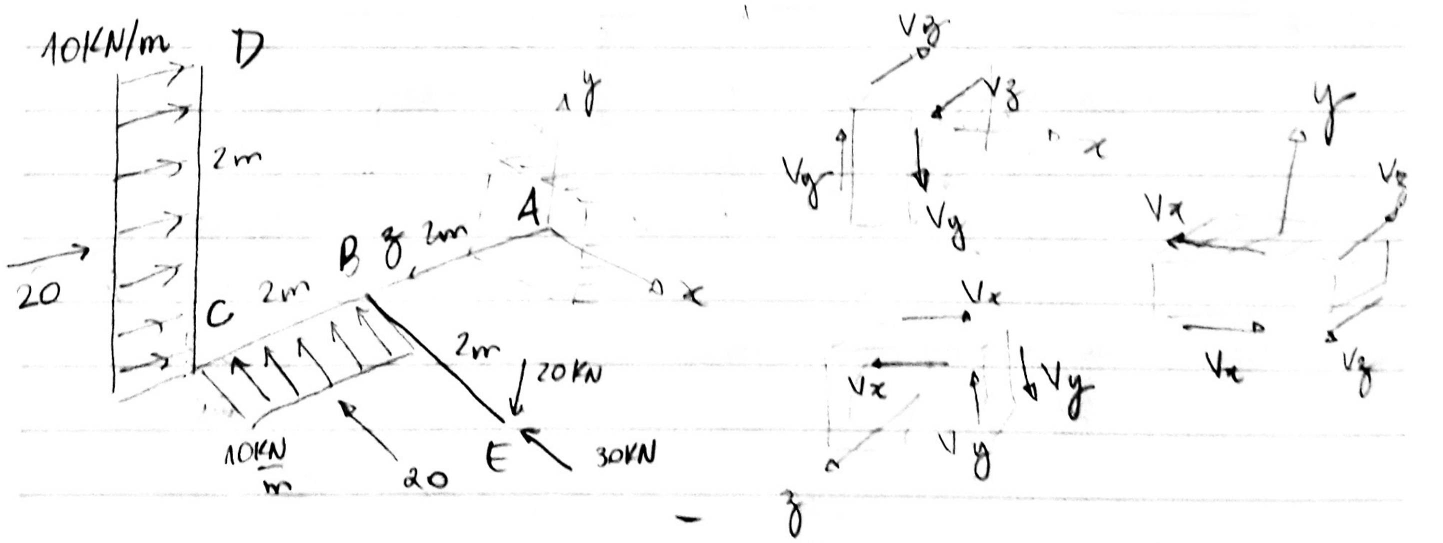
$\varepsilon_2 \leq \bar{\varepsilon}$

$\frac{N_2}{EA_{eq}} \leq \bar{\varepsilon}$.

$|N_2 \leq 1277 KN|$

Usando equilíbrio: $P \leq 2660 KN$

$P_{max} = 2660 KN$



Forças de Reação:

Em x: $-20 - 30 + R_x = 0 \Rightarrow R_x = 50KN$

Em y: $-20 + R_y = 0 \Rightarrow R_y = 20KN$

Em z: $-20 + R_z = 0 \Rightarrow R_z = 20KN$

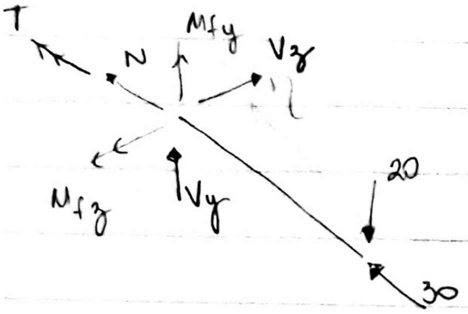
Momentos:

Em x: $-20.1 + 20.2 + M_x = 0 \Rightarrow M_x = -20KNm$

Em y: $-20.3 - 30.2 + M_y = 0 \Rightarrow M_y = 120 KNm$

Em z: $-20.2 + M_z = 0 \Rightarrow M_z = 40KNm$

Barra BE $\quad 0 < \eta < 2$

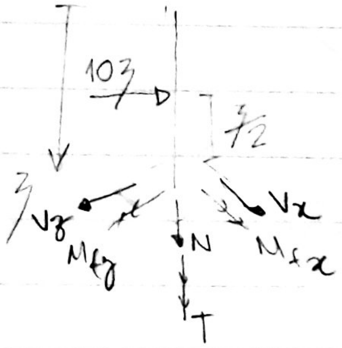


$N = -30 \qquad T = 0$

$V_z = 0 \qquad M_{fz} = 20\eta$ ⊕ cima

$V_y = 20 \qquad M_{fy} = 0$

Barra DC

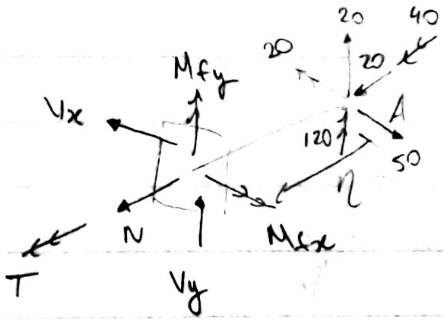


$N = 0 \qquad T = 0$

$V_x = 0 \qquad M_{fx} = 5z^2$ ⊕ direita

$V_z = 10z \qquad M_{fz} = 0$

Barra AB



$N = -R_z = -20 \qquad T = -M_z = -40$

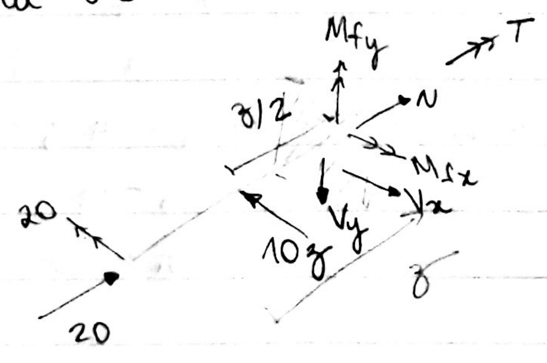
$V_y = -R_y = -20 \qquad M_{fy} + M_y - R_x \cdot \eta = 0$

