Lista 1A | Paula Xavier 2017.1

1) a) $\mathbb{P}$(segundo) = 0,1 ∴ $\mathbb{P}$(primeira) = 0,9

C1 = classificado como 1º

C2 = classificado como 2º

$\mathbb{P}(C2|1) = 0,3 \therefore \mathbb{P}(C1|1) = 0,7$

$\mathbb{P}(C1|2) = 0,4 \therefore \mathbb{P}(C2|2) = 0,6$

$$\mathbb{P}(2|C1) = \frac{\mathbb{P}(C1|2)\cdot\mathbb{P}(2)}{\mathbb{P}(C1)}$$

$\mathbb{P}(C1) = \mathbb{P}(C1|2)\cdot\mathbb{P}(2) + \mathbb{P}(C1|1)\cdot\mathbb{P}(1) =$

$= 0,4\cdot 0,1 + 0,7\cdot 0,9 = 0,04 + 0,63 = 0,67$

Pelo Teorema de Bayes temos que:

$$\mathbb{P}(2|C1) = \frac{0,4\cdot 0,1}{0,67} \cong 0,06$$

b) $\mathbb{P}(\text{erro}) = \mathbb{P}(C2|1)\cdot\mathbb{P}(1) + \mathbb{P}(C1|2)\cdot\mathbb{P}(2) =$

$= 0,3\cdot 0,9 + 0,4\cdot 0,1 = 0,27 + 0,04 = 0,31$

erro identificado

0,31 → 0,15 → 0,25

ou (+)

0,31 → 0,85 → 0,17

erro não identificado

$\mathbb{P}$(treinamento) =

$= (0,31\cdot 0,15\cdot 0,25) + (0,31\cdot 0,85\cdot 0,17) =$

$= 0,05642$

2) a) Como são distribuídos uniformemente entre [18, 19]

então $\mathbb{P}(X = x) = \frac{1}{60}$, com $x = \{1, 2, \dots, 60\}$. Assim

$\mathbb{P}(18h30 < X \leq 18h45) = \frac{15}{60} = \frac{1}{4}$

$\mathbb{P}(\text{Milton e Victor}) = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$

b) $\mathbb{P}(18h00 < X < 18h30) = \frac{30}{60} = \frac{1}{2}$

$\mathbb{P}(18h30 < X < 19h00) = \frac{30}{60} = \frac{1}{2}$

$\mathbb{P}(\text{antes e depois}) = \left(\frac{1}{2}\right)^2\cdot\left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$

$\mathbb{P}(\text{alunos}) = C_{10,8}\cdot\frac{1}{1024} = \frac{10!}{8!\,2!}\cdot\frac{1}{1024} = \frac{10\cdot 9}{2}\cdot\frac{1}{1024} =$

$= \frac{45}{1024}$

c) $\mathbb{P}(18h00 < X < 18h45) = \frac{45}{60} = \frac{3}{4}$

$\mathbb{P}(\text{Milton e Victor}) = \frac{3}{4}\cdot\frac{3}{4} = \frac{9}{16}$

$\mathbb{P}(18h45 < X < 19h00) = \frac{15}{60} = \frac{1}{4}$

$\mathbb{P}(3\text{ alunos}) = C_{8,3}\cdot(1/4)^3 = \frac{8!}{3!5!}\cdot(1/4)^3 = 8\cdot 7\cdot(1/4)^3$

$\mathbb{P}(\text{restantes}) = \left(\frac{3}{4}\right)^5$

$\mathbb{P}(\text{total}) = \left(3/4\right)^2\cdot 56\cdot\left(1/4\right)^3\cdot\left(3/4\right)^5 = \frac{15309}{131072}$

3) a) T: tempo de vida do chip

$$\mathbb{P}(-\infty < x < +\infty) = \int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{+\infty}\frac{k}{x^2}\,dx = 1 \Rightarrow \int_{100}^{+\infty}\frac{k}{x^2}\,dx =$$

$$= -\frac{k}{x}\Big|_{100}^{\infty} = 1 \Rightarrow \frac{k}{100} = 1 \Rightarrow k = 100$$

b) Para $t > 100$: $\mathbb{P}(T > t) = 1 - \mathbb{P}(100 \leq T \leq t) \Rightarrow$

$$\Rightarrow \mathbb{P}(T > t) = 1 - \int_{100}^{t}\frac{100}{x^2}\,dx = 1 - \left[-\frac{100}{x}\right]_{100}^{t} = 1 + \frac{100}{t} - 1 = \frac{100}{t}$$

Para $t < 100$: $\mathbb{P}(T > t) = 1 - \mathbb{P}(T \leq t) = 1 - 0 = 1$.

c) $\mathbb{P}$(defeito nas 150 primeiras horas) $= 1 - \mathbb{P}(T > 150) =$

$= 1 - \frac{100}{150} = \frac{5\cancel{0}}{15\cancel{0}} = \frac{1}{3}$

$\mathbb{P}(\text{chips}) = (1/3)^2\,(2/3)^3$

$\mathbb{P}(\text{total}) = C_{5,2}\cdot\mathbb{P}(\text{chips}) = \frac{5!}{2!\,3!}\cdot(1/3)^2\,(2/3)^3 =$

$= 10\cdot\frac{8}{3^5} = \frac{80}{243}$

8)

$\frac{9}{25}$ → $\frac{1}{4}$

ou (+)

