Lista 1 Paulo Akira 2017.1

2) $I_{xx}^G = \frac{MR^2}{4}$, $I_{yy}^G = \frac{MR^2}{4} + Ml^2$, $I_{zz}^G = \frac{MR^2}{2} + Ml^2$

Note que o baricentro G do disco não tem coordenadas nem em z e nem em y. Assim: $I_{xy} = 0 + m\,l\,0 = 0$, $I_{zy} = 0 + 0 \cdot 0 \cdot m = 0$ e $I_{xz} = 0 + m\,l \cdot 0 = 0$. Também cabe ressaltar que os momentos e produtos de inércia estão sendo calculado em relação ao polo O. A matriz de inércia será, portanto, $J_A = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & -I_{zz} \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$

Consideramos que o polo O não translada, portanto, $\vec{v_0} = 0$

$\vec{H_0} = m(G-O) \wedge \vec{v_0} + [\vec{i}\ \vec{j}\ \vec{k}][J_A][\omega] \Rightarrow$

$\Rightarrow H_0 = [\vec{i}\ \vec{j}\ \vec{k}] \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} 0 \\ \omega_1 \\ \omega_2 \end{bmatrix} =$

$= [I_{xx}\vec{i}\ \ I_{yy}\vec{j}\ \ I_{zz}\vec{k}] \begin{bmatrix} 0 \\ \omega_1 \\ \omega_2 \end{bmatrix} = I_{yy}\omega_1\vec{j} + I_{zz}\omega_2\vec{k} \Rightarrow$

$\Rightarrow \vec{H_0} = M\left[\left(\frac{R^2}{4} + l^2\right)\omega_1\vec{j} + \left(\frac{R^2}{2} + l^2\right)\omega_2\vec{k}\right]$

3) $I_{xx}^G = \frac{2MR^2}{5}$, $I_{yy}^G = \frac{2MR^2}{5} + \frac{ML^2}{4}$, $I_{zz}^G = \frac{2MR^2}{5} + \frac{ML^2}{4}$

Novamente o baricentro da esfera sólida não tem coordenadas em y e z. Portanto, $I_{xy} = I_{zy} = I_{zx} = 0$. A matriz de inércia será $J_A = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} =$

$= \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$

Consideramos que o polo O não translada, portanto, $\vec{v_0} = \vec{0}$

$\vec{H_0} = m(G-O) \wedge \vec{v_0} + [\vec{i}\ \vec{j}\ \vec{k}] \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} \omega \\ \Omega \\ 0 \end{bmatrix} =$

$= [I_{xx}\vec{i}\ \ I_{yy}\vec{j}\ \ I_{zz}\vec{k}] \begin{bmatrix} \omega \\ \Omega \\ 0 \end{bmatrix} = I_{xx}\omega\vec{i} + I_{yy}\Omega\vec{j} =$

$= \frac{2MR^2}{5}\omega\vec{i} + \left(\frac{2MR^2}{5} + \frac{ML^2}{4}\right)\Omega\vec{j} \Rightarrow$

$\Rightarrow \vec{H_0} = M\left[\frac{2R^2}{5}\omega\vec{i} + \left(\frac{2R^2}{5} + \frac{L^2}{4}\right)\Omega\vec{j}\right]$

4) a) $X_G = \frac{3m\,0 - m\,L/2 + m\,L/2}{5m} = 0$

$Y_G = \frac{3m\,0 + m\,0 + m\,L/2}{5m} = \frac{L}{10}$

$Z_G = \frac{3m\,0 + m\,L/2 + m\,0}{5m} = \frac{L}{10}$

b) $I_x = \left(\frac{mL^2}{12} + \frac{L^2}{4}\right) + \left(\frac{mL^2}{12} + \frac{L^2}{4}\right) = \frac{2mL^2}{3}$

$I_y = \frac{3m\,9L^2}{12} + \left[\frac{mL^2}{12} + \underbrace{\left(\frac{mL^2}{4}\right) + \left(\frac{mL^2}{4}\right)}_{\text{distância do centro de massa até y}^2}\right] + \frac{mL^2}{4} =$

$= \frac{37mL^2}{12}$

$I_z = \frac{3m\,9L^2}{12} + \frac{mL^2}{4} + \left[\frac{mL^2}{12} + \left(\frac{mL^2}{4}\right) + \left(\frac{mL^2}{4}\right)\right] =$

$= \frac{37mL^2}{12}$

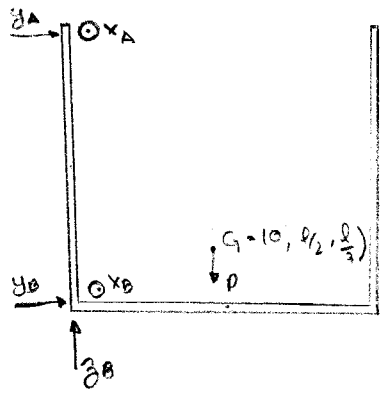
$I_{xy} = 0 + 0 + 0 + m\frac{L}{2}\cdot\frac{L}{2} = \frac{mL^2}{4}$

$I_{xz} = 0 + 0 + m\left(-\frac{L}{2}\right)\frac{L}{2} + 0 = -\frac{mL^2}{4}$

$I_{yz} = 0 + 0 + 0 = 0$

$J = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} \frac{2mL^2}{3} & -\frac{mL^2}{4} & \frac{mL^2}{4} \\ -\frac{mL^2}{4} & \frac{37mL^2}{12} & 0 \\ \frac{mL^2}{4} & 0 & \frac{37mL^2}{12} \end{bmatrix}$

5)



$X_G = 0$

$Y_G = \frac{m\,l/2 + m\,l}{3m} = \frac{3ml}{2}\,\frac{1}{3m} = \frac{l}{2}$

$Z_G = \frac{m\,l/2 + m\,l/2}{3m} = m\,l\,\frac{1}{3m} = \frac{l}{3}$

Como B é uma articulação então $\vec{v_B} = \vec{0}$ e $\vec{a_B} = \vec{0}$

Então: $\vec{0}$ $\vec{0}$, pois, ω é cte.

$$\vec{a_G} = \cancel{\vec{a_B}} + \cancel{\dot{\vec{\omega}}} \wedge (G-B) + \vec{\omega} \wedge [\vec{\omega} \wedge (G-B)] \Rightarrow$$

$$\Rightarrow \vec{a_G} = \omega \vec{k} \wedge [\omega \vec{k} \wedge (\tfrac{l}{2}\vec{j} + \tfrac{l}{3}\vec{k})] \Rightarrow$$

$$\Rightarrow \vec{a_G} = \omega \vec{k} \wedge [-\tfrac{\omega l}{2}\vec{i}] \Rightarrow \vec{a_G} = -\tfrac{\omega^2 l}{2}\vec{j}$$

TMB: $3m \cdot \vec{a_G} = \vec{R} \Rightarrow 3m\vec{a_G} = (x_A + x_B)\vec{i} + (y_A + y_B)\vec{j} + (z_B - 3mg)\vec{k}$

$$\begin{cases} x_A + x_B = 0 & (1) \\ y_A + y_B = -\frac{\omega^2 l}{2} 3m & (2) \\ z_B - 3mg = 0 & (3) \end{cases}$$

TMA: vamos tomar o polo B como referência para o calculo da quantidade de movimento angular, pois, assim os momento e produtos de inércia serão calculados em relação a este polo.

