S T Q Q S S D

P2 - 2010

Q2 a) TMI $\sum \vec{M}_G^{ext} = \Delta K_G$;

$(O-G) \wedge I\vec{j} = \{\vec{i}\,\vec{j}\,\vec{k}\} [J_G] \begin{bmatrix} 0 \\ 0 \\ -\omega' \end{bmatrix} \Rightarrow (-\frac{L}{2}\vec{i}) \wedge I\vec{j} = \{\vec{i}\,\vec{j}\,\vec{k}\} \begin{bmatrix} 0 & & \\ & \frac{mL^2}{12} & \\ & & \frac{mL^2}{12} \end{bmatrix} [\,-\omega'\,]$

$\Rightarrow +\frac{L}{2} I \vec{k} = \frac{mL^2}{12}\omega'\vec{k} \Rightarrow \boxed{+I = \frac{mL}{6}\omega'}$ (I)

TRI $\sum \vec{I}^{ext} = m\Delta V_G \Rightarrow I\vec{j} = m(\vec{V}_G' - \vec{V}_G) \Rightarrow$

$I = m(V_G' - (-U)) \Rightarrow I = m(V_G' + U)$;

$\vec{V}_G' = -\frac{L}{2}\omega'\vec{j} \Rightarrow \boxed{I = m\left(-\frac{L}{2}\omega' + U\right)}$ (II)

$\therefore$ (I) e (II) $\Rightarrow \cancel{m}(V_G' + U) = +\frac{\cancel{m}L}{6}\omega' \Rightarrow$

$V_G' + U = +\frac{L}{6}\omega' \Rightarrow -\frac{L}{2}\omega' + U = +\frac{L}{6}\omega' \Rightarrow$

$U = \omega' L\left(\frac{1}{2} + \frac{1}{6}\right) \Rightarrow U = \omega' L\left(\frac{3+1}{6}\right) \Rightarrow U = \frac{\omega' L}{3} \cdot 2 \Rightarrow$

$\boxed{\omega' = \frac{3}{2}\frac{U}{L}} \Rightarrow \vec{\omega}' = -\frac{3}{2}\frac{U}{L}\vec{k}$

b) $I = \frac{mL}{6}\omega' \Rightarrow I = \frac{mL}{\cancel{6}_2} \cdot \frac{\cancel{3}}{2} \cdot \frac{U}{L} \Rightarrow \boxed{I = \frac{mU}{4}}$

$\Rightarrow \boxed{\vec{I} = \frac{mU}{4}\vec{j}}$

c) $T = \frac{1}{2} m V_G^2 = \frac{1}{2} m U^2$; $T' = +\frac{1}{2} J_{zzO}\, \omega'^2$

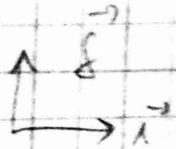
$\Rightarrow T' = \frac{1}{2}\left(\frac{mL^2}{12} + \frac{L^2}{4}\right)\omega'^2 = \frac{1}{2} mL^2 \left(\frac{1+3}{12}\right)\omega'^2 = \frac{1}{6} mL^2 \omega'^2$

$\Rightarrow \Delta T = T' - T = \frac{1}{2}\left(\frac{1}{3} mL^2\omega'^2 - mU^2\right)$

$$= \frac{1}{2} m \left(\frac{1}{3} (\omega' L)^2 - v^2 \right) = \frac{1}{2} m \left(\frac{1}{3} \cdot \frac{9}{4} v^2 - v^2 \right)$$

$$= \frac{1}{2} m v^2 \left(\frac{3}{4} - 1 \right) = \frac{1}{2} m v^2 \cdot \left(-\frac{1}{4} \right) = -\frac{1}{8} m v^2$$

$\boxed{\text{P2 - 2009}}$



Q3 a) conservação de energia:

$$T' - T = W_g \Rightarrow T' = W_g \Rightarrow \frac{1}{2} J_{zz_O} \Omega^2 = M g \frac{L}{2} \Rightarrow$$

$$\frac{1}{2} \left(\frac{M L^2}{12} + M \frac{L^2}{4} \right) \Omega^2 = M g \frac{L}{2} \Leftrightarrow \left(\frac{L}{12} + \frac{L}{4} \right) \Omega^2 = g \Leftrightarrow$$

$$\frac{1+3}{12} L \Omega^2 = g \Leftrightarrow \frac{1}{3} L \Omega^2 = g \Leftrightarrow \Omega = \sqrt{\frac{3g}{L}}$$

b) TMI no polo O (fixo)

$$\sum \vec{M}_O^{ext} = \Delta K_O \Leftrightarrow (A - O) \wedge (-I \vec{i}) = \{\vec{i}\ \vec{j}\ \vec{k}\} [J_O] \Delta \omega \Rightarrow$$

$$(-a \vec{j}) \wedge (-I \vec{i}) = \{\vec{i}\ \vec{j}\ \vec{k}\} \begin{bmatrix} J_{xx} & & \\ & J_{yy} & \\ & & J_{zz} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \Omega' - \Omega \end{bmatrix}$$

$$\Rightarrow -aI \vec{k} = (\Omega' - \Omega) J_{zz} \vec{k} \Rightarrow$$

$$-aI = (\Omega' - \Omega) \frac{mL^2}{3} \quad (I)$$

TRI no corpo A

$$\sum \vec{I}_A^{ext} = m \Delta \vec{V}_A = m V_A' \vec{i} \Rightarrow I \vec{i} = m V_A' \Rightarrow m V_A' = I$$

(II) em (I): $-a(m V_A') = (\Omega' - \Omega) \frac{mL^2}{3}$

Hipótese de Newton

$$u' = -e_1 u \; ; \quad u' = (\vec{V_P}' - \vec{V_A}') \cdot \vec{n} \Leftrightarrow$$

$$u' = (\vec{V_P}' - \vec{V_A}') \cdot \vec{i} \Leftrightarrow u' = V_P' - V_A'$$

$$u = (\vec{V_P} - \vec{V_A}) \cdot \vec{n} \Leftrightarrow u = (\vec{V_P}) \cdot \vec{i} \Leftrightarrow u = V_P \; ;$$

$$\vec{V_P} = \vec{V_O} + \Omega \vec{k} \wedge (P - O) \Rightarrow \vec{V_P} = \Omega \vec{k} \wedge (-a \vec{j}) \Rightarrow$$

$$\vec{V_P} = \Omega a \vec{i} \; ; \quad \vec{V_P}' = \Omega' a \vec{i} \; ;$$

$$\therefore \quad u' = \Omega' a - V_A' \; ; \quad u = \Omega a \; ;$$

$$\therefore \quad \Omega' a - V_A' = -e_1 \Omega a$$

$$-a (m V_A') = (\Omega' - \Omega) \frac{ML^2}{3} \Rightarrow$$

$$-V_A' = (\Omega' - \Omega) \frac{ML^2}{3ma} \Rightarrow V_A' = (\Omega - \Omega') \frac{ML^2}{3ma}$$