$\frac{16}{25}$ → $\frac{7}{8}$

$\mathbb{P}(\max) = \frac{9}{25}\cdot\frac{1}{4} + \frac{16}{25}\cdot\frac{7}{8} =$

$= 0,09 + \frac{14}{25} = 0,65$

10) Soma possível: 1 + 2 + 4

Essa soma pode ser 3! vezes. Podendo ser 1+2+4, 1+4+2, 2+1+4, 2+4+1, 4+1+2, 4+2+1. Como os números não podem se repetir então $\mathbb{P}(\text{soma } 7) = 6\cdot\frac{1}{6}\cdot\frac{1}{5}\cdot\frac{1}{4} = \frac{1}{20}$

5) $\mathbb{P}(3\text{ caras ou } 3\text{ coroas}) = (1/2)^3 + (1/2)^3 = 1/4$

$\mathbb{P}(\overline{3\text{ caras ou } 3\text{ coroas}}) = 1 - \mathbb{P}(3\text{ caras ou } 3\text{ coroas}) = 3/4$

$\mathbb{P}(\text{branca urna } 1) = \frac{9}{14}$, $\mathbb{P}(\text{branca urna } 2) = \frac{8}{11}$

1/4 → 9/14

ou (+)

3/4 → 8/11

$\mathbb{P}(\text{branca}) = \frac{1}{4}\cdot\frac{9}{14} + \frac{3}{4}\cdot\frac{8}{11} =$

$= \frac{9(44+56)}{56\cdot 44} = \frac{900}{56\cdot 44} = \frac{225}{616}$

6) a) Devemos ter $\int_{-\infty}^{\infty} f(x)\,dx = 1$, como para $x > 1$ e $x < 0$

$\int f(x)\,dx = 0$ ($\Rightarrow$ a área será zero) temos $\int_{-\infty}^{\infty} f(x)\,dx =$

$= \int_0^1 f(x)\,dx = 1 \Rightarrow \frac{\alpha x^3}{3} + \frac{\beta x^2}{2}\Big|_0^1 = \frac{\alpha}{3} + \frac{\beta}{2} = 1$

Sabemos também que $F_X(1/2) = 1/5$, portanto:

$\int_0^{1/2} f(x)\,dx = \frac{\alpha x^3}{3} + \frac{\beta x^2}{2}\Big|_0^{1/2} = \underbrace{\frac{\alpha}{24} + \frac{\beta}{8}}_{F_X(1/2)} - \underbrace{\left(\frac{\alpha\cdot 0}{3} + \frac{\beta\, 0}{2}\right)}_{F_X(0)} =$

$= \frac{\alpha}{24} + \frac{\beta}{8} = \frac{1}{5}$

$\begin{cases} \frac{\alpha}{24} + \frac{\beta}{8} = \frac{1}{5} \Rightarrow \alpha + 3\beta = \frac{24}{5} \\ \frac{\alpha}{3} + \frac{\beta}{2} = 1 \Rightarrow \alpha = \frac{6-3\beta}{2} \end{cases}$

$\frac{6 - 3\beta + 6\beta}{2} = \frac{24}{5} \Rightarrow 3\beta = \frac{48}{5} - 6 \Rightarrow \beta = \frac{6}{5} \therefore \alpha = \frac{12}{10} = \frac{6}{5}$

b) $P(-1/4 \leq x \leq 1/4) = \int_{-1/4}^{1/4} f(x)\,dx = \int_{-1/4}^{0} f(x)\,dx + \int_0^{1/4} f(x)\,dx =$

$= 0 + \frac{6x^3}{15} + \frac{6x^2}{10}\Big|_0^{1/4} = \frac{\cancel{6}}{\cancel{15}\cdot\cancel{64}} + \frac{6}{160} = \frac{1}{160} + \frac{6}{160} = \frac{7}{160}$

12) $P(+1000 \text{ visit}) = 0{,}25\cdot 0{,}2 + 0{,}5\cdot 0{,}25 + 0{,}75\cdot 0{,}3 + 0{,}75\cdot 0{,}25$

$= 0{,}5875$

$P(25\text{-}30 \mid +1000 \text{ visit}) = \frac{P(+1000 \text{ visit} \mid 25\text{-}30)\cdot P(25\text{-}30)}{P(+1000 \text{ visit})} =$

$= \frac{0{,}75\cdot 0{,}3}{0{,}5875} = 0{,}382979$

13) $P(\text{adulterado}) = 0{,}05$ ⊕ acerta ⊖ erra

$P(+ \mid \text{adulterado}) = 0{,}9$ ; $P(- \mid \text{adulterado}) = 0{,}1$

$P(- \mid \overline{\text{adulterado}}) = 0{,}05$ ; $P(+ \mid \overline{\text{adulterado}}) = 0{,}95$

$P(\text{adulterada} \mid +) = \frac{P(+ \mid \text{adulterada})\, P(\text{adulterada})}{P(+)}$

$P(+) = 0{,}9\cdot 0{,}05 + 0{,}95\cdot 0{,}05 = 0{,}0925$

$P(\text{adulterada} \mid +) = \frac{0{,}9\cdot 0{,}05}{0{,}0925} = 0{,}4865$

15) $P(\text{spam}) = P(\text{spam} \mid A)\cdot P(A) + P(\text{spam} \mid B)\, P(B) +$