$$\vec{H_B} = m(G-B) \wedge \cancel{\vec{v_B}}^{0} + [\vec{i}\ \vec{j}\ \vec{k}][I_B][0\ 0\ \omega]^t$$

$$= [\vec{i}\ \vec{j}\ \vec{k}] \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} =$$

$$= [\vec{i}\ \vec{j}\ \vec{k}] \begin{bmatrix} -I_{xz}\omega \\ -I_{yz}\omega \\ I_{zz}\omega \end{bmatrix} = (-I_{xz}\vec{i} - I_{yz}\vec{j} + I_{zz}\vec{k})\omega$$

$$\dot{\vec{H_B}} = (-I_{xz}\dot{\vec{i}} - I_{yz}\dot{\vec{j}} + I_{zz}\dot{\vec{k}})\omega + (-I_{xz}\vec{i} - I_{yz}\vec{j} + I_{zz}\vec{k})\cancel{\dot{\omega}}^{0}$$

$$\dot{\vec{i}} = \omega\vec{k} \wedge \vec{i} = \omega\vec{j}, \quad \dot{\vec{j}} = \omega\vec{k} \wedge \vec{j} = -\omega\vec{i}, \quad \dot{\vec{k}} = \omega\vec{k} \wedge \vec{k} = \vec{0}$$

$$\dot{\vec{H_B}} = (-I_{xz}\omega\vec{j} + I_{yz}\omega\vec{i})\omega$$

$$\dot{\vec{H_B}} = m\vec{v_G} \wedge \cancel{\vec{v_B}}^{0} + \vec{M}_B^{ext} = (-y_A l - P l/2)\vec{i} + x_A l \vec{j}$$

$$\therefore (-y_A l - Pl/2)\vec{i} + x_A l \vec{j} = -I_{xz}\omega^2\vec{j} + I_{yz}\omega^2\vec{i}$$

$$\begin{cases} -y_A l - \frac{Pl}{2} = I_{yz}\omega^2 & (4) \\ x_A \cdot l = -I_{xz}\omega^2 & (5) \end{cases}$$

Note que $-I_{xz} = -(0 + m \cdot 0 \cdot \frac{l}{2} + 0 + m \cdot 0 \cdot 0 + 0 + m \cdot 0 \cdot \frac{l}{2})$

$\Rightarrow I_{xz} = 0$

$\therefore x_A \cdot l = 0 \Rightarrow x_A = 0$

Note que: $I_{yz} = 0 + m \cdot 0 \cdot l/2 + 0 + m \cdot l/2 \cdot 0 + 0 + m \cdot l \cdot \frac{l}{2} =$

$= \frac{ml^2}{2}$

$$\therefore -y_A l - \frac{3mgl}{2} = \frac{ml^2}{2}\omega^2 \quad (6)$$

Para que as reações na articulação sejam nulas temos que $x_B = 0$ e $y_B = 0$. Portanto, de (2) em (6)

$$\frac{3ml\omega^2}{2} - \frac{3mgl}{2} = \frac{ml^2\omega^2}{2} \Rightarrow \cancel{m}l^{\cancel{2}}\omega^2 = \frac{3\cancel{m}g\cancel{l}}{2} \Rightarrow$$

$$\Rightarrow \omega^2 = \frac{3g}{2l} \Rightarrow \omega = \sqrt{\frac{3g}{2l}}$$

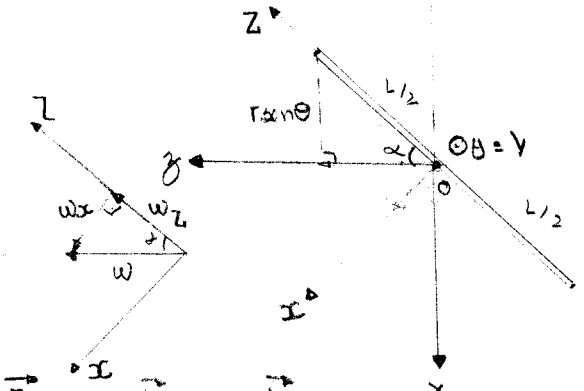
Como $x_A = 0$ então $x_B = 0$. Lembremos que $m = \rho l$

De (2): $y_A = -\frac{3g}{2\cancel{l}} \cdot \frac{\cancel{l}}{2} \cdot 3m = -\frac{9gm}{4} = -\frac{9g\rho l}{4}$

De (3): $z_B = 3mg \Rightarrow z_B = 3\rho l g$

a) $\omega = \sqrt{\frac{3g}{2l}}$ b) $x_A = x_B = 0$, $y_A = -\frac{9g\rho l}{4}$ e $z_B = 3\rho l g$

4) Vamos rotacionar os eixos x e z em um angulo α de forma que os momentos de inercia possam ser calculados com maior facilidade.



$\vec{I} = \cos\alpha\,\vec{i} + \operatorname{sen}\alpha\,\vec{k}$; $\vec{k} = -\operatorname{sen}\alpha\,\vec{i} + \cos\alpha\,\vec{k}$

$\vec{J} = \vec{j}$

Como O está fixo então $\vec{v_O} = \vec{0}$.

$$\vec{H_O} = m(G-O) \wedge \vec{v_O} + [\vec{I}\ \vec{J}\ \vec{k}] \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ 0 \\ \omega_L \end{bmatrix}$$

$$I_{xx} = \frac{mL^2}{12} + m(r\cos\theta)^2$$

$$I_{yy} = \frac{mL^2}{12}$$

$$I_{zz} = 0 + m(r\cos\theta)^2 =$$

$$= m(r\cos\theta)^2$$

Como o centro de massa se encontra no eixo de simetria de massa na posição $(0, r\cos\theta, 0)$ Então: $I_{xy} = I_{zx} = I_{zy} = 0$.

$$\vec{H_0} = [\vec{I}\ \vec{J}\ \vec{K}] \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} \omega \operatorname{sen}\alpha \\ 0 \\ \omega\cos\alpha \end{bmatrix} =$$

$$= [\vec{I}\ \vec{J}\ \vec{K}] \begin{bmatrix} I_{xx}\,\omega\operatorname{sen}\alpha \\ 0 \\ I_{zz}\,\omega\cos\alpha \end{bmatrix} = I_{xx}\omega\operatorname{sen}\alpha\,\vec{I} +$$

$$+ I_{zz}\omega\cos\alpha\,\vec{K} = \left(\frac{mL^2}{12} + m(r\cos\theta)^2\right)(\omega\operatorname{sen}\alpha)\cdot(\cos\alpha\,\vec{i} +$$

$$\operatorname{sen}\alpha\,\vec{k}) + (mr\cos\theta)^2(\omega\cos\alpha)(-\operatorname{sen}\alpha\,\vec{i} + \cos\alpha\,\vec{k}) =$$

$$= \left[m\omega\left(\frac{L^2}{12} + r^2\cos^2\theta\right)\operatorname{sen}\alpha\cdot\cos\alpha - m\omega(r^2\cos^2\theta)\right.$$

$$\left.\cos\alpha\operatorname{sen}\alpha\right]\vec{i} + \left[m\omega\left(\frac{L^2}{12} + r^2\cos^2\theta\right)\operatorname{sen}^2\alpha + m\omega\right.$$

$$\left.\cdot(r^2\cos^2\theta)\cos^2\alpha\right]\vec{k} = \left[\frac{m\omega L^2}{12}\operatorname{sen}\alpha\cos\alpha\right]\vec{i} +$$

$$+ \left[\frac{m\omega L^2}{12}\operatorname{sen}^2\alpha + m\omega r^2\cos^2\theta\operatorname{sen}^2\alpha + m\omega r^2\cos^2\theta\cos^2\alpha\right]\vec{k}$$

$$= \left[\frac{m\omega L^2}{12}\operatorname{sen}\alpha\cos\alpha\right]\vec{i} + \left[\frac{m\omega L^2}{12}\operatorname{sen}^2\alpha + m\omega r^2\cos^2\theta\cdot(\right.$$

$$\left.\operatorname{sen}^2\alpha + \cos^2)\right]\vec{k} = \left[\frac{m\omega L^2}{12}\operatorname{sen}\alpha\cos\alpha\right]\vec{i} +$$

$$+ \left[\frac{m\omega L^2}{12}\operatorname{sen}^2\alpha + m\omega r^2\cos^2\theta\right]\vec{k}$$