$V_x = R_x = 50. \qquad M_{fy} = 50\eta - 120$ ⊕ trás

$M_{fx} + M_x + R_y\,\eta = 0$

$M_{fx} = 20 - 20\eta$ ⊕ cima

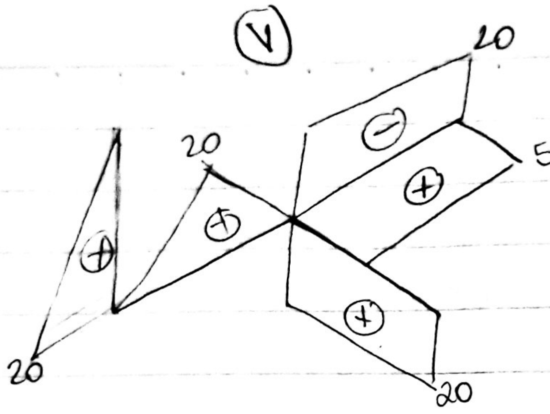
Barra BC

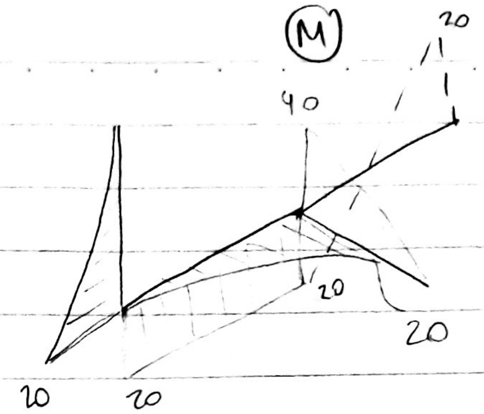


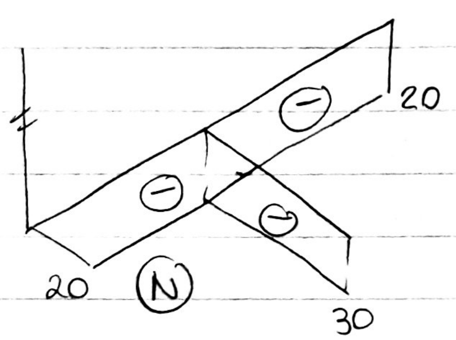
$N = -20 \qquad T = 0$

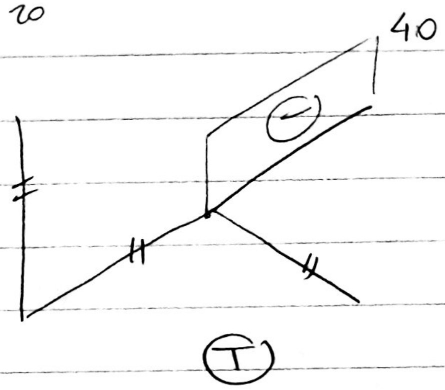
$V_x = 10z \qquad M_{fx} = 20$ ⊕ baixo

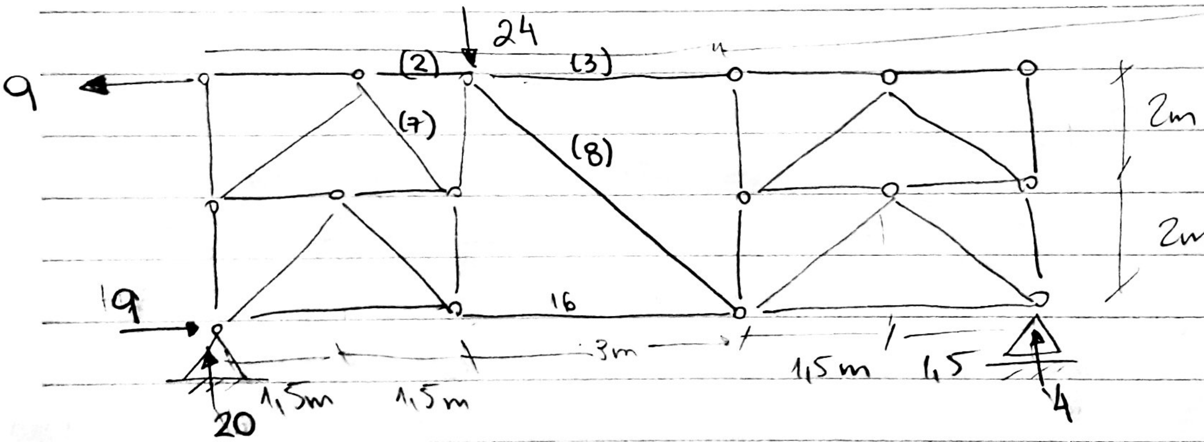
$V_y = 0 \qquad M_{fy} = 5z^2$ ⊕ frente











Barra	N [KN]
2	−6
3	−3
7	−20
8	−5

(1) Reações de Apoio

$\sum F_x = 0 \Rightarrow \boxed{H_A = 9}$

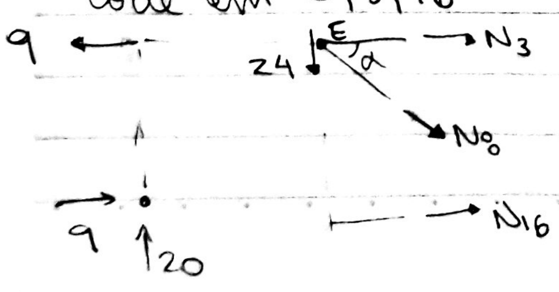
$\sum F_y = 0 \Rightarrow V_A + V_B = 24$

$\sum M_{(A)} = 0 \Rightarrow V_B \cdot 9 - 24 \cdot 3 + 9 \cdot 4 = 0$

$\boxed{V_B = 4}$

$\boxed{V_A = 20}$

Corte em 3, 8, 16



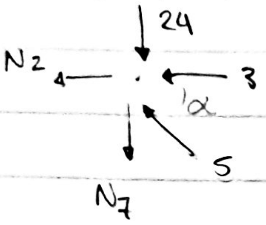
$\sum F_y = 0: \; 20 - 24 - N_8 \operatorname{sen} \alpha = 0$

$N_8 = \frac{-4}{0{,}8} = -5 \text{ KN}$

$\sum M_{(E)} = 0: \; 9 \cdot 4 - 20 \cdot 3 + N_{16} \cdot 4 = 0$

$N_{16} = 6 \Rightarrow N_3 = -3$

Nó E



$$N_2 + 3 + 3 = 0 \rightarrow N_2 = -6$$

$$N_7 + 24 = 4$$

$$\boxed{N_7 = -20}$$