$+ P(\text{spam} \mid C)\, P(C) = 0{,}01\cdot 0{,}6 + 0{,}02\cdot 0{,}3 + 0{,}05\cdot 0{,}1 \Rightarrow$

$\Rightarrow P(\text{spam}) = 0{,}017$

$P(\overline{\text{spam}}) = 1 - P(\text{spam}) = 0{,}983$

16) $\sum_{x=0}^{5} P(X = x) = 1 \Rightarrow a\cdot 0 + a\cdot 1 + a\cdot 2 + a\cdot 3 + a\cdot 4 + a\cdot 5 = 1$

$\Rightarrow 15a = 1 \Rightarrow a = \frac{1}{15} \quad P(X = x) = \frac{x}{15}$

$P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5) =$

$= \frac{3}{15} + \frac{4}{15} + \frac{5}{15} = \frac{12}{15} = \frac{4}{5}$

17) Do gráfico temos que $P(X \geq 9) = \frac{(1+3)\cdot 1/8}{2} = \frac{1}{4}$

$P(\text{Jolol}) = C_{3,2}\,(1/4)(1/4)(3/4) + (1/4)(1/4)(1/4) =$

$= \frac{3\cdot 3}{64} + \frac{1}{64} = \frac{10}{64} = \frac{5}{32}$

19) $P(X=0) = \frac{12}{42}$, $P(X=1) = \frac{10}{42}$, $P(X=2) = \frac{8}{42}$,

$P(X=3) = \frac{6}{42}$, $P(X=4) = \frac{4}{42}$, $P(X=5) = \frac{2}{42}$

$P(X=3) = \frac{4}{42} + \frac{2}{42} = P(X=4) + P(X=5)$

$P(X=1) \neq P(X=0) + \frac{1}{36} \Rightarrow \frac{10}{42} \neq \frac{12}{42} + \frac{1}{36}$

$P(X>1) = \frac{20}{42}$ e $P(X<1) = \frac{12}{42}$

$P(X>1) \neq 1 - \frac{12}{42} = \frac{30}{42}$

20) $X$ := reais

$P(X=1) = C_{3,2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{3}{8}$

$P(X=8) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$

$P(X=x) = \begin{cases} \frac{3}{8}, & \text{se } x = 1 \\ \frac{1}{8}, & \text{se } x = 8 \\ 0, & \text{caso contrário} \end{cases} \quad \therefore P(X<7) = 1 - P(X=8) \Rightarrow P(X<7) = 7/8$

21) $P(x \geq \frac{1}{2a}) = 1 - P(x \leq \frac{1}{2a}) \Rightarrow P(x \geq \frac{1}{2a}) = 1 - 1 + e^{-\frac{1}{2}}$

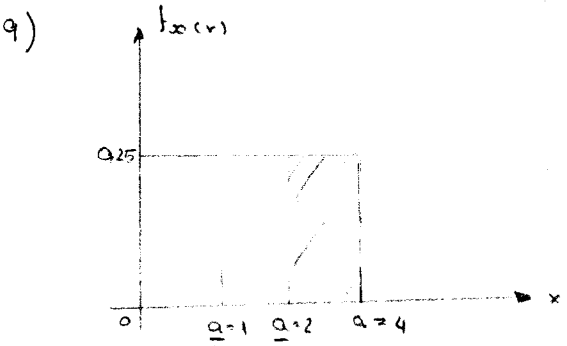
$\Rightarrow P(x \geq 1/2a) = e^{-1/2}$

11) $P(S) + P(M) + P(C) = 1 \Rightarrow$

$\Rightarrow P(M) = 1 - P(S) - P(C) \Rightarrow P(M) = 1 - 0{,}648 - 0{,}02$

$\Rightarrow P(M) \cong 0{,}32$.

9)




$$\int_0^a f(x)\,dx = \frac{1}{4}\cdot a = 1 \Rightarrow a = 4$$

$\mathbb{P}(x>2 \text{ dado } x>1) = \frac{2}{3}$

14) $\mathbb{P}(HH) = 1{,}21\,\mathbb{P}(TT)$

$\Omega = \{HH, TT, HT, TH\}$

$\mathbb{P}(HH) + \mathbb{P}(TT) = \frac{1}{2} \Rightarrow 2{,}21\,\mathbb{P}(TT) = \frac{1}{2} \Rightarrow \mathbb{P}(TT) = 0{,}226$

$\mathbb{P}(T) = \mathbb{P}(TT) + \mathbb{P}(HT) = 0{,}226 + 0{,}25 = 0{,}476 = \frac{10}{21}$

7) $\mathbb{P}(\text{Vaca}) = \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot 0 = \frac{5}{8}$

$\mathbb{P}(\text{Vaca Selecionada}) = 1\cdot\frac{1}{4} + 1\cdot\frac{1}{4} = \frac{1}{2}$

$\mathbb{P}(\text{Total}) = \frac{1}{\cancel{2}}\cdot\frac{\cancel{8}}{5} = \frac{4}{5}$

23) a) $\mathbb{P}(A_1 \text{ e correto}) = 0{,}3\cdot 0{,}8\cdot 0{,}8 + 0{,}3\cdot 0{,}2\cdot 0{,}6 =$