Note que: $\operatorname{sen}\alpha = \dfrac{r\operatorname{sen}\theta}{L/2} = \dfrac{2r\operatorname{sen}\theta}{L}$

$\therefore \cos^2\alpha = 1 - \operatorname{sen}^2\alpha \Rightarrow \cos\alpha = \sqrt{1 - \dfrac{4r^2\operatorname{sen}^2\theta}{L^2}}$

Substituindo:

$$\vec{H_0} = \left[\frac{m\omega L^{\cancel{2}}}{\cancel{12}_{\,6}}\cdot\frac{\cancel{2}r\operatorname{sen}\theta}{\cancel{L}}\sqrt{1 - \frac{4r^2\operatorname{sen}^2\theta}{L^2}}\right]\vec{i} +$$

$$+ \left[\frac{m\omega\cancel{L^2}}{\cancel{12}_{\,3}}\cdot\frac{\cancel{4}r^2\operatorname{sen}^2\theta}{\cancel{L^2}} - m\omega r^2(1 - \operatorname{sen}^2\theta)\right]\vec{k} =$$

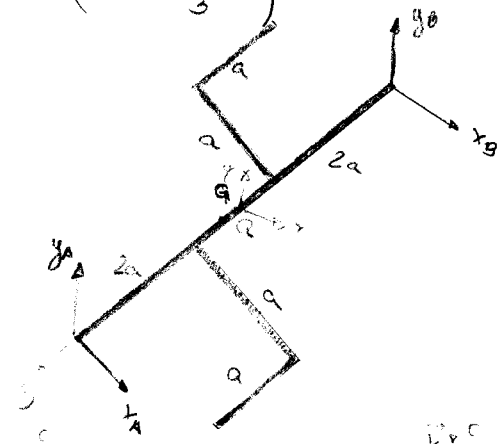
$$= \left[\frac{m\omega L r}{6}\operatorname{sen}\theta\sqrt{1 - \frac{4r^2\operatorname{sen}^2\theta}{L^2}}\right]\vec{i} +$$

$$+ \left[\frac{m\omega r^2}{3}\operatorname{sen}^2\theta + m\omega r^2 - m\omega r^2\operatorname{sen}^2\theta\right]\vec{k} \Rightarrow$$

$$\Rightarrow \vec{H_0} = \frac{m\omega L r}{6}\operatorname{sen}\theta\sqrt{1 - \frac{4r^2\operatorname{sen}^2\theta}{L^2}}\,\vec{i} +$$

$$+ m\omega r^2\left(1 - \frac{2\operatorname{sen}^2\theta}{3}\right)\vec{k}.$$

7)



$$\vec{a_G} = \cancel{\vec{a_A}} + \cancel{\dot{\vec{\omega}}}\wedge(G - A) + \vec{\omega}\wedge[\vec{\omega}\wedge(G - A)] = \vec{0}$$

TMB $a_m\cdot\vec{a_G} = \vec{R}^{ext} \Rightarrow \vec{R}^{ext} = \vec{0}$ $\begin{cases} x_A + x_B = 0 \\ y_A + y_B = 0 \end{cases}$

TMA:

$$\vec{H_G} = [\vec{i}\ \vec{j}\ \vec{k}] \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} =$$

$$= [\vec{i}\ \vec{j}\ \vec{k}] \begin{bmatrix} -I_{xz}\omega \\ -I_{yz}\omega \\ I_{zz}\omega \end{bmatrix} = -I_{xz}\omega\vec{i} - I_{yz}\omega\vec{j} + I_{zz}\omega\vec{k}$$

Vamos considerar o sistema de eixos solidário ao sólido

$$\dot{\vec{H_G}} = (-I_{xz}\dot{\vec{i}} - I_{yz}\dot{\vec{j}} + I_{zz}\dot{\vec{k}})\omega + (-I_{xz}\vec{i} - I_{yz}\vec{j} + I_{zz}\vec{k})\cancel{\dot{\omega}}^{\,0}$$

$$\dot{\vec{i}} = \omega\vec{k}\wedge\vec{i} = \vec{j};\ \dot{\vec{j}} = \omega\vec{k}\wedge\vec{j} = -\vec{i};\ \dot{\vec{k}} = \omega\vec{k}\wedge\vec{k} = \vec{0}$$

$$\dot{\vec{H_G}} = -I_{xz}\omega^2\vec{j} + I_{yz}\omega^2\vec{i}$$

$$\dot{\vec{H_G}} = \vec{M_G} \Rightarrow \vec{H_G} = y_B\frac{5a}{2}\vec{i} + y_A\cdot\frac{5a}{2}\vec{i} + x_A\frac{5a}{2}\vec{j} - x_B\frac{5a}{2}\vec{j}$$

$$\dot{\vec{H_G}} = \frac{5a(y_B - y_A)}{2}\vec{i} + \frac{5a(x_A - x_B)}{2}\vec{j}$$

Note que nenhuma barra tem coordenada de um em y, portanto, $I_{yz} = 0$. Por outro lado:

$$I_{xy} = 5\rho a\cdot 0\cdot 0 + \rho a\,\frac{a}{2}\cdot\frac{a}{2} + \rho a\cdot a\cdot a + \rho a\left(-\frac{a}{2}\right)\left(-\frac{a}{2}\right) +$$

$$+ \rho a(-a)(-a) = \frac{\rho a^3}{4} + \rho a^3 + \frac{\rho a^3}{4} + \rho a^3 =$$

$$= \frac{\rho a^2}{2} + 2\rho a^3 = \frac{5\rho a^2}{2}$$

$$\therefore \frac{5a(y_B-y_A)}{2}\vec{i} + \frac{5a(x_A-x_B)}{2}\vec{j} = -I_{zy}\omega^2\vec{j} + I_{yz}\omega^2\vec{i}$$

$$\begin{cases} y_B - y_A = 0 \Rightarrow y_A = y_B \\ \dfrac{5a(x_A-x_B)}{\cancel{2}} = \dfrac{\cancel{5}\rho a^3}{\cancel{2}}\omega^2 \end{cases}$$

Do TMB: $x_B = -x_A$

$$y_A - y_B = 0 \Rightarrow 2y_A = 0 \Rightarrow y_A = y_B = 0$$

$$2x_A = -\rho a^2\omega^2 \Rightarrow x_A = -\frac{\rho a^2\omega^2}{2} = x_B = \frac{\rho a^2\omega^2}{2}$$

8) a) Considerando D fixo:

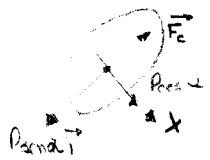
$$\vec{H_D} = [\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}\begin{bmatrix} -\omega\cos\alpha \\ \omega\operatorname{sen}\alpha \\ 0 \end{bmatrix} =$$

$$= [\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix} -I_{xx}\omega\cos\alpha \\ I_{yy}\omega\operatorname{sen}\alpha \\ 0 \end{bmatrix} = -I_{xx}\omega\cos\alpha\,\vec{i} +$$

$$+ I_{yy}\omega\operatorname{sen}\alpha\,\vec{j} = -\left(\frac{ma^2}{2}\right)\omega\cos\alpha\,\vec{i} + \left(\frac{ma^2}{4} + \frac{ma^2}{tg^2\alpha}\right)\omega\operatorname{sen}\alpha\,\vec{j}$$

b) $\vec{D} = P\cos\alpha\,\vec{i} - P\operatorname{sen}\alpha\,\vec{j}$

Disco



TMA:

$$\vec{H_D} = -\left(\frac{ma^2}{2}\right)\omega\cos\alpha\,\vec{i} + \left(\frac{ma^2}{4} + \frac{ma^2}{tg^2\alpha}\right)\omega\operatorname{sen}\alpha\,\vec{j}$$

Considerando que o ω seja constante:

$$\dot{\vec{H_D}} = -\left(\frac{ma^2}{2}\right)\omega\cos\alpha\,\dot{\vec{i}} + \left(\frac{ma^2}{4} + \frac{ma^2}{tg^2\alpha}\right)\omega\operatorname{sen}\alpha\,\dot{\vec{j}}$$

$$\dot{\vec{i}} = (-\omega\cos\alpha\,\vec{i} + \omega\operatorname{sen}\alpha\,\vec{j}) \wedge \vec{i} = -\omega\operatorname{sen}\alpha\,\vec{k}$$

$$\dot{\vec{j}} = (-\omega\cos\alpha\,\vec{i} + \omega\operatorname{sen}\alpha\,\vec{j}) \wedge \vec{j} = -\omega\cos\alpha\,\vec{k}$$

$$\dot{\vec{H_D}} = \vec{M_D^{ext}} \Rightarrow \dot{\vec{H_D}} = -mg\operatorname{sen}\alpha\,\frac{a}{tg\alpha}\,\vec{k} + F_c\,\frac{a}{tg\alpha}\,\vec{k}$$

Para $\vec{F_c} = \vec{0}$, temos que:

$$\left(\frac{ma^2}{2}\right)\omega^2\cos\alpha\operatorname{sen}\alpha - \left(\frac{ma^2}{4} + \frac{ma^2}{tg^2\alpha}\right)\omega^2\operatorname{sen}\alpha\cos\alpha =$$

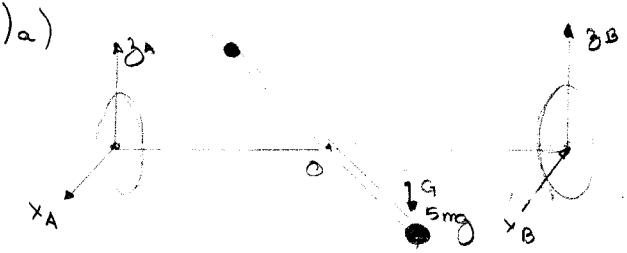
$$= -mg\operatorname{sen}\alpha\,\frac{a}{tg\alpha} \Rightarrow$$

$$\Rightarrow \cancel{m}a^{\cancel{2}}\omega^2\cancel{\cos\alpha}\,\cancel{\operatorname{sen}\alpha}\left(\frac{1}{2} - \frac{1}{4} - \frac{1}{tg^2\alpha}\right) = -\cancel{m}g\cancel{a}\cancel{\operatorname{sen}\alpha}\,\frac{\cancel{\cos\alpha}}{\cancel{\operatorname{sen}\alpha}}$$

$$\Rightarrow a\omega^2\left(\frac{1}{4} - \frac{1}{tg^2\alpha}\right) = -\frac{g}{\operatorname{sen}\alpha} \Rightarrow$$

$$\Rightarrow \omega^2 = \frac{g}{a\operatorname{sen}\alpha\left(\frac{1}{tg^2\alpha} - \frac{1}{4}\right)} \Rightarrow \omega = \sqrt{\frac{g}{a\operatorname{sen}\alpha\left(\frac{1}{tg^2\alpha} - \frac{1}{4}\right)}}$$

9) a)



$$x_G = \frac{m\,0 + m\cdot 0 + m\,0 + 2m\cdot 0}{5m} = 0$$

$$y_G = \frac{m(-2a) + m(-a) - 2ma + m(2a)}{5m} = \frac{-2ma - ma + 2ma + 2ma}{5m}$$

$$= \frac{ma}{5m} = \frac{a}{5}$$

$$z_G = \frac{m\cdot 0 + ma - 2ma + m\cdot 0}{5m} = -\frac{ma}{5m} = -\frac{a}{5}$$

$\therefore G(0, a/5, -a/5)$

$$\vec{a_G} = \overset{0}{\vec{a_O}} + \overset{0}{\dot{\vec{\omega}}} \wedge (G-O) + \vec{\omega} \wedge [\vec{\omega} \wedge (G-O)] \Rightarrow$$

$$\Rightarrow \vec{a_G} = \omega\vec{j} \wedge [\omega\vec{j} \wedge (a/5\,\vec{j} - a/5\,\vec{k})] \Rightarrow$$

$$\Rightarrow \vec{a_G} = \omega\vec{j} \wedge [\omega a/5\,\vec{i}] = -\frac{\omega^2 a}{5}\,\vec{k}$$

TMB: $5m\,\vec{a_G} = \vec{R} \Rightarrow -\frac{\cancel{5}ma\omega^2}{\cancel{5}}\vec{k} = (x_A - x_B)\vec{i} + (z_A + z_B - 5mg)\vec{k}$

$$\begin{cases} x_A - x_B = 0 & (1) \\ -ma\omega^2 = z_A + z_B - 5mg & (2) \end{cases}$$

TMA:

$$\vec{H_O} = m(G-O) \wedge \overset{0}{\vec{V_O}} + [\vec{i}\ \vec{j}\ \vec{k}]\,[I_O]\,[\omega_x\ \omega_y\ \omega_z]^T$$

$$I_{xy} = I_{xz} = 0$$

$$I_{yz} = m\cdot(-2a)\,0 + m(-a)(a) + 2m(a)(-a) + m\,2a\cdot 0 =$$

$$= -ma^2 - 2ma^2 = -3ma^2$$

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}\begin{bmatrix} 0 \\ \omega \\ 0 \end{bmatrix} = \begin{bmatrix} -\overset{0}{\cancel{I_{xy}}}\omega \\ I_{yy}\omega \\ -I_{yz}\omega \end{bmatrix}$$

$I_{yy} = \frac{ma^2}{2} + ma^2 + ma^2 + \frac{ma^2}{2} = 3ma^2$

$\vec{H_O} = I_{yy}\omega\vec{j} - I_{yz}\omega\vec{k}$

$\dot{\vec{H_O}} = (I_{yy}\dot{\vec{j}} - I_{yz}\dot{\vec{k}})\vec{\omega} + (I_{yy}\vec{j} - I_{yz}\vec{k})\cancel{\dot{\vec{\omega}}}^{\,0}$

$\dot{\vec{j}} = \omega\vec{j} \wedge \vec{j} = 0$

$\dot{\vec{k}} = \omega\vec{j} \wedge \vec{k} = -\omega\vec{i}$ $\quad \therefore \dot{\vec{H_O}} = I_{yz}\omega^2\vec{i}$

$\dot{\vec{H_O}} = m\vec{V_a} \wedge \cancel{\vec{V_O}}^{\,0} + \vec{M_O}^{ext} = (-z_A\, 2a + z_B \cdot 2a - \frac{mga}{5})\vec{i} +$

$+ (x_A 2a - x_B \cdot 2a)\vec{k}$

$$\begin{cases} -3ma^2\omega^2 = -z_A 2a + z_B 2a - mg & (3) \\ x_A 2a + x_B \cdot 2a = 0 & (4) \end{cases}$$