$$\Omega' + (\Omega' - \Omega) \frac{ML^2}{3ma^2} = -e_1 \Omega \Rightarrow$$

$$\Rightarrow \Omega' + \frac{\Omega' ML^2}{3ma^2} = -e_1 \Omega + \frac{\Omega ML^2}{3ma^2} \Rightarrow$$

$$\Omega' \left(\frac{3ma^2 + ML^2}{3ma^2} \right) = \Omega \left(-e_1 + \frac{ML^2}{3ma^2} \right)$$

$$\Rightarrow \Omega' = \frac{\Omega \cdot 3ma^2}{3ma^2 + ML^2} \left(-e_1 + \frac{ML^2}{3ma^2} \right) \Rightarrow$$

$$\boxed{\Omega' = -\frac{\Omega}{ma^2 + \frac{ML^2}{3}} \left(e_1 \cdot ma^2 - \frac{ML^2}{3} \right)} \checkmark$$

$$V_A' = \Omega \left[\frac{e_1 ma^2 - \frac{ML^2}{3} + ma^2 + \frac{ML^2}{3}}{ma^2 + \frac{ML^2}{3}} \right] = \frac{\Omega ma^2 (e_1 + 1)}{ma^2 + \frac{ML^2}{3}}$$

P2 - 2008 / Q1

Conservação de momento:

$$m v = (m+M) v' \Rightarrow v' = \frac{m v}{m+M}$$

Conservação de energia:

$$\frac{1}{2} \cancel{(m+M)} v'^2 = \cancel{(m+M)} g h \Rightarrow$$

$$\frac{1}{2} \cdot \frac{m^2 v^2}{(m+M)^2} = g h \Rightarrow v = \frac{m+M}{m} \sqrt{2 g h}$$

Q2 a) * TMI no polo G

$$\sum \vec{M}^{ext}_G = \Delta \vec{K}_G = \{\vec{i}\ \vec{j}\ \vec{k}\} [J_G] \Delta \underset{\sim}{\omega}$$

$$\Rightarrow (A-G) \wedge I_A \vec{j} + (B-G) \wedge I_B \vec{j} = \{\vec{i}\ \vec{j}\ \vec{k}\} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{mL^2}{12} & 0 \\ 0 & 0 & \frac{mL^2}{12} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \omega' \end{bmatrix}$$

$$\Rightarrow (-L\vec{i}) \wedge I_A \vec{j} + (L\vec{i}) \wedge I_B \vec{j} = \frac{mL^2}{12} \omega' \vec{k} \Rightarrow$$

$$-L I_A \vec{k} + L I_B \vec{k} = \frac{mL^2}{12} \omega' \vec{k} \Rightarrow$$

$$-I_A + I_B = \frac{mL}{12} \omega' \quad (I)$$

$(\vec{V}_G' = V_G \vec{j})$

* TRI na barra

$$\sum \vec{I}^{ext} = m \Delta \vec{V}_G \Rightarrow I_A \vec{j} + I_B \vec{j} = m(-v\vec{j} - V_G' \vec{j}) \Rightarrow$$

$$I_A + I_B = -m(v + V_G') \quad (II)$$

* Hipótese de Newton

$$\vec{V}_A = \vec{V}_B = \vec{v} = -v\vec{j} \ ; \quad \vec{V}_A' = \vec{V}_G' + \omega' \vec{k} \wedge \underbrace{(A-G)}_{-L\vec{i}} \Rightarrow$$

$$\vec{V}_A' = \vec{V}_G' - \omega' L \vec{j} \ ;$$

$$\vec{V}_B' = \vec{V}_G' + \omega' \vec{k} \wedge (B-G)$$

$$\vec{V}_B' = \vec{V}_G' + \omega' L \vec{j}$$

Hip Newton p/o A : $u'_A = -e_A u_A$;

$u'_A = V'_A$; $u_A = V_A$; $\Rightarrow$ $V'_A = -e_A V_A \Rightarrow$

$V'_A = -e_A(-v) \Rightarrow V'_A = e_A V$;

Hip. Newton pto B : $V'_B = +e_B V$;

$e_B V \vec{j} = \vec{V}'_G + \omega' L \vec{j} \Rightarrow V'_G = -\omega' L + e_B V$

$e_A V \vec{j} = \vec{V}'_G - \omega' L \vec{j} \Rightarrow V'_G = \omega' L + e_A V$

$\therefore \omega' L + e_A V = -\omega' L + e_B V \Rightarrow$

$2\omega' L = V(e_B - e_A) \Rightarrow \vec{\omega}' = \frac{V}{2L}(e_B - e_A)\vec{k}$

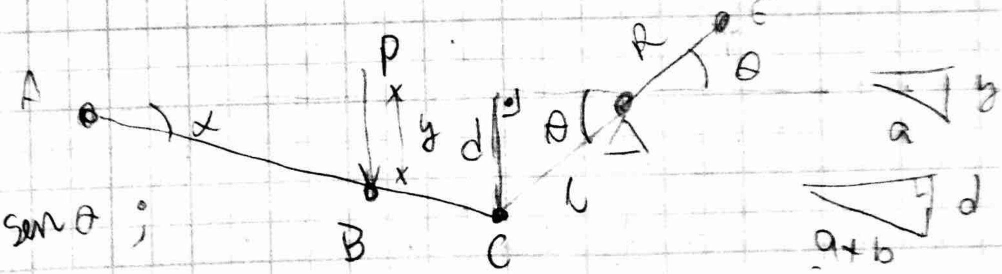
$\therefore V'_G = \left[\frac{V}{2}(e_B - e_A) + e_A V\right]\vec{j}$

$x = R\theta$

δx

b) Se $e_B = e_A = e \Rightarrow \begin{cases} \omega' = 0 \\ V'_G = eV \end{cases}$

P2-2007



$d = c \operatorname{sen}\theta$;

$\frac{a+b}{a} = \frac{d}{y} \Rightarrow y = \frac{a d}{a+b} \Rightarrow y = \frac{a}{a+b} \cdot c \operatorname{sen}\theta$

$\Rightarrow \delta y = \frac{a}{a+b} \cdot c \cos\theta \, \delta\theta$;

$\delta W_P = P \delta y \Rightarrow \delta W_P = P \cdot \frac{a}{a+b} c \cos\theta \, \delta\theta$

$\delta W_E = F_E \cdot \delta x = -KR\theta R \delta\theta = -KR^2\theta \, \delta\theta$