$= 0{,}192 + 0{,}036 = 0{,}228$

$\mathbb{P}(A_2 \text{ e correto}) = 0{,}7\cdot 0{,}8\cdot 0{,}9 + 0{,}7\cdot 0{,}2\cdot 0{,}8 =$

$= 0{,}504 + 0{,}112 = 0{,}616$

$\mathbb{P}(\text{correto}) = \mathbb{P}(A_1 \text{ e correto}) + \mathbb{P}(A_2 \text{ e correto}) = 0{,}844$

b) $\mathbb{P}(A_1 \text{ e normal} \mid \text{Correto}) = \dfrac{\mathbb{P}(A_1 \text{ e normal e correto})}{\mathbb{P}(\text{correto})} =$

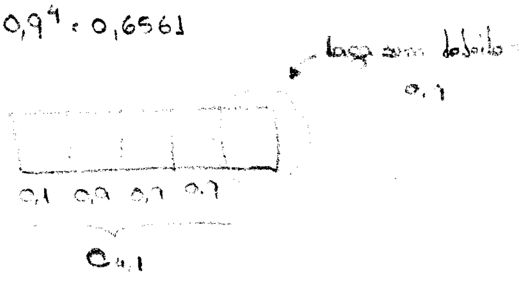
$= \dfrac{0{,}192}{0{,}844} = 0{,}227.$

25) a) $\mathbb{P}(1 \text{ com defeito}) = C_{5,1}\cdot 0{,}1\cdot 0{,}9^4 = 5\cdot 0{,}1\cdot 0{,}9^4 =$

$= 0{,}32805$

b) $\mathbb{P}(4^{\circ} \text{ laço}) = 0{,}9^4 = 0{,}6561$

Quinta laço:




$\mathbb{P}(5^{\circ} \text{ laço}) = C_{4,1}\cdot 0{,}1\cdot 0{,}9^3\cdot 0{,}9 = 0{,}26244$

22) a) $c + \frac{c}{2} + \frac{c}{3} + \frac{c}{4} + \frac{c}{5} = 1 \Rightarrow$

$\Rightarrow \dfrac{60c + 30c + 20c + 15c + 12c}{60} = 1 \Rightarrow \dfrac{137c}{60} = 1 \Rightarrow c = \dfrac{60}{137}$

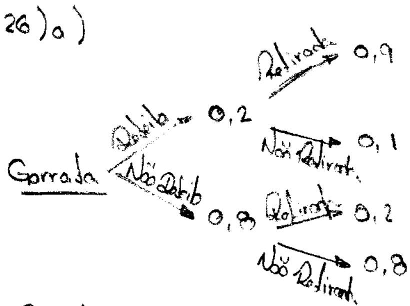
c) $\mathbb{P}(x>1) = \dfrac{30c + 20c + 15c + 12c}{60} = \dfrac{77c}{60} = \dfrac{77}{\cancel{60}}\cdot\dfrac{\cancel{60}}{137} = \dfrac{77}{137}$

23) a) $\mathbb{P}(\text{padrão}) = \frac{1}{3}\cdot 1\cdot 1 + \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{4}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{2} =$

$= \frac{1}{3} + \frac{3}{16} + \frac{1}{12} = \frac{16 + 9 + 4}{48} = \frac{29}{48}$

b) $\mathbb{P}(3^{\circ} \text{ lado}) = \dfrac{\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{2}}{\frac{29}{48}} = \dfrac{1/12}{29/48} = \dfrac{4}{29}$

26) a)




$\mathbb{P}(\text{retirada}) = 0{,}2\cdot 0{,}9 + 0{,}8\cdot 0{,}2 = 1{,}8 + 1{,}6 = 3{,}4$

$\mathbb{P}(\text{defeito} \mid \text{retirada}) = \dfrac{\mathbb{P}(\text{defeito e retirada})}{\mathbb{P}(\text{retirada})} = \dfrac{0{,}2\cdot 0{,}9}{3{,}4} =$

$= \dfrac{1{,}8}{3{,}4} = \dfrac{18}{34}$

b) $\mathbb{P}(\text{não retirada} \mid \text{defeito}) = \dfrac{\mathbb{P}(\text{não retirada e defeito})}{\mathbb{P}(\text{defeito})}$

$= \dfrac{0{,}2\cdot 0{,}1}{0{,}2} = 0{,}1 = \dfrac{1}{10}$

Lista 1 | (2014) Paulo Akira 2017 1

1) a) O espaço amostral é $S = \{AC, MC, BC, AL, ML, BL\}$

b) $A_1 = \{MC, ML\}$

c) $A_2 = \{AC, MC, BC\}$

d) $A_3 = \{AC, AL, BC, BL\}$

e) Não são mutualmente exclusivos, pois, apenas os eventos $A_1$ e $A_3$ são disjuntos

2) a) O espaço amostral é $\{aaa, daa, ada, aad, ddd, add, dad, dda\}$

b) $Z_d = \{aad, ddd, add, dad\}$ e $X_A = \{aaa, ada, aad, add\}$

c) Não são mutualmente exclusivos, pois $Z_d \cap X_A = \{aad, add\}$

5) O espaço amostral é $S = \{GR, GL, PR, PL\}$

| | Grandes (G) | Pequenos (P) |
|---|---|---|
| Rápidos (R) | 0,2 | 0,5 |
| Lentos (L) | 0,2 | 0,1 |

a) Da Tabela: $\mathbb{P}(L) = 0,3$

b) Da Tabela: $\mathbb{P}(G) = 0,4$

c) $\mathbb{P}(L \cup G) = \mathbb{P}(L) + \mathbb{P}(G) - \mathbb{P}(L \cap G) = 0,3 + 0,4 - 0,2 = 0,5$