De (2) $\quad z_A = -ma\omega^2 - z_B + 5mg \quad (5)$

De (5) em (3):

$-3ma^2\omega^2 = -2a(-ma\omega^2 - z_B + 5mg) + 2az_B - mg \Rightarrow$

$\Rightarrow -3ma^2\omega^2 = 2ma^2\omega^2 + 2az_B - 10mg + 2az_B - mg \Rightarrow$

$\Rightarrow -5ma^2\omega^2 = 4az_B - 11mg \Rightarrow z_B = \frac{11mg - 5ma\omega^2}{4} \quad (6)$

De (6) em (5): $z_B = \frac{-4ma\omega^2 - 11mg + 5ma\omega^2 + 20mg}{4} =$

$= \frac{ma\omega^2 + 9mg}{4}$

De (4): $x_A = -x_B \quad (7)$

De (7) em (1): $2x_A = 0 \Rightarrow x_A = 0 \therefore x_B = 0$

As reações nos mancais são $z_A = \frac{ma\omega^2 + 9mg}{4}$, $z_B = \frac{11mg - 5ma\omega^2}{4}$,

$x_A = x_B = 0$

b) $\overline{I_{yz}} = 0 \Rightarrow I_{yz} + m_A(-2a)(-a) + m_B(2a)\cdot a = 0 \Rightarrow$

$\Rightarrow -3ma^2 + 2m_A a^2 + 2m_B a^2 = 0 \quad (8)$

$\overline{I_{xy}} = 0 \Rightarrow I_{xy} + m_A \cdot 0 \cdot (-2a) + m_B \cdot 0 \cdot 2a = 0$

$\overline{x_G} = 0$

$\overline{y_G} = 5my_G - 2m_A a + 2m_B \cdot a$ não importa! (eixo de rotação)

$\overline{z_G} = 5mz_G - m_A a + m_B a = 0 \Rightarrow a(m_A + m_B) = \frac{\cancel{3}ma}{\cancel{3}}$

$\Rightarrow m_A + m_B = m \Rightarrow m_A = m - m_B \quad (9)$

De (9) em (8):

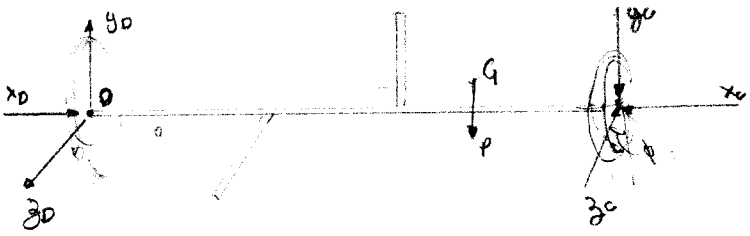
$-3m + 2(-m + m_B) + 2m_B = 0 \Rightarrow 4m_B = 5m \Rightarrow$

$\Rightarrow m_B = \frac{5m}{4} \quad (10)$

De (10) em (9): $m_A = \frac{5m}{4} - m = \frac{m}{4}$

Posicionamos as massas em $(0, -2a, -a)$ e $(0, 2a, a)$ tal que $m_A = \frac{m}{4}$ e $m_B = \frac{5m}{4}$

10) a)



$x_G = \frac{M\cdot 0 + m\cdot 3l + m\cdot 6l + M\cdot 12l}{2(M+m)} = \frac{9ml + 12Ml}{2(M+m)}$

$y_G = \frac{M\cdot 0 + m\cdot 0 + ml + M\cdot 0}{2(M+m)} = \frac{ml}{2(M+m)}$

$z_G = \frac{M\cdot 0 + ml + m\cdot 0 + M\cdot 0}{2(M+m)} = \frac{ml}{2(M+m)}$

$\therefore G = \left(\frac{9ml + 12Ml}{2(M+m)}, \frac{ml}{2(M+m)}, \frac{ml}{2(M+m)}\right)$

Suponhamos dito.

$\vec{a_G} = \vec{a_D} + \dot{\vec{\omega}} \wedge (G - O) + \vec{\omega} \wedge [\vec{\omega} \wedge (G - O)] =$

$= \omega\vec{i} \wedge [\omega\vec{i} \wedge (x_G\vec{i} + y_G\vec{j} + z_G\vec{k})] =$

$= \omega\vec{i} \wedge [\omega y_G\vec{k} - \omega z_G\vec{j}] = -\omega^2 y_G\vec{j} - \omega^2 z_G\vec{k}$

TMB: $2(M+m)\vec{a_G} = \vec{R}$

$$\begin{cases} x_D - x_C = 0 & (1) \\ 2(M+m)(-\omega^2 y_G) = y_D - y_C - mg & (2) \\ 2(M+m)(-\omega^2 z_G) = z_D - z_C & (3) \end{cases}$$

TMA: $\vec{H_O} = [\vec{i}\ \vec{j}\ \vec{k}][I_O][\omega_x\ \omega_y\ \omega_z]^t =$

$= I_{xx}\omega\vec{i} - I_{xy}\omega\vec{j} - I_{xz}\omega\vec{k}$

$I_{xx} = \frac{M(3l)^2}{2} + \frac{M(3l)^2}{2} + \frac{m(2l)^2}{3} + \frac{m(2l)^2}{3} =$

$= 9Ml^2 + \frac{8ml^2}{3}$

$I_{xy} = m \cdot 6l \cdot l = 6ml^2$

$I_{xz} = m \cdot 3l \cdot l = 3ml^2$

$\dot{\vec{i}} = \omega\vec{i} \wedge \vec{i} = 0 \qquad \dot{\vec{k}} = \omega\vec{i} \wedge \vec{k} = -\omega\vec{j}$

$\dot{\vec{j}} = \omega\vec{i} \wedge \vec{j} = \omega\vec{k}$

$\dot{\vec{H}}_O = -I_{xy}\omega^2\vec{k} + I_{xz}\omega^2\vec{j}$

$\dot{\vec{H}}_O = \vec{M}_O^{ext} = mg\, z_G\vec{i} + z_C 9l\vec{j} + (-mg\, x_G - y_C 12l)\vec{k}$

$z_C = \dfrac{I_{xz}\omega^2}{9l}$ (4)

$-y_C 12l - mg x_G = -I_{xy}\omega^2 \Rightarrow y_C 12l = I_{xy}\omega^2 - mg x_G \Rightarrow$

$\Rightarrow y_C = \dfrac{I_{xy}\omega^2 - mg x_G}{12l}$ (5)

De (5) em (2): $y_O = \dfrac{I_{xy}\omega^2 - mg x_G}{12l} + mg - \omega^2 l$

De (4) em (3): $z_O = \dfrac{I_{xz}\omega^2}{9l} - \omega^2 l$

b) Suponha, que a posição angular em relação ao eixo horizontal z seja dada por θ e ϕ respectivamente nos eixos R e C, temos

$\overline{y_G} = 0 \Rightarrow 2(M+m)y_G + m_1 \cdot 3l\,\text{sen}\theta + m_2\, 3l\,\text{sen}\phi = 0$ (6)

$\overline{z_G} = 0 \Rightarrow 2(M+m)z_G + m_1\, 3l\cos\theta + m_2\, 3l\cos\phi = 0$ (7)

$\overline{I_{xy}} = 0 \Rightarrow I_{xy} + m_1 \cdot 0 \cdot 3l\,\text{sen}\theta + m_2 \cdot 12l\, 3l\,\text{sen}\phi = 0$ (8)

$\overline{I_{xz}} = 0 \Rightarrow I_{xz} + m_1 \cdot 0 \cdot 3l\cos\theta + m_2\, 12l\, 3l\cos\phi = 0$ (9)