$\cancel{\delta W = \delta W_E + \delta W_P = 0 \Rightarrow P \frac{ac}{a+b} \operatorname{sen}\theta - KR \, \delta\theta}$

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~~$\operatorname{sen}\theta = \frac{kR}{P}\,(a+b)\, ac$~~

$\delta W = \delta W_G + \delta W_P = 0$

$\Rightarrow -kR^2\theta + P\frac{ac}{a+b}\cos\theta\,\delta\theta = 0$

P2 - 2007

TMI no polo O

$\sum \vec{M}_O^{ext} = \Delta K_O \Rightarrow (A-O)\wedge I\vec{i} = \{\vec{i}, \vec{j}, \vec{k}\}\begin{bmatrix} J_{xx} & & \\ & J_{yy} & \\ & & J_{zz} \end{bmatrix}\,\omega'$

$h\vec{j}\wedge I\vec{i} = J_{zz}(\omega'-\omega)\vec{k} \Rightarrow$

$-hI = J_{zz}(\omega'-\omega) \Rightarrow -hI = \left[\frac{ML^2}{12} + \frac{ML^2}{4}\right](\omega'-\omega)$

$\rightarrow I = -\frac{Ma^2}{h}(\omega'-\omega)$ (I)

TRI na partícula

$\sum \vec{I}^{ext} = m\Delta\vec{V} = m(V'-V) = -I \rightarrow$

$\Rightarrow \frac{Ma^2}{h}(\omega'-\omega) = m(V'-V)$

Hipótese de Newton: $e=0$

$\Rightarrow \mu'=0 \Rightarrow V_A'-V'=0 \Rightarrow V_A'=V'$,

$\vec{V}_A' = \cancel{\vec{V}_O}^{\,0} + \omega'\vec{k}\wedge\underbrace{(A-O)}_{h\vec{j}} \Rightarrow \vec{V}_A = -\omega' h\vec{i} \Rightarrow$

$\frac{Ma^2}{h}(\omega'-\omega) = m(\omega' h - V) \Rightarrow \omega'=0 \Rightarrow$

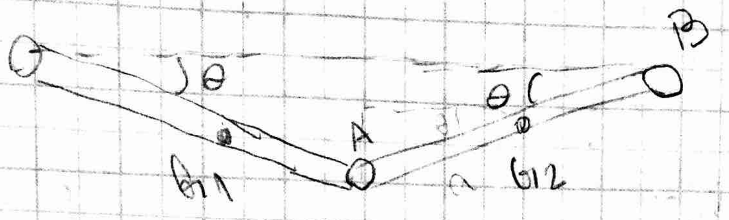
$\frac{Ma^2}{h}\omega = mV \Rightarrow h = \frac{Ma^2\omega}{mV} < 2a$ ✓

S T Q Q S S D

P2-2012

Q1

a)



$(G_1-O) = a\cos\theta\,\vec{i} - a\,\text{sen}\,\theta\,\vec{j}$

$(A-O) = 2a\cos\theta\,\vec{i} - 2a\,\text{sen}\,\theta\,\vec{j}$

$(G_2-O) = (G_2-A)+(A-O) = a\cos\theta\,\vec{i} + a\,\text{sen}\,\theta\,\vec{j} + 2a\cos\theta\,\vec{i} - 2a\,\text{sen}\,\theta\,\vec{j}$

$(G_2-O) = 3a\cos\theta\,\vec{i} - a\,\text{sen}\,\theta\,\vec{j}$

$(B-O) = 2\,[2a\cos\theta]\,\vec{i} = 4a\cos\theta\,\vec{i}$

$\delta G_1 = -a\,\text{sen}\,\theta\,\delta\theta\,\vec{i} - a\cos\theta\,\delta\theta\,\vec{j} = -a\,\delta\theta(\text{sen}\,\theta\,\vec{i} + \cos\theta\,\vec{j})$

$\delta G_2 = -3a\,\text{sen}\,\theta\,\delta\theta\,\vec{i} - a\cos\theta\,\delta\theta\,\vec{j} = -a\,\delta\theta(3\,\text{sen}\,\theta\,\vec{i} + \cos\theta\,\vec{j})$

$\delta B = -4a\,\text{sen}\,\theta\,\delta\theta\,\vec{i}$

b) $\delta W_P = (-mg\,\vec{j}) \cdot [\delta\vec{G_1} + \delta\vec{G_2}] =$

$(-mg)\cdot(-a\cos\theta\,\delta\theta - a\cos\theta\,\delta\theta) =$

$2mga\cos\theta\,\delta\theta\,;$

$\delta W_F = (-F\,\vec{i}) \cdot \delta\vec{B} = 4Fa\,\text{sen}\,\theta\,\delta\theta$

$\delta W_T = -K_T\,\theta\,\delta\theta$

$\Rightarrow \delta W_P + \delta W_F + \delta W_T = 0 \Rightarrow$

$2mga\cos\theta\,\delta\theta + 4Fa\,\text{sen}\,\theta\,\delta\theta - K_T\,\theta\,\delta\theta = 0$

$\delta\theta\,(2mga\cos\theta + 4Fa\,\text{sen}\,\theta - K_T\,\theta) = 0$

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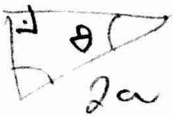
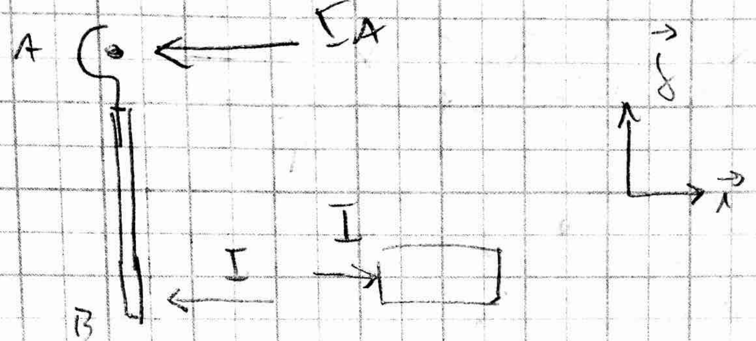
[P2-2012] $\frac{kx^2}{2}$  θ $2a$ __/__/__

(Q2) a) considerando as forças peso muito menores que as forças de interação produzidas pelo choque temos:



$\frac{m(2l)^2}{12} + ml^2$

$= \frac{m4l^2}{12} + ml^2$

$\frac{ml^2}{3} + ml^2 = \frac{4}{3}ml^2$

TMI na barra, polo A:

$\sum \vec{M}^{ext}_A = \Delta K_A \Rightarrow (B-A) \wedge (-I\vec{i}) = \{\vec{i}\,\vec{j}\,\vec{k}\}[J_A]\Delta \underset{\sim}{\omega}$