6) O espaço amostral é $S = \{NC, NL, SC, SL\}$

Nota que como $\mathbb{P}(C) = 0,5$ então $\mathbb{P}(NC) - \mathbb{P}(SC) = 0,5$

| | Curtas (C) | Longas (L) |
|---|---|---|
| Smart (S) | 0,3 | 0,1 |
| Normal (N) | 0,2 | 0,4 |

a) Da tabela: $\mathbb{P}(L) = \mathbb{P}(SL) + \mathbb{P}(NL) = 0,5$

b) Da tabela: $\mathbb{P}(SC) = 0,5 - \mathbb{P}(NC) = 0,3$

c) Da tabela: $\mathbb{P}(N) = \mathbb{P}(NC) + \mathbb{P}(NL) = 0,6$

7) Da tabela: $\mathbb{P}(H_0) = \mathbb{P}(H_0L) + \mathbb{P}(H_0C) = 0,5$,

$\mathbb{P}(C) = \mathbb{P}(H_0C) + \mathbb{P}(H_1C) + \mathbb{P}(H_2C) = 0,1 + 0,1 + 0,1 = 0,$[illegible]

$\mathbb{P}(L) = \mathbb{P}(H_0L) + \mathbb{P}(H_1L) + \mathbb{P}(H_2L) = 0,4$

$\mathbb{P}(H_2) = \mathbb{P}(H_2C) + \mathbb{P}(H_2L) = 0,3$

$\mathbb{P}(H_2 \cap L) = \mathbb{P}(H_2L) = 0,2$

$\mathbb{P}(L \cup H_2) = \mathbb{P}(L) + \mathbb{P}(H_2) - \mathbb{P}(H_2 \cap L) = 0,5$

8) a) Dado que o resultado do lançamento é maior do que 1: $\mathbb{P}(R_3|M_1) = \dfrac{\mathbb{P}(R_3 \cap M_1)}{\mathbb{P}(M_1)} = \dfrac{1/6}{5/6} = \dfrac{1}{5}$

b) $\mathbb{P}(R_6|M_3) = \dfrac{\mathbb{P}(R_6 \cap M_3)}{\mathbb{P}(M_3)} = \dfrac{1/6}{3/6} = \dfrac{1}{3}$

c) $\mathbb{P}(M_3|P) = \dfrac{\mathbb{P}(M_3 \cap P)}{\mathbb{P}(P)} = \dfrac{2/6}{3/6} = \dfrac{2}{3}$

d) $\mathbb{P}(P|M_3) = \dfrac{\mathbb{P}(P \cap M_3)}{\mathbb{P}(M_3)} = \dfrac{2/6}{3/6} = \dfrac{2}{3}$

4) (Lista de 2015)

a) $\mathbb{P}(s/\text{mulheres}) = \dfrac{C_{4,3}\, C_{4,0}}{C_{8,3}} = \dfrac{\frac{4!}{3!}}{\frac{8!}{5!\,3!}} = \dfrac{4}{56} \cong 0,0714$

$\therefore \mathbb{P} = 1 - 0,0714 = 0,9286$

b) $\mathbb{P} = \dfrac{C_{4,4} \cdot C_{4,1}}{C_{8,5}} = \dfrac{4!}{3!} \cdot \dfrac{1}{56} \cong 0,0714$

c) $\mathbb{P} = \dfrac{1}{8!} = \dfrac{1}{40326}$

Lista 1B | Paulo Akira 2017 1

4) a) Não é uma partição, pois os $A_i$ não conseguem cobrir todo $\mathbb{R}$.

b) Sim, é uma partição, pois, os conjuntos $A_i$ são disjuntos e cobrem todo $S$

c) Não é uma partição, pois, os conjuntos $A_i$ não são todos disjuntos

10) a) Como $A$ e $B$ são disjuntos $\mathbb{P}(A \cap B) = 0$

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

$\mathbb{P}(A \cap \overline{B}) = \mathbb{P}(A) + \mathbb{P}(\overline{B}) - \mathbb{P}(A \cup \overline{B}) = \mathbb{P}(A) + \mathbb{P}(\overline{B}) - \mathbb{P}(\overline{B}) = \frac{1}{4}$

$\mathbb{P}(A \cup \overline{B}) = \mathbb{P}(\overline{B}) = \frac{7}{8}$

b) $\mathbb{P}(A|B) = \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = 0 \neq \mathbb{P}(A) = \frac{1}{4}$

$\mathbb{P}(B|A) = \dfrac{\mathbb{P}(B \cap A)}{\mathbb{P}(A)} = 0 \neq \mathbb{P}(B) = \frac{1}{8}$

$\therefore A$ e $B$ não são independentes.

c) $\mathbb{P}(C|D) = \mathbb{P}(C) \Rightarrow \frac{\mathbb{P}(C\cap D)}{\mathbb{P}(D)} = \mathbb{P}(C) \Rightarrow \mathbb{P}(C\cap D) = \frac{15}{64}$

$\mathbb{P}(\bar D) = 1 - \frac{3}{8} = \frac{5}{8}$

Como C e D são independentes então C e $\bar D$ também serão.