De (8) e (9): $m_2 \cdot 36l^2 \cdot \text{sen}\phi = -6ml^2$ e

$m_2 \cdot 36l^2 \cdot \cos\phi = -3ml^2$

$\therefore \dfrac{\text{sen}\phi}{\cos\phi} = 2 \Rightarrow \text{tg}\phi = 2 \quad \dfrac{1}{\cos^2\phi} = 1 + 4 \Rightarrow$

$\Rightarrow \cos\phi = \dfrac{\sqrt{5}}{5} \quad \text{sen}\phi = \dfrac{2\sqrt{5}}{5}$

De (9): $\overset{12}{\cancel{36}}\cancel{l^2} m_2 \cdot \dfrac{\sqrt{5}}{5} = -\cancel{3}m\cancel{l^2} \Rightarrow m_2 = -\dfrac{5m}{12\sqrt{5}} \Rightarrow$

$\Rightarrow m_2 = -\dfrac{m\sqrt{5}}{12}$

De (6) e (7): $ml + m_1\, 3l\,\text{sen}\theta - \dfrac{m\cancel{\sqrt{5}}}{\cancel{12}_2}\cancel{3}l\,\dfrac{\cancel{2}\cancel{\sqrt{5}}}{\cancel{5}} = 0 \Rightarrow$

$\Rightarrow ml + m_1\, 3l\,\text{sen}\theta - \dfrac{ml}{2} = 0 \Rightarrow m_1\, \cancel{3l}\,\text{sen}\theta = -\dfrac{m\cancel{l}}{2} \Rightarrow$

$\Rightarrow m_1\,\text{sen}\theta = -\dfrac{m}{6}$ (10)

$ml + m_1\, 3l\cos\theta - \dfrac{m\cancel{\sqrt{5}}}{\cancel{12}_4}\,\dfrac{\cancel{\sqrt{5}}}{\cancel{5}} \cdot \cancel{3}l$

$\Rightarrow ml - \dfrac{ml}{4} + m_1\, 3l\cos\theta = 0 \Rightarrow m_1\, \cancel{3l}\cos\theta = -\dfrac{\cancel{3}m\cancel{l}}{4}$

$\Rightarrow m_1\cos\theta = -\dfrac{m}{4}$ (11)

De (10) e (11):

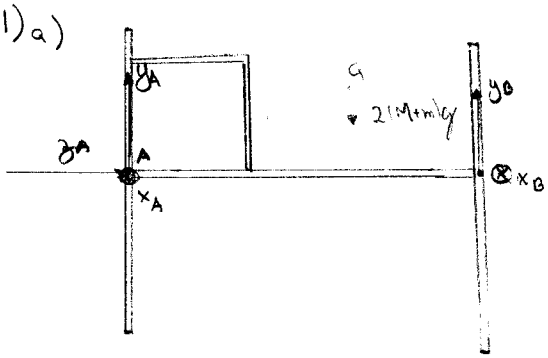
$\dfrac{\text{sen}\theta}{\cos\theta} = \dfrac{\cancel{m}}{6} \cdot \dfrac{4}{\cancel{m}} = \dfrac{2}{3} \Rightarrow \text{tg}\theta = \dfrac{2}{3} \quad \dfrac{1}{\cos^2\theta} = 1 + \dfrac{4}{9} \Rightarrow$

$\Rightarrow \cos^2\theta = \dfrac{9}{13} \Rightarrow \cos\theta = \dfrac{3}{\sqrt{13}} \quad \text{sen}\theta = \dfrac{2}{\sqrt{13}}$

De (10): $m_1 \dfrac{2}{\sqrt{13}} = -\dfrac{m}{6} \Rightarrow m_1 = -\dfrac{m\sqrt{13}}{12}$

Devemos colocar massas em $\left(0, -\dfrac{6l}{\sqrt{13}}, \dfrac{9l}{\sqrt{13}}\right)$ e em $\left(12l, -\dfrac{6l\sqrt{5}}{5}, -\dfrac{3l\sqrt{5}}{5}\right)$ nos valores de $m_1 = \dfrac{m\sqrt{13}}{12}$ e $m_2 = \dfrac{m\sqrt{5}}{12}$

11) a)



$x_G = 0$

$y_G = \dfrac{mL + mL/2}{2(m+M)} = \dfrac{3}{4}\,\dfrac{mL}{m+M}$

$z_G = \dfrac{mL/2 + mL + M \cdot 3L}{2(m+M)} = \dfrac{(6M+3m)L}{4(M+m)}$

$\therefore G\left(0, \dfrac{3}{4}\,\dfrac{mL}{m+M}, \dfrac{(6M+3m)L}{4(M+m)}\right)$ zerar

$\vec{a}_G = \overset{=0}{\cancel{\vec{a}_A}} + \overset{=0}{\cancel{\dot{\vec{\omega}}}} \wedge (G-A) + \omega\vec{k} \wedge [\omega\vec{k} \wedge (x_G\vec{i} + y_G\vec{j} + z_G\vec{k})]$

$\vec{a}_G = \omega\vec{k} \wedge (\overset{=0}{\cancel{\omega x_G}}\vec{j} - \omega y_G\vec{i}) = -\omega^2 y_G\vec{i}$

TMB:

$$\begin{cases} x_A = -x_B & (1) \\ y_A + y_B - 2(M+m)g = -\omega^2\,\frac{3}{2}mL & (2) \\ z_A = 0 & (3) \end{cases}$$

TMA:

$$\vec{H_A} = [\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix} -I_{xz}\omega \\ -I_{yz}\omega \\ I_{zz}\omega \end{bmatrix} = -I_{xz}\omega\vec{i} - I_{yz}\omega\vec{j} + I_{zz}\omega\vec{k}$$

$$\dot{\vec{i}} = \omega\vec{k}\wedge\vec{i} = \omega\vec{j},\ \dot{\vec{j}} = \omega\vec{k}\wedge\vec{j} = -\omega\vec{i},\ \dot{\vec{k}} = \omega\vec{k}\wedge\vec{k} = \vec{0}$$

$$\dot{\vec{H_A}} = -I_{xz}\omega^2\vec{j} + I_{yz}\omega^2\vec{i}$$

$$\dot{\vec{H_A}} = \vec{M_A}^{ext} = [2(M+m)g\cdot\frac{3L}{2} - y_B\cdot 2L]\vec{i} - x_B\cdot 3L\,\vec{j}$$

$I_{xz} = 0$

$I_{yz} = mL\cdot\frac{L}{2} + m\frac{L}{2}\cdot L = mL^2$

$$\begin{cases} mL^2\omega^2 = \frac{3(2M+m)L}{2}g - y_B\cdot 3L & (4) \\ x_B = 0 & (5) \end{cases}$$

De (4): $y_B = \frac{1}{3\cancel{L}}\left(\frac{3(2M+m)\cancel{L}}{2}g - mL^{\cancel{2}}\omega^2\right) \Rightarrow$

$\Rightarrow y_B = \left(M + \frac{m}{2}\right)g - \frac{mL\omega^2}{3}$ (6)

De (6) em (2): $y_A = \left(M + \frac{3m}{2}\right)g - \frac{7mL}{6}\omega^2$.