$\Rightarrow (-2l\vec{j}) \wedge (-I\vec{i}) = \{\vec{i}\,\vec{j}\,\vec{k}\} \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \omega' - \omega \end{bmatrix}$

$= -2lI\vec{k} = J_{zz}(\omega' - \omega)\vec{k} \Rightarrow \boxed{-2lI = J_{zz}(\omega' - \omega)}$

TRI no bloco $\Rightarrow \quad I = \frac{-2}{3}ml(\omega' - \omega)$ (I)

$\sum I^{ext} \Rightarrow I\vec{i} = M(\vec{V_c}' - \vec{V_c}) \Rightarrow \boxed{I = MV_c'}$ (II)

Hipótese de Newton: $u' = -eu$

$\vec{V_B} = \vec{V_A} + \omega\vec{k} \wedge (B-A) \Rightarrow V_B = \omega\vec{k} \wedge (-2l\vec{j}) \rightarrow$

$\vec{V_B} = 2l\omega\vec{i}$; $\vec{V_B}' = 2l\omega'\vec{i}$;

$u' = (\vec{V_B}' - \vec{V_c}') \cdot \vec{n} \Rightarrow u' = V_B' - V_c'$;

$u = (\vec{V_B} - \vec{V_c}) \cdot \vec{n} \Rightarrow u = V_B$;

~~$\therefore V_B = -e(V_B' - V_c') \Rightarrow 2l\omega = -e(2l\omega' - V_c')$~~

~~$2l\omega = -e2l\omega' + e\left[\frac{-2}{3}\frac{m}{M}l(\omega' - \omega)\right] \Rightarrow$~~

~~$2l\omega = -e2l\omega' - e\frac{2}{3}\frac{m}{M}l(\omega') + e\frac{2}{3}\frac{m}{M}l\omega \Rightarrow$~~

$$2lw = -e2lw' - e\frac{2}{3}\frac{m}{M}lw' + e\frac{2}{3}\frac{m}{M}lw \Rightarrow$$

$$w = -ew' - \frac{e}{3}\frac{m}{M}w' + \frac{e}{3}\frac{m}{M}w \Rightarrow$$

$$w\left(1-\frac{e}{3}\frac{m}{M}\right) = -ew'\left(1+\frac{1}{3}\frac{m}{M}\right) \Rightarrow$$

$$-\frac{w}{e}\left(\frac{1-\frac{e}{3}\frac{m}{M}}{1+\frac{1}{3}\frac{m}{M}}\right) = w'$$

$$-\frac{w}{e}\left(\frac{\frac{3M-em}{3M}}{\frac{3M+1m}{3M}}\right) = -\frac{w}{e}\cdot\frac{3M-em}{3M+m} = \frac{em-3M}{3M+m}\cdot\frac{w}{e}$$

$$w' = -ew \Rightarrow$$

$$v_B' - v_C' = -e v_B \Rightarrow 2lw' - v_c' = -e2lw \Rightarrow$$

$$\cancel{2}\cancel{l}w' + \frac{2}{3}\frac{m}{M}\cancel{l}(w'-w) = -e\cancel{2}\cancel{l}w \Rightarrow$$

$$w' + \frac{1}{3}\frac{m}{M}w' - \frac{1}{3}\frac{m}{M}w = -ew \Rightarrow$$

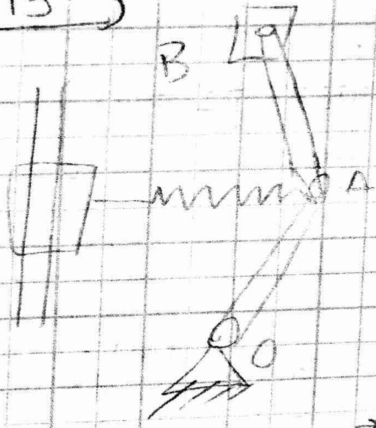
$$w'\left(1+\frac{1}{3}\frac{m}{M}\right) = w\left(\frac{1}{3}\frac{m}{M} - e\right) \Rightarrow$$

$$w' = \frac{\frac{1}{3}\frac{m}{M} - e}{\frac{1}{3}\frac{m}{M}+1}\,w \Rightarrow w' = \frac{\frac{3Me-m}{3M}}{\frac{3M+m}{3M}}\cdot w \Rightarrow$$

$$w' = \frac{3Me-m}{3M+m}\circ w$$

2-2013

(21)



$(B-O) = 2\,L\cos\theta\,\vec{j}$

$(A-O) = L\cos\theta\,\vec{j} + L\,\text{sen}\,\theta\,\vec{i}$

$\Rightarrow \delta\vec{B} = -2L\,\text{sen}\,\theta\,\delta\theta\,\vec{j}$

$\delta\vec{A} = -2\,L\cos\theta\,\delta\theta\,\vec{j} + L\cos\theta\,\delta\theta\,\vec{i}$

$\delta W_{mola} = -K\Delta x\,\vec{i}\cdot\delta\vec{A}\;;\quad \Delta x = L\,\text{sen}\,\theta$

$= (-KL\,\text{sen}\,\theta\,\vec{i})\cdot\delta\vec{A} = -K\,L\,\text{sen}\,\theta\cdot L\cos\theta\,\delta\theta$

$= -K\,L^2\,\text{sen}\,\theta\cos\theta\;\delta\theta$

$\delta W_F = -F\vec{j}\cdot\delta\vec{B} = +F\,2\,L\,\text{sen}\,\theta\,\delta\theta$

$\delta W_T = \delta W_F + \delta W_{mola} = 0 \Rightarrow L\,\delta\theta\cdot\text{sen}\,\theta\,(-K\,L\,\text{sen}\,\theta + 2F) = 0$

$\forall\,\delta\theta \Rightarrow \quad \text{sen}\,\theta = 0 \quad (\theta = 0^\circ)$

ou $\dfrac{2F}{KL} = \cos\theta$

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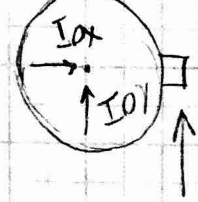
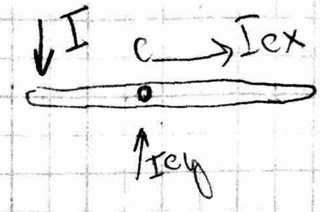
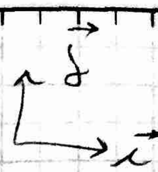
P2-2006

Q1) a)