$\mathbb{P}(C\cap\bar D) = \mathbb{P}(C)\cdot\mathbb{P}(\bar D) = \frac58\cdot\frac58 = \frac{25}{64}$

$\mathbb{P}(\bar C) = 1-\frac58 = \frac38$

Como C e D são independentes então $\bar C$ e $\bar D$ também serão.

$\mathbb{P}(\bar C\cap\bar D) = \mathbb{P}(\bar C)\cdot\mathbb{P}(\bar D) = \frac38\cdot\frac58 = \frac{15}{64}$

d) $\mathbb{P}(\bar C|\bar D) = \frac{\mathbb{P}(\bar C\cap\bar D)}{\mathbb{P}(\bar D)} = \frac58 = \mathbb{P}(\bar C)$

$\mathbb{P}(\bar D|\bar C) = \frac{\mathbb{P}(\bar D\cap\bar C)}{\mathbb{P}(\bar C)} = \frac38 = \mathbb{P}(\bar D)$

∴ $\bar C$ e $\bar D$ são independentes

12) a) $\mathbb{P}(C_1)=\mathbb{P}(C_2)=1/4 \therefore \mathbb{P}(K_1)=\mathbb{P}(K_2)=3/4$

$\mathbb{P}(C_1 \text{ e } C_2) = \frac14\cdot\frac14=\frac1{16}$

$\mathbb{P}(C_1|C_2) = \frac{\mathbb{P}(C_1 \text{ e } C_2)}{\mathbb{P}(C_2)} = \frac1{16}\cdot 4 = \frac14$

b) $\mathbb{P}(C_1 \text{ e } K_2) = \mathbb{P}(C_1)\cdot P(K_2) = \frac14\cdot\frac34=\frac3{16}$

13) $\mathbb{P}(HIV) = \frac1{5000} = 0{,}2\cdot10^{-3} = 0{,}0002$

$\mathbb{P}(+|HIV)=0{,}99$ e $\mathbb{P}(-|\overline{HIV})=0{,}99$

$\mathbb{P}(-|HIV) = 1-\mathbb{P}(+|HIV) = 0{,}01$

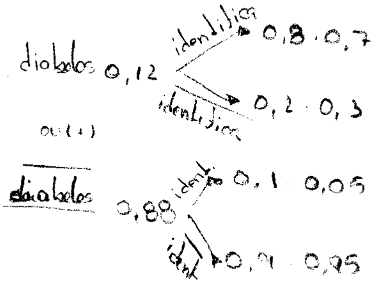
$\mathbb{P}(+|\overline{HIV}) = 1-\mathbb{P}(-|\overline{HIV}) = 0{,}01$

$\mathbb{P}(+) = \mathbb{P}(+|HIV)\cdot\mathbb{P}(HIV) + \mathbb{P}(+|\overline{HIV})\cdot\mathbb{P}(\overline{HIV}) =$

$= 0{,}99\cdot0{,}0002 + 0{,}01\cdot0{,}9998 \cong 0{,}0102$

$\mathbb{P}(HIV|+) = \frac{\mathbb{P}(+|HIV)\cdot\mathbb{P}(HIV)}{\mathbb{P}(+)} = \frac{0{,}99\cdot0{,}0002}{0{,}0102} \cong 0{,}02$

14) a) $\mathbb{P}(\text{diabetes})=0{,}12 \therefore \mathbb{P}(\overline{\text{diabetes}}) = 0{,}88$



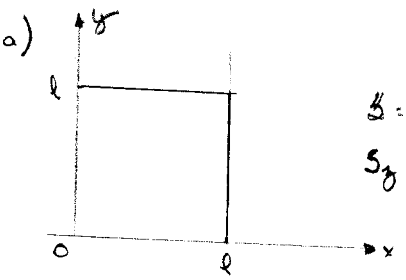

$P(\text{identifica}) = 0{,}12\cdot0{,}8\cdot0{,}7 + 0{,}88\cdot0{,}1\cdot0{,}05 =$

$= 0{,}716$

b) $\mathbb{P}(+|\text{diabetes}) = 1-\mathbb{P}(-|\text{diabetes}) =$

$= 1 - 0{,}2\cdot0{,}3 \quad 0{,}2\cdot0{,}3 = 0{,}9964$

16) a)



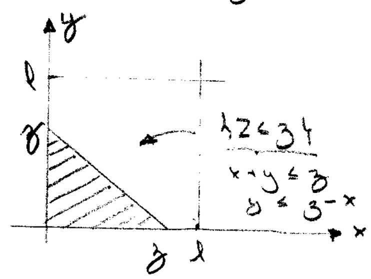

$S = [0,l]\times[0,l]$

$S_Z = [0,2l]$

$Z: S \longrightarrow S_Z$

$(x,y) \longrightarrow x+y$

b) $x+y = z \Rightarrow y = z-x$




c) $\mathbb{P}(Z\le z) =$

$= \int_0^z z-x\,dx =$

$= zx - \frac{x^2}{2}\Big|_0^z =$

$= \frac{z^2}{2}$

17) a) $\sum_{n=0}^{\infty} c(1/2)^n = c\sum_{n=0}^{\infty} \frac1{2^n} = c\cdot\frac{1}{1-1/2} = 1 \Rightarrow$ pg infinita