De (1) e (5): $x_A = x_B = 0$

b) $\overline{I_{yz}} = 0 \Rightarrow mL^2 + m_1(-R)\cdot 0 + m_2(-R)(3L) = 0$ (7)

$\overline{y_G} = 0 \Rightarrow \frac{3mL}{2} - m_1 R - m_2 R = 0$ (8)

De (7): $mL^2 - m_2 R\cdot 3L = 0 \Rightarrow m_2 = \frac{mL}{3R}$ (9)

De (9) em (8): $\frac{3mL}{2} - m_1 R - \frac{mL}{3} = 0 \Rightarrow$

$\Rightarrow m_1 R = \frac{9mL}{6} - \frac{2mL}{6} \Rightarrow m_1 R = \frac{7mL}{6} \Rightarrow m_1 = \frac{7mL}{6R}$

Note que como $x_G = 0$ deviamos adicionar (ou retirar) massa de modo que o cm se deslocasse na direção do eixo y ($y_G \neq 0$) e também porque o deslocamento no eixo z não é relevante (eixo de rotação). Desse modo qualquer massa adicionada (ou retirada) nesse balanceamento não alteraria o produto de inércia I_{xz} e nem o produto I_{xy}

14) a) $\vec{\omega_{abs}} = \Omega\vec{j} + \omega\vec{i}$

$\vec{a_{a,rel}} = \vec{0} \Rightarrow \vec{V_{a,rel}} = 0$

$\vec{a_{a,arr}} = \vec{a_0} + \vec{\Omega}\wedge\vec{\Omega}\wedge(G-O) = a\vec{j} + \Omega\vec{k}\wedge\Omega\vec{k}\wedge l\vec{i} \Rightarrow$

$\Rightarrow \vec{a_{a,arr}} = a\vec{j} - \Omega\vec{k}\wedge\Omega l\vec{j} \Rightarrow \vec{a_{a,arr}} = a\vec{j} - \Omega^2 l\vec{i}$

$\vec{a_{a,cor.}} = 2\Omega\vec{k}\wedge\vec{V_{a,rel}} = \vec{0}$

$\therefore \vec{a_a} = \vec{a_{a,arr}} + \vec{a_{a,rel}} + \vec{a_{a,cor}} \Rightarrow \vec{a_a} = a\vec{j} - \Omega^2 l\vec{i}$

Obs: G é um ponto da barra, então poderíamos ter aplicado Poisson direto na barra.

b) Note que os produtos de inércia são todos nulos.

TMA:

$$\vec{H_O} = m(G-O)\wedge\vec{V_O} + [\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}\begin{bmatrix} \omega \\ \Omega \\ 0 \end{bmatrix} =$$

$= m(G-O)\wedge\vec{V_O} + I_{xx}\omega\vec{i} + I_{yy}\Omega\vec{j}$

$\dot{\vec{H_O}} = m(V_G - V_O)\wedge\cancel{\vec{V_O}}^{0} + m(G-O)\wedge\vec{a_O} + I_{xx}\cancel{\dot{\omega}}^{0}\vec{i} + I_{xx}\omega\dot{\vec{i}} +$

$+ I_{yy}\cancel{\dot{\Omega}}^{0}\vec{j} + I_{yy}\Omega\dot{\vec{j}} \Rightarrow$

$\Rightarrow \dot{\vec{H_O}} = m l\vec{i}\wedge a\vec{j} + I_{xx}\omega\dot{\vec{i}} + I_{yy}\Omega\dot{\vec{j}}$

$\dot{\vec{i}} = \Omega\vec{j}\wedge\vec{i} = -\Omega\vec{k}$, $\dot{\vec{j}} = \Omega\vec{j}\wedge\vec{j} = \vec{0}$

$\dot{\vec{H_O}} = ma l\vec{k} - I_{xx}\omega\Omega l\vec{k}$

$\dot{\vec{H_O}} = m\vec{V_G}\wedge\cancel{\vec{V_O}}^{0} + \vec{M_O}^{ext} = -mg\,l$

$\therefore -mg\,l\vec{k} = (mal - I_{xx}\omega\Omega)\vec{k} \Rightarrow -mgl = mal - I_{xx}\omega\Omega \Rightarrow$

$\Rightarrow I_{xx}\omega\Omega = ml(a+g) \Rightarrow \Omega = \frac{ml(a+g)}{I_{xx}\omega}$

$I_{xx} = \frac{mR^2}{2}$; $\Omega = \frac{2ml(a+g)}{m\omega R^2}$

c) TMB: $m\vec{a_G} = \vec{R}^{ext} \Rightarrow m(a\vec{j} - \Omega^2 l\vec{i}) = x_0\vec{i} + y_0\vec{j} + z_0\vec{k} - mg$

$$\begin{cases} -m\Omega^2 l = x_0 \\ ma = y_0 - mg \\ 0 = z_0 \end{cases}$$

$x_0 = -m\Omega^2 l \therefore \vec{x_0} = -m\Omega^2 l\vec{i}$

$y_0 = m(a+g) \therefore \vec{y_0} = m(a+g)\vec{j}$; $\vec{z_0} = \vec{0}$

d) Se o elevador estivesse em queda livre, então $a = -g$. Assim o movimento de precessão seria zero e o movimento não seria possível

13) a) $\vec{V_G} = \vec{V_O} + \Omega\vec{k}\wedge(G-O) \Rightarrow \vec{V_G} = \Omega\vec{k}\wedge R\vec{i} \Rightarrow$

$\Rightarrow \vec{V_G} = \Omega R\vec{j} \therefore \Omega = \frac{V}{R}$

$\vec{\omega}_{abs} = \vec{\Omega} + \omega\vec{k} \Rightarrow \vec{\omega}_{abs} = \frac{V}{R}\vec{k} - \omega\vec{j}$

$$\vec{H}_G = [\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix} J_{xG} & 0 & 0 \\ 0 & J_{yG} & 0 \\ 0 & 0 & J_{zG} \end{bmatrix}\begin{bmatrix} 0 \\ -\omega \\ V/R \end{bmatrix} =$$

$$= -J_{yG}\,\omega\vec{j} + J_{zG}\frac{V}{R}\vec{k} = -\frac{Mr^2}{2}\omega\vec{j} + \frac{V}{R}\,\frac{19Mr^2}{12}\,\vec{k}$$

b) TMA:

$\dot{\vec{j}} = \frac{V}{R}\vec{k}\wedge\vec{j} = -\frac{V}{R}\vec{i}$; $\dot{\vec{k}} = \frac{V}{R}\vec{k}\wedge\vec{k} = \vec{0}$

$$\frac{VMr^2}{2R}\vec{i} = m\cancel{\vec{V}_G\wedge\vec{V}_G}^{\,0} + \vec{M}_G \therefore \vec{M}_G^{\text{rotor lvogão}} = -\frac{VMr^2}{2R}\vec{i}$$

Binário Giroscópico

c) Como V é constante então $\dot{\vec{\Omega}} = \vec{0}$.

$$\vec{a}_{G,arr} = \cancel{\vec{a}_O}^{\,0} + \cancel{\dot{\vec{\Omega}}}^{\,0}\wedge(G-O) + \vec{\Omega}\wedge\vec{\Omega}\wedge(G-O) =$$

$$= \Omega\vec{k}\wedge\Omega\vec{k}\wedge R\vec{i} = \Omega\vec{k}\wedge\Omega R\vec{j} = -\Omega^2 R\vec{i} = -\frac{V^2}{R}\vec{i}$$

$\vec{a}_{G,rel} = \vec{0}$

$\vec{a}_{G,cor} = 2\vec{\Omega}\wedge\vec{V}_{G,rel} = \vec{0}$ $\quad\therefore \vec{a}_G = -\frac{V^2}{R}\vec{i}$

d) Surgirão forças na direção de $\vec{k}$, pois, o momento é na direção $\vec{i}$