$\vec{\omega} = -\omega \vec{k}$

$\vec{\omega}' = \omega' \vec{k}$

$\vec{\Omega}' = \Omega' \vec{k}$

b) TMI no disco, polo O:

$$\sum \vec{M}_O^{ext} = \Delta K_O \Rightarrow (B-O) \wedge I\vec{j} = \{\vec{i}\,\vec{j}\,\vec{k}\} [J_O] \Delta \underset{\sim}{W} \Rightarrow$$

$$2a\vec{i} \wedge I\vec{j} = \{\vec{i}\,\vec{j}\,\vec{k}\} \begin{bmatrix} J_{xx} & & \\ & J_{yy} & \\ & & J_{zz} \end{bmatrix} \begin{bmatrix} \\ \\ \omega'+\omega \end{bmatrix} \Rightarrow$$

$$2aI\vec{k} = J_{zz}(\omega'+\omega)\vec{k} \Rightarrow 2aI = (\omega'+\omega)\frac{m(2a)^2}{2}$$

$$\Rightarrow \cancel{2}\cancel{a} I = (\omega'+\omega) \cdot m \cdot \frac{\cancel{4}a^{\cancel{2}}}{\cancel{2}} \Leftrightarrow$$

$$\boxed{I = (\omega'+\omega) \cdot ma} \quad (I)$$

TMI na barra, polo C

$$\sum \vec{M}_C^{ext} = \Delta K_C \Rightarrow (B-C) \wedge (-I\vec{j}) = \{\vec{i}\,\vec{j}\,\vec{k}\} [J_C] \Delta \Omega \Rightarrow$$

$$(-a\vec{i}) \wedge (-I\vec{j}) = \{\vec{i}\,\vec{j}\,\vec{k}\} \begin{bmatrix} J_{xx} & & \\ & J_{yy} & \\ & & J_{zz} \end{bmatrix} \begin{bmatrix} \\ \\ \Omega'-0 \end{bmatrix}$$

$$\Rightarrow aI\vec{k} = J_{zz}\Omega'\vec{k} \; ; \; J_{zz} = \frac{m(4a)^2}{12} + ma^2 = \frac{7}{3}ma^2$$

$$\Rightarrow aI = \frac{7}{3}ma^2\Omega' \Rightarrow \boxed{I = \frac{7}{3}ma\Omega'} \quad (II)$$

$$\therefore \frac{7}{3}\cancel{m}\cancel{a}\Omega' = (\omega'+\omega)\cancel{m}\cancel{a} \Rightarrow \omega'+\omega = \frac{7}{3}\Omega'$$

$$\Rightarrow \boxed{\omega = \frac{7}{3}\Omega' - \omega'} \quad (III) \Rightarrow \boxed{\omega' = \frac{7}{3}\Omega' - \omega}$$

Hipótese de Newton

$u' = -e u$;

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$(A \equiv B)$

$\vec{V_A} = \cancel{\vec{V_0}} + \vec{\omega} \wedge (A-O) \Rightarrow \vec{V_A} = -\omega\vec{k} \wedge (2a\vec{i}) = -2a\omega\vec{j}$

$\vec{V_A}' = \vec{V_0} + \vec{\omega}' \wedge (A-O) \Rightarrow \vec{V_A}' = \omega'\vec{k} \wedge (2a\vec{i}) = 2a\omega'\vec{j}$

$\vec{V_B} = \vec{0}$;

$\vec{V_B}' = \vec{V_C} + \vec{\Omega}' \wedge (B-C) \Rightarrow \vec{V_B}' = \Omega'\vec{k} \wedge (-a\vec{i}) = -a\Omega'\vec{j}$

$\vec{n} = \vec{j}$

$\therefore \quad u' = (\vec{V_A}' - \vec{V_B}') \cdot \vec{n} = (2a\omega' + a\Omega')$

$u = (\vec{V_A} - \vec{V_B}) \cdot \vec{n} = (-2a\omega)$

$\Rightarrow 2a\omega' + a\Omega' = +e2a\omega \Rightarrow$

$2\omega' + \Omega' = 2e\omega \Rightarrow 2\left(\frac{7}{3}\Omega' - \omega\right) + \Omega' = 2e\omega \Rightarrow$

$\frac{14}{3}\Omega' - 2\omega + \Omega' = 2e\omega \Rightarrow \Omega'\left(\frac{14}{3} + 1\right) = 2\omega(e+1)$

$\frac{17\Omega'}{3} = 2\omega(e+1) \Rightarrow \Omega' = \frac{6\omega}{17}(e+1) \Rightarrow \vec{\Omega}' = \frac{6\omega(e+1)}{17}\vec{k}$

$\omega' = \frac{7}{\cancel{3}} \cdot \frac{\cancel{6}^{2}\omega}{17}(e+1) - \omega \Rightarrow \omega' = \frac{14}{17}\omega(e+1) - \omega \Rightarrow$

$\omega' = \frac{14}{17}\omega e + \frac{14}{17}\omega - \omega \Rightarrow \omega' = \frac{14}{17}\omega e - \frac{3}{17}\omega \Rightarrow$

$\vec{\omega}' = \frac{\omega}{17}(14e-3)\vec{k}$

e) $\quad I = (\omega' + \omega)ma = \left[\frac{\omega}{17}(14e-3) + \omega\right]ma$

$\left[\frac{\omega}{17}(14e-3+17)\right]ma = \frac{14}{17}\omega ma(1+e)$

TRI no disco : $\sum \vec{I}_O^{ext} = m\Delta\vec{V_G} = \vec{0} \Rightarrow$

$I_{Ox} = 0$; $I_{Oy} = -\frac{14}{17}\omega ma(1+e)$

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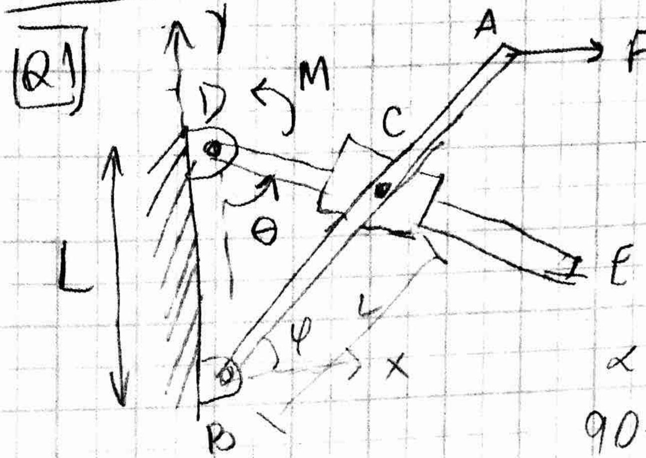
$\cos(a-b) = \cos a \cos b + \operatorname{sen} a \operatorname{sen} b$