$\Rightarrow 2c = 1 \Rightarrow c = 1/2$

b) $p_N(n) = \begin{cases}\frac12\left(\frac12\right)^n, & n=0,1,\dots\\ 0, & \text{caso contrário}\end{cases}$

$\mathbb{P}(N\le1) = p_N(0)+p_N(1) = \frac12+\frac14 = \frac34$

18) a) A área de $-\infty$ até $+\infty$ deve ser 1 mas o único local em que a área não é zero é no intervalo $[0;2]$. Portanto: $\int_{-\infty}^{+\infty} f(y)\,dy = 1 \Rightarrow \int_0^2 f(y)\,dy = 1 \Rightarrow$

$\Rightarrow \frac{cy^2}{2}\Big|_0^2 = 1 \Rightarrow 2c=1\Rightarrow c=\frac12$.

b) $f_Y(y) = \begin{cases}\frac y2, & 0\le y\le 2\\ 0, & \text{caso contrário}\end{cases}$

$\mathbb{P}(0\le y\le1) = \int_0^1 f_Y(y)\,dy = \frac{y^2}{4}\Big|_0^1 = \frac14$.

c) $\mathbb{P}(-1/2\le y\le 1/2) = \int_{-1/2}^{1/2} f_Y(y)\,dy = \int_{-1/2}^{0} f_Y(y)\,dy + \int_0^{1/2} f_Y(y)\,dy =$

$= 0 + \int_0^{1/2}\frac y2\,dy = \frac{y^2}{4}\Big|_0^{1/2} = \frac1{16}$.

d) $F_Y(y) = \int f_Y(y)\,dy$ . $F_Y(y) = \begin{cases} 1, & y \geq 2 \\ \frac{y^2}{4}, & 0 < y < 2 \\ 0, & 0 \leq y \end{cases}$

20) b) $P(Y = y) = \begin{cases} 1/11, & \text{se } y = 5, \dots, 15 \\ 0, & \text{caso contrário} \end{cases}$

b) $P(Y < 10) = P(Y=5) + P(Y=6) + P(Y=7) + P(Y=8) +$

$+ P(Y=9) = \frac{1}{11} + \frac{1}{11} + \frac{1}{11} + \frac{1}{11} + \frac{1}{11} = \frac{5}{11}$

c) $P(Y > 12) = P(Y=13) + P(Y=14) + P(Y=15) = \frac{3}{11}$

d) $P(8 \leq Y < 12) = P(Y=8) + P(Y=9) + P(Y=10) +$

$+ P(Y=11) + P(Y=12) = \frac{5}{11}$

19) $P(X = x) = \begin{cases} (1-p)^{x-1} \cdot p, & \text{se } x = 1, 2, 3 \\ 0, & \text{caso contrário} \end{cases}$

$P(X \leq 3) = 0{,}95 \Rightarrow p + (1-p)p + (1-p)^2 p = 0{,}95 \Rightarrow$

$\Rightarrow p + p - p^2 + p - 2p^2 + p^3 = 0{,}95 \Rightarrow p(p^2 - 3p + 3) = 0{,}95$

$\Rightarrow p = 0{,}6316$

24) $\sum_{n=1}^{n} c(n) \frac{1}{x} = 1 \Rightarrow c(n) = \frac{1}{\sum_{n=1}^{n} \frac{1}{x}}$

$c(1) = 1$

$c(2) = \frac{1}{1 + \frac{1}{2}} = \frac{2}{3}$

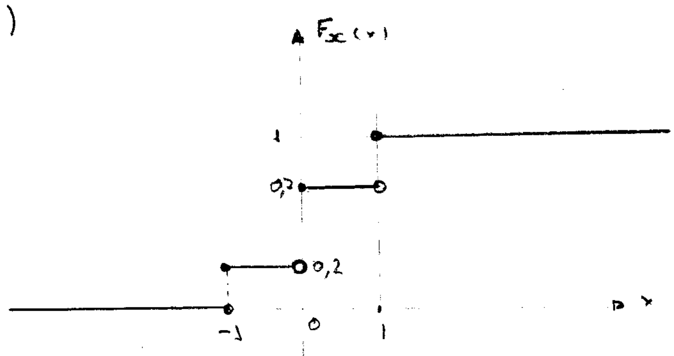
$c(3) = \frac{1}{1 + \frac{1}{2} + \frac{1}{3}} = \frac{6}{11}$

$c(4) = \frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = \frac{12}{25}$

$c(5) = \frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}} = \frac{60}{137}$

$c(6) = \frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}} = \frac{147}{60}$

26) a)




b) $P(X = 1) = F_X(1) - F_X(0) = 0{,}3$

$P(X = 0) = F_X(0) - F_X(-1) = 0{,}5$

$P(X = -1) = 0{,}2$

$P(X = x) = \begin{cases} 0{,}3, & \text{se } x = 1 \\ 0{,}5, & \text{se } x = 0 \\ 0{,}2 & \text{se } x = -1 \\ 0, & \text{caso contrário} \end{cases}$

27) $P(N = n) = \begin{cases} \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^n, & n = 1, \dots, 99 \\ \left(\frac{1}{3}\right)^n & n = 100 \\ 0, & \text{caso contrário} \end{cases}$

$F_N(n) = \begin{cases} \sum_{i=0}^{n} \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^i, & \text{se } 1 \leq n < 100 \\ 1, & \text{se } n \geq 100 \\ 0, & \text{se } n < 0. \end{cases}$