15) Seja O o centro da curva feita pelo helicóptero

$\vec{V}_G = V\vec{k} \therefore \vec{V}_G = \Omega\vec{i}\wedge R\vec{j} \Rightarrow V\vec{k} = \Omega R\vec{k} \Rightarrow \Omega = \frac{V}{R} \therefore \vec{\Omega} = \frac{V}{R}\vec{i}$

$$\vec{H}_G = [\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix} J_{xG} & 0 & 0 \\ 0 & J_{yG} & 0 \\ 0 & 0 & J_{zG} \end{bmatrix}\begin{bmatrix} V/R \\ 0 \\ \omega \end{bmatrix} = \frac{V}{R}J_{xG}\vec{i} + \omega J_{zG}\vec{k}$$

$\dot{\vec{i}} = \frac{V}{R}\vec{i}\wedge\vec{i} = \vec{0}$, $\dot{\vec{j}} = \frac{V}{R}\vec{i}\wedge\vec{k} = -\frac{V}{R}\vec{j}$

$\therefore \vec{M}_{G/R} = \vec{\Omega}\wedge\vec{H}_G = -J_{zG}\cdot\frac{\omega V}{R}\vec{j}$

16) a) Seja O o centro da curva.

$\vec{V}_G = \vec{V}_O + \vec{\Omega}\wedge(G-O) \Rightarrow \vec{V}_G = \Omega\vec{k}\wedge(-R\vec{i}) \Rightarrow$

$\Rightarrow V\vec{j} = -\Omega R\vec{j} \therefore \Omega = -\frac{V}{R} \Rightarrow \vec{\Omega} = -\frac{V}{R}\vec{k}$

$\vec{\omega}_{abs} = \omega_d\vec{i} - \frac{V}{R}\vec{k}$

$$\vec{H}_G = [\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix} \overset{J}{I_{xx}} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & \underset{J}{I_{zz}} \end{bmatrix}\begin{bmatrix} \omega_d \\ 0 \\ -V/R \end{bmatrix} = J\omega_d\vec{i} - \frac{JV}{R}\vec{k}$$

b) $\dot{\vec{H}}_G = J\omega_d\left(-\frac{V}{R}\right)(\vec{k}\wedge\vec{i}) - \frac{JV}{R}\cdot\frac{V}{R}\cancel{(\vec{k}\wedge\vec{k})}^{\,0} \Rightarrow$

$\Rightarrow \dot{\vec{H}}_G = -J\frac{\omega_d\cdot V}{R}\vec{j}$

$\vec{M}_G^{ext} = \frac{F_A\cdot l}{2}\vec{j} - \frac{F_B l}{2}\vec{j}$

c) Pelo TMA: $-J\frac{\omega_d\cdot V}{R} = \frac{(F_A-F_B)\,l}{2} \Rightarrow$

$\Rightarrow R = \frac{2J\omega_d\cdot V}{(F_B-F_A)\,l}$. Como $F_B > F_A$ então a curva é para a direita

20) a) Valor de rotação absoluta: ψ

$\vec{\psi} = \Omega\vec{j} + \omega\vec{i}$

b) $\vec{H}_O = m(G-O)\wedge\cancel{\vec{V}_O}^{\,0} + [\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}\begin{bmatrix} \omega \\ \Omega \\ 0 \end{bmatrix} =$

$= I_x\omega\vec{i} + I_y\Omega\vec{j} = \frac{mR^2}{2}\omega\vec{i} + \left(\frac{mR^2}{4} + ml^2\right)\Omega\vec{j}$

c) Momento no disco em rotação ao polo O:

$\vec{M}_O^{ext} = mgl$

$\dot{\vec{H}}_O = \frac{mR^2}{2}\omega\Omega(\vec{j}\wedge\vec{i}) + \left(\frac{mR^2}{4} + ml^2\right)\Omega^2(\vec{j}\wedge\vec{j}) \Rightarrow$

$\Rightarrow \dot{\vec{H}}_O = -\frac{mR^2}{2}\omega\Omega\vec{k}$

TMA: $-\frac{\cancel{m}R^2}{2}\omega\Omega = \cancel{m}gl \Rightarrow \Omega = \frac{2l}{\omega R^2}$

d) $x_G = \frac{2ml/2 + ml}{3m} = \frac{2l}{3}$, $y_G = z_G = 0$

$\vec{a}_G = \cancel{\vec{a}_O}^{\,0} + \cancel{\dot{\vec{\Omega}}}^{\,0}\wedge(G-O) + \vec{\Omega}\wedge[\vec{\Omega}\wedge(G-O)] \Rightarrow$

$\Rightarrow \vec{a}_G = \Omega\vec{j}\wedge\left[\Omega\vec{j}\wedge\frac{2l}{3}\vec{i}\right] = \Omega\vec{j}\wedge\left(-\frac{2\Omega l}{3}\vec{k}\right) =$

$= -\frac{2\Omega^2 l}{3}\vec{i}$

$\cancel{3}m\left(-\frac{2\Omega^2 l}{\cancel{3}}\vec{i}\right) = x_O\vec{i} + y_O\vec{j} + z_O\vec{k} - 3mg\vec{j}$

$x_O = -2\Omega^2 lm$, $y_O = 3mg$, $z_O = 0$.

21) a) $\vec{\Omega} = \omega_1\vec{i} + \omega_2\vec{j}$.

b) $\vec{H}_G = [\vec{i}\ \vec{j}\ \vec{k}]\begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}\begin{bmatrix} \omega_1 \\ \omega_2 \\ 0 \end{bmatrix} =$

$= I_x\omega_1\vec{i} + I_y\omega_2\vec{j}$

$\dot{\vec{H}}_G = I_y\omega_2\omega_1(\vec{i}\wedge\vec{j}) = I_y\omega_1\omega_2\vec{k}$

$\therefore \vec{M}_{G/R} = -I_y\omega_1\omega_2\vec{k}$

Se $I_x = J$ então $I_y = 2J$ $\therefore \vec{M}_{G/R} = -2J\omega_1\omega_2\vec{k}$.

19) a) $\vec{V_G} = \vec{V_O} - \Omega\vec{k} \wedge (G-O) \Rightarrow V\vec{z} = \Omega\vec{k} \wedge (-R\vec{u}) \Rightarrow$

$\Rightarrow V\vec{z} = -\Omega R\vec{z} \therefore \Omega = \frac{V}{R} \therefore \vec{\Omega} = -\frac{V}{R}\vec{k}$

$\vec{\Psi} = \omega\vec{u} - \frac{V}{R}\vec{k}$, $\vec{\Psi} \equiv$ velocidade de rotação absoluta.

$$\vec{K_G} = [\vec{u}\ \vec{z}\ \vec{k}] \begin{bmatrix} J_u & 0 & 0 \\ 0 & J_z & 0 \\ 0 & 0 & J_k \end{bmatrix} \begin{bmatrix} \omega \\ 0 \\ -V/R \end{bmatrix} =$$

$= J_u \cdot \omega\vec{u} - \frac{V}{R} J_k \cdot \vec{k}$

b) $\overset{\circ}{\vec{K_G}} = J_u \cdot \omega \cdot \Omega(\vec{k} \wedge \vec{u}) - \frac{V}{R} J_k \cdot \Omega(\vec{k} \wedge \vec{k}) \Rightarrow$

$\Rightarrow \dot{\vec{K_G}} = -\frac{J_u \cdot \omega \cdot V}{R}\vec{z}$

<u>TMA</u>: $\vec{M_G^{ext}} = -\frac{J_u \cdot \omega \cdot V}{R} \cdot \vec{z}$

c) Para a esquerda, devido ao sentido do momento que os mancais realizam no carro.