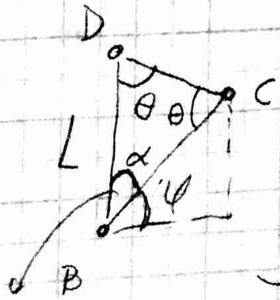
$\cos(2\theta) = \cos^2\theta - \operatorname{sen}^2\theta$

__/__/__

P3 - 2006

Q1





$$\alpha = 180 - 2\theta \Rightarrow 90 - \overbrace{(180-2\theta)}^{\alpha} \Rightarrow$$

$$90 - 180 + 2\theta \Rightarrow \varphi = 2\theta - 90$$

$$\Rightarrow (A-B) = 2L\cos\varphi\,\vec{i} + 2L\operatorname{sen}\varphi\,\vec{j} \;;$$

$$\cos\varphi = \cos(2\theta - 90) = \cos(2\theta)\cdot\cos 90 + \operatorname{sen} 2\theta \cdot \operatorname{sen} 90$$

$$\Rightarrow \cos\varphi = \operatorname{sen} 2\theta \Rightarrow \cos\varphi = 2\operatorname{sen}\theta\cos\theta \;;$$

$$\operatorname{sen}\varphi = \operatorname{sen}(2\theta - 90) = \operatorname{sen}(2\theta)\cos(90) - \operatorname{sen}(90)\cos(2\theta)$$

$$= -\operatorname{sen}(90)\cos(2\theta) = -\cos(2\theta) = -\cos^2\theta + \operatorname{sen}^2\theta$$

$$= \operatorname{sen}\varphi$$

$$\Rightarrow (A-B) = 2L\cdot 2\operatorname{sen}\theta\cos\theta\,\vec{i} + 2L(-\cos^2\theta + \operatorname{sen}^2\theta)\,\vec{j}$$

$$A_x = 4L\operatorname{sen}\theta\cos\theta \Rightarrow \delta A_x = 4L[\cos\theta\cdot\cos\theta\,\delta\theta + \operatorname{sen}\theta(-\operatorname{sen}\theta)\,\delta\theta]$$

$$\Rightarrow \delta A_x = 4L\,\delta\theta[\cos^2\theta - \operatorname{sen}^2\theta] = 4L\,\delta\theta\cos(2\theta)$$

$$\delta W_A = F\,\delta A_x = 4LF\cos(2\theta)\,\delta\theta$$

$$\delta W_M = M\,\delta\theta$$

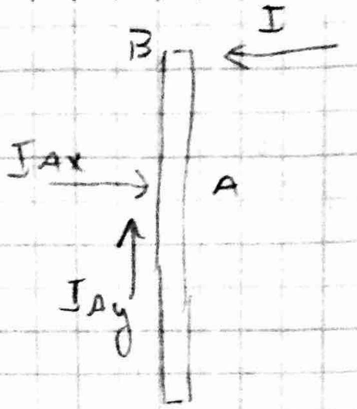
$$\Rightarrow \delta W_A + \delta W_M = 0 \Rightarrow \delta\theta\,(4LF\cos(2\theta) + M) = 0$$

$$\Rightarrow 4LF\cos(2\theta) + M = 0$$

$$\Rightarrow M = -4LF\cos 2\theta$$

$\frac{2+1}{6}$

PZ-2005 (Q2)



$\vec{\omega}' = \omega' \vec{k}$

$\vec{V}' = V' \vec{i}$

TMI na barra pelo A:

$\sum \vec{M}_A^{ext} = (B-A) \wedge (-I\vec{i}) = \Delta \vec{K_A} \Rightarrow$

$\left(\frac{l}{3}\vec{j}\right) \wedge (-I\vec{i}) = \{\vec{i}\ \vec{j}\ \vec{k}\} \begin{bmatrix} J_{xx}^A & & \\ & J_{yy}^A & \\ & & J_{zz}^A \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \omega' \end{bmatrix} \Rightarrow$

$\frac{l}{3} I \vec{k} = J_{zz}^A \omega' \vec{k}$; $J_{zz}^A = J_{zz}^G + m\left(\frac{l}{6}\right)^2 \Rightarrow$

$J_{zz}^A = \frac{ml^2}{12} + \frac{ml^2}{36} = \frac{3+1}{36} ml^2 = \frac{1}{9} ml^2$

$\therefore \frac{l}{3} I = \frac{1}{9} ml^2 \omega' \Rightarrow I = \frac{ml}{3}\omega'$ (1)

TRI na bala

$\sum I_{bala}^{ext} = \frac{m}{2} \Delta \vec{V}_{bala} = \frac{m}{2}\left(v'\vec{i} - (-v\vec{i})\right) \Rightarrow$

$\frac{m}{2}(v'+v)\vec{i} = I\vec{i} \Rightarrow \frac{m}{2}(v'+v) = I$ (2)

$\therefore$ (2) e (1): $\frac{ml}{3}\omega' = \frac{m}{2}(v'+v) \Rightarrow$

$\frac{l}{3}\omega' = \frac{1}{2}(v'+v)$ (3)

$\vec{V_B}' = \vec{V_A} + \omega'\vec{k} \wedge \underbrace{(B-A)}_{\frac{l}{3}\vec{j}} \Rightarrow \vec{V_B}' = -\omega'\frac{l}{3}\vec{i}$; $\vec{V_B} = \vec{0}$;

Hip Newton: $u' = -e\cancel{u}^{\,0} \Rightarrow u' = 0 \Rightarrow$

$\vec{V_B}' = \vec{V}' \Rightarrow v' = -\omega'\frac{l}{3} \Rightarrow$ (3): $\frac{l}{3}\omega' = -\frac{\omega' l}{6} + \frac{v}{2}$

$\Rightarrow \omega' l\left(\frac{1}{3}+\frac{1}{6}\right) = \frac{v}{2} \Rightarrow \omega' l \cdot \frac{1}{2} = \frac{v}{2} \Rightarrow \vec{\omega}' = \frac{v\vec{k}}{l}$

$\vec{V_D} = -v$ …

$\vec{V}_B = \vec{V}_A + \Omega' \vec{k} \wedge (B-A)$ $\quad \omega' > 0 \ \vec{k}$

$\Omega' \vec{k} \wedge (-t\,2L)$ $\quad \omega' < -\vec{k}$ $\quad$ __/__/__

$$\vec{V}_G' = \vec{V}_B' + \omega' \vec{k} \wedge (G-B) \Rightarrow \vec{V}_G' = -\frac{v}{3}\vec{i} + \frac{v}{\ell}\vec{k} \wedge \left(-\frac{\ell}{2}\vec{j}\right)$$

$$\Rightarrow \vec{V}_G' = -\frac{v}{3}\vec{i} + \frac{v}{2}\vec{i} \Rightarrow \vec{V}_G' = \frac{3-2}{6} v\vec{i} \Rightarrow$$

$$\underline{\vec{V}_G' = \frac{v}{6}\vec{i}}$$

(1) $\quad I = \frac{m\ell}{3}\omega' \Rightarrow I = \frac{m\ell}{3}\frac{v}{\ell} \Rightarrow \underline{I = \frac{mv}{3}}$