29) Para que $f_X(x)$ seja uma função densidade devemos garantir que $\begin{cases} f_X(x) \geq 0, \forall x \\ \int_{-\infty}^{+\infty} f(x)\,dx = 1 \end{cases}$

$\int_0^1 ax^2 + bx = 1 \Rightarrow \frac{ax^3}{3} + \frac{bx^2}{2} \Big|_0^1 = 1 \Rightarrow \frac{a}{3} + \frac{b}{2} = 1 \Rightarrow$

$\Rightarrow \frac{b}{2} = 1 - \frac{a}{3} \Rightarrow b = 2 - \frac{2a}{3}$

Para $x = 1$ temos: $a + b \geq 0 \Rightarrow a - \frac{2a}{3} + 2 \geq 0 \Rightarrow$

$\Rightarrow \frac{a}{3} \geq -2 \Rightarrow a \geq -6$

$b \geq 0 \Rightarrow 2 - \frac{2a}{3} \geq 0 \Rightarrow \frac{2a}{3} \leq 2 \Rightarrow a \leq 3$

28) $\int a^2 w e^{-\frac{a^2 w^2}{2}}\,dw$

$u = w^2 \Rightarrow du = 2w\,dw \Rightarrow w\,dw = \frac{du}{2}$

$\int \frac{a^2}{2} \cdot e^{-\frac{a^2 u}{2}}\,du = -\frac{a^2}{2} \cdot \frac{e^{-\frac{a^2 u}{2}}}{a^2/2} = -e^{-\frac{a^2 w^2}{2}} + c$

$F_W(w) \longrightarrow 1$
$w \longrightarrow \infty$ $\therefore F_W(w) = \begin{cases} 1 - e^{-\frac{a^2 w^2}{2}}, & w \geq 0 \\ 0, & w < 0 \end{cases}$

Lista 1C | Paulo Akira 2017.1

5) $\mathbb{P}(1 \text{ cometer erro}) = 0{,}1 \cdot 0{,}85 \cdot 0{,}8 + 0{,}9 \cdot 0{,}15 \cdot 0{,}8 +$

$+ 0{,}9 \cdot 0{,}85 \cdot 0{,}2 = 0{,}329$

$\mathbb{P}(2 \text{ cometerem erro}) = 0{,}1 \cdot 0{,}15 \cdot 0{,}8 + 0{,}1 \cdot 0{,}85 \cdot 0{,}2 +$

$+ 0{,}9 \cdot 0{,}15 \cdot 0{,}2 = 0{,}056$

$\mathbb{P}(3 \text{ cometerem erro}) = 0{,}1 \cdot 0{,}15 \cdot 0{,}2 = 0{,}003$

$\mathbb{P}(\text{total}) = 0{,}329 + 0{,}056 + 0{,}003 = 0{,}388$

6) $\mathbb{P}(A_1 \cup A_2) = p_1(1-p_2) + p_2(1-p_1)$

7) $\mathbb{P}(\text{aceito e } \overline{\text{aceito}}) = 0{,}9 \cdot 0{,}1 + 0{,}1 \cdot 0{,}9 = 0{,}18$

3) a) $\mathbb{P}(\text{perfeita e perfeita}) = \frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90}$

b) $\mathbb{P}(\text{perfeita e defeituoso}) = 2 \cdot \frac{6}{10} \cdot \frac{4}{9} = \frac{48}{90}$

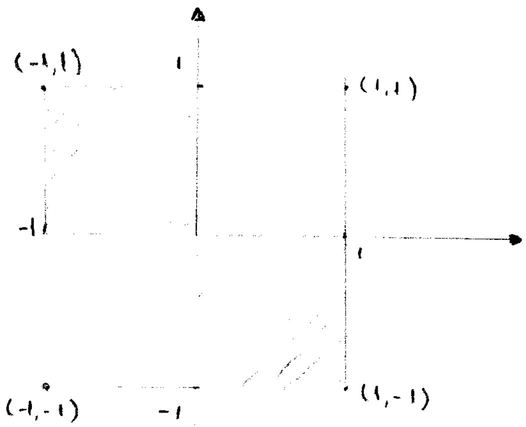
9) $\int_{-\infty}^{\infty} f(x)\,dx = -a + b + b + a + a + b + b - a = 1 \Rightarrow$

$\Rightarrow 4b = 1 \Rightarrow b = 1/4$

$f(x) \geq 0 \therefore f(2) \geq 0 \Rightarrow b - a \geq 0 \Rightarrow b \geq a$

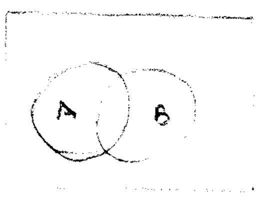
Como $b > 0$ então $b \geq a \geq 0$.

4)




$\mathbb{P}(\text{total}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

2)




$\mathbb{P}(A \cup B) + \mathbb{P}(A^c \cap B^c) = 1 \Rightarrow$

$\Rightarrow \mathbb{P}(A^c \cap B^c) = 1 - \mathbb{P}(A \cup B) \Rightarrow$

$\Rightarrow \mathbb{P}(A^c \cap B^c) =$

$= 1 - \mathbb{P}(A) - \mathbb{P}(B) + \mathbb{P}(A \cap B)$