TRI na barra

$$\sum \vec{I}^{ext}_{barra} = m\Delta\vec{V}_G = m \cdot \frac{v}{6}\vec{i} = \vec{I} + \vec{I}_{Ax} + \vec{I}_{Ay} \Rightarrow$$

$$\vec{I}_{Ay} = \vec{0} \ ; \quad m\frac{v}{6} = -I + I_{Ax} \Rightarrow$$

$$I_{Ax} = mv\left(\frac{1}{6} + \frac{1}{3}\right) \Rightarrow I_{Ax} = mv\left(\frac{1+2}{6}\right)$$

$$\Rightarrow \vec{I}_{Ax} = \frac{mv}{2}\vec{i}$$

P2-2014 $\quad$ 17:02

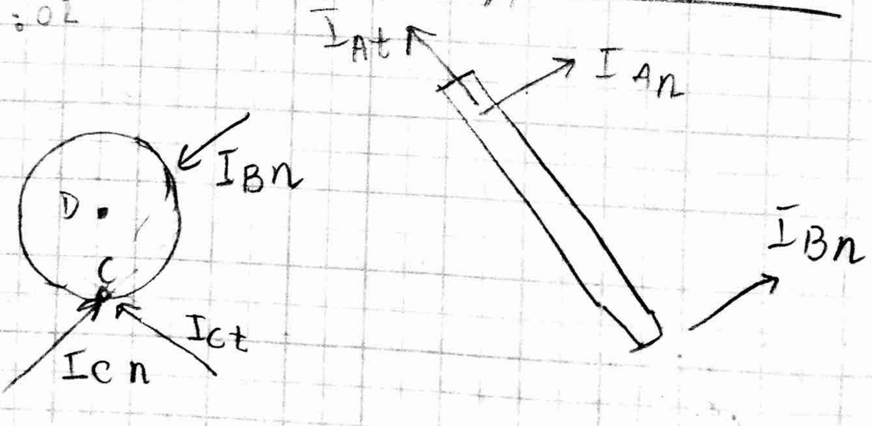
$\vec{\Omega} = -\Omega\vec{k}$

$\vec{\Omega}' = \Omega'\vec{k}$

$\vec{\omega}' = \omega'\vec{k}$

$\vec{v}_D = v_D\vec{i}$

① a)



b) $\quad$ TMI na barra polo A $\quad -2L\vec{t}$

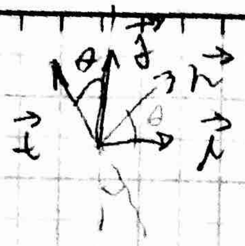
$$\sum \vec{M}^{ext}_A = \Delta\vec{K}_A \Rightarrow \overrightarrow{(B-A)} \wedge I_{Bn}\vec{n} = \{\vec{i}\,\vec{j}\,\vec{k}\}[K_A]\Delta\omega$$

$$\Rightarrow 2L\, I_{Bn}\vec{k} = J^A_{zz}(\Omega' + \Omega)\vec{k} \Rightarrow 2L\, I_{Bn} = \frac{4}{3}mL^2 \Delta\omega$$

$$\Rightarrow I_{Bn} = \frac{2}{3}mL(\Omega' + \Omega) \quad (1)$$

$\frac{m(2L)^2}{12} + mL^2 = \frac{m4L^2}{12} + mL^2$

$\frac{mL^2}{3} + mL^2 = \frac{4}{3}$



$\Rightarrow \quad \vec{j} = \vec{t}\cos\theta + \vec{n}\,\text{sen}\,\theta$

$\vec{i} = \vec{n}\cos\theta - \vec{t}\,\text{sen}\,\theta$

$\therefore \quad B-C = \underbrace{(B-D)}_{R\vec{n}} + \underbrace{(D-C)}_{R\vec{j} \,=\, R(\vec{t}\cos\theta + \vec{n}\,\text{sen}\,\theta)}$

TMI no disco polo C

$\sum \vec{M}_C^{ext} = (B-C) \wedge (-I_B n\,\vec{n}) =$

$R(\vec{t}\cos\theta + \vec{n}\,\text{sen}\,\theta + \vec{n}) \wedge (-I_B n\,\vec{n}) =$

$= R\cos\theta\, I_B n\,\vec{k} = \{\vec{i}\,\vec{j}\,\vec{k}\}\,[J_C]\,[\underset{\sim}{\omega'}] \Rightarrow$

$R\cos\theta\, I_B n = J^C_{zz}\,\omega' \Rightarrow \cancel{R}\cos\theta\, I_B n = \frac{3}{2} m R^{\cancel{2}}\omega' \Rightarrow$

$I_B n = \frac{3}{2\cos\theta} m R \omega'$ | (2)

$\therefore$ (1) e (2) $\Rightarrow \frac{2}{3}\cancel{m} L(\Omega' + \Omega) = \frac{3}{2\cos\theta}\cancel{m} R\omega' \Rightarrow$

$\frac{4\cos\theta}{9}\frac{L}{R}(\Omega' + \Omega) = \omega'$ | (3)

$\vec{V_B}$ barra $= +2L\Omega'\vec{n}$

$\vec{V_B}'$ barra $= -2L\Omega\vec{n}$; $\vec{V_B}'$ disco $= \vec{V_D} + \omega'\vec{k} \wedge \overbrace{(B-D)}^{R\vec{n}} \Rightarrow$

$\vec{V_B}'$ disco $= V_D\vec{i} + \omega' R\vec{t}$; Mas $V_D = +\omega' R$;

$\Rightarrow \vec{V_B}'$ disco $= +\omega' R(\vec{n}\cos\theta - \vec{t}\,\text{sen}\,\theta) + \omega' R\vec{t} \Rightarrow$

Hip Newton ; $u' = -eu$;

$u' = (\vec{V_B}\text{disco}' - \vec{V_B}'\text{barra}) \cdot \vec{n} = +\omega' R\cos\theta - 2L\Omega'$

$u = (\vec{V_B}\text{disco} - V_B\text{barra}) \cdot \vec{n} = 2L\Omega$

$\Rightarrow +\omega' R\cos\theta - 2L\Omega' = -e\,2L\Omega \Rightarrow$

$-\frac{2\cos^2\theta}{9}(\Omega' + \Omega) + \Omega' = +e\Omega$

$-\frac{2}{9}\cos^2\theta\,\Omega' - \frac{2\cos^2\theta}{9}\Omega + \Omega' = e\Omega \Rightarrow$

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$$-\frac{2\cos^2\theta}{9}\Omega' - \frac{2\cos^2\theta}{9}\Omega + \Omega' = e\Omega \Rightarrow$$

$$\Omega'\left(-\frac{2}{9}\cos^2\theta + 1\right) = -\Omega\left(e + \frac{2}{9}\cos^2\theta\right) \Rightarrow$$

$$\Omega' = -\Omega\left(\frac{e + \frac{2}{9}\cos^2\theta}{-\frac{2}{9}\cos^2\theta + 1}\right) = \Omega\left(\frac{\frac{9e + 2\cos^2\theta}{9}}{\frac{-2\cos^2\theta + 9}{9}}\right)$$

$$\vec{\Omega}' = -\Omega\left(\frac{9e + 2\cos^2\theta}{9 - 2\cos^2\theta}\right)\vec{k} \;;$$

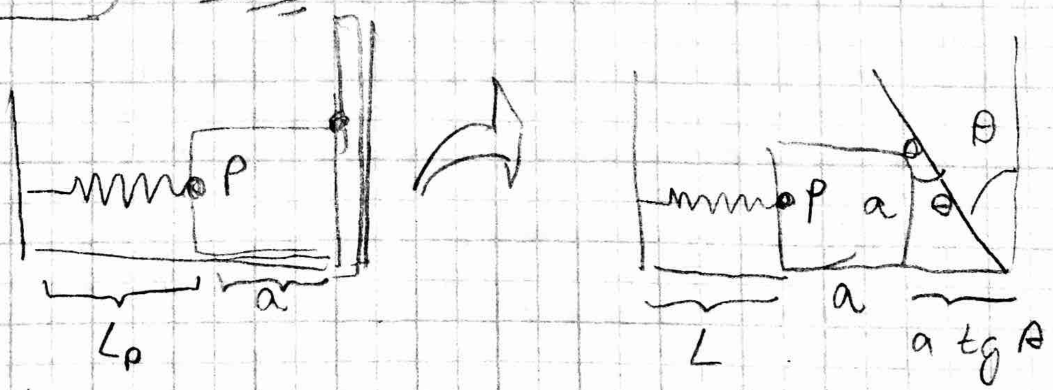
$$w' = \frac{4\cos\theta}{9}\frac{L}{R}\Omega\left(\frac{9e + \cancel{2\cos^2\theta} + 9 - \cancel{2\cos^2\theta}}{9 - 2\cos^2\theta}\right)$$

$$w' = \frac{4\cos\theta}{\cancel{9}}\frac{L}{R}\Omega\left(\frac{1+e}{9 - 2\cos^2\theta}\right) ✓$$

$$I_{Bn} = \frac{3}{\cancel{2}\cancel{\cos\theta}} \cdot m\cancel{R} \cdot \cancel{{}^{2}4}\cancel{\cos\theta}\,\frac{L}{\cancel{R}}\,\Omega\left(\frac{1+e}{9 - 2\cos^2\theta}\right)$$

$$I_{Bn} = 6mL\Omega\left(\frac{1+e}{9 - 2\cos^2\theta}\right)$$

(P3 - Q1) - 2005



$$\cancel{a} + L_0 = L + \cancel{a} + a\,tg\,\theta \Rightarrow \underline{L = L_0 - a\,tg\,\theta} \Rightarrow \Delta L = -a\,tg\,\theta$$

$$B_x = -a\,tg\theta\,\vec{i} \Rightarrow \delta B_x = -a\sec^2\theta\,\vec{i}\,\delta\theta$$

$$P_x = (-a - a\,tg\,\theta)\vec{i} \Rightarrow \delta P_x = -a\sec^2\theta\,\vec{i}\,\delta\theta$$

$\delta W_{mola} = -k\Delta L_i\, \delta \vec{P}_x = +ka\,tg\theta\,(-a\sec^2\theta) = -ka^2\,tg\theta\sec^2\theta\,\delta\theta$

$\delta W_F = -F\vec{i}\;\delta \vec{B}_x = Fa\sec^2\theta\,\delta\theta$

$\Rightarrow \delta W_{mola} + \delta W_F = 0 \Rightarrow \left(-ka^2\,tg\theta\sec^2\theta + Fa\sec^2\theta\right)\delta\theta = 0$

$\Rightarrow \quad F\sec^2\theta = ka\;tg\theta\sec^2\theta$

$\boxed{\text{P2 - 2003}}$

$\boxed{\text{Q1}}$ a) $\frac{1}{2}mV_B^2 = mgh$ (conserv. de energia)

$V_B = \sqrt{2gh}$;

Cons. de momento angular com rel ao polo A:

$K = -mLV_B$; $K' = -mLV_B'\vec{k} + K_A$;

$V_B' = \omega' L \Rightarrow \omega' = \frac{V_B'}{L}$;

$\vec{K}_A = \{\vec{i}\;\vec{j}\;\vec{k}\}[J_A]\,\underset{\sim}{\omega'} = -\frac{ML^2}{3}\omega'\vec{k}$

$\Rightarrow K' = -m\cancel{L}V_B' - \frac{M\cancel{L}^2}{3}\frac{V_B'}{\cancel{L}} = -m\cancel{L}V_B \Rightarrow$

$-mV_B' - \frac{M}{3}V_B' = -mV_B \Rightarrow mV_B = V_B'\left(m + \frac{M}{3}\right) \Rightarrow$

$mV_B = V_B'\left(\frac{3m+M}{3}\right) \Rightarrow V_B' = \frac{3mV_B}{3m+M} \Rightarrow$

$\vec{V}_B = -\sqrt{2gh}\,\vec{j}$

$\vec{V}_B' = -\frac{3m}{3m+M}\sqrt{2gh}\,\vec{j}$

$\cancel{b)\ T_{barra\,A}' = \frac{1}{2}J_{zz}^A\,\omega'^2 = \frac{1}{2}\frac{M}{3}\frac{9m^2}{(3m+M)^2}2gh} = \frac{3Mm^2}{(3m+M)^2}gh$

$\cancel{= k\ \ \ \max \Rightarrow \ \ \max = \frac{6Mm^2}{\ldots}gh}$

(cont)

$$T_{sist}' = \frac{1}{2} m v_B'^2 + \frac{1}{2} \frac{ML^2}{3} \left(\frac{v_B'}{L}\right)^2 \Rightarrow$$

$$T_{sist}' = \frac{1}{2} m v_B'^2 + \frac{1}{6} M v_B'^2 \Rightarrow$$

$$T_{sist}' = v_B'^2 \cdot \left(\frac{1}{2} m + \frac{1}{6} M\right);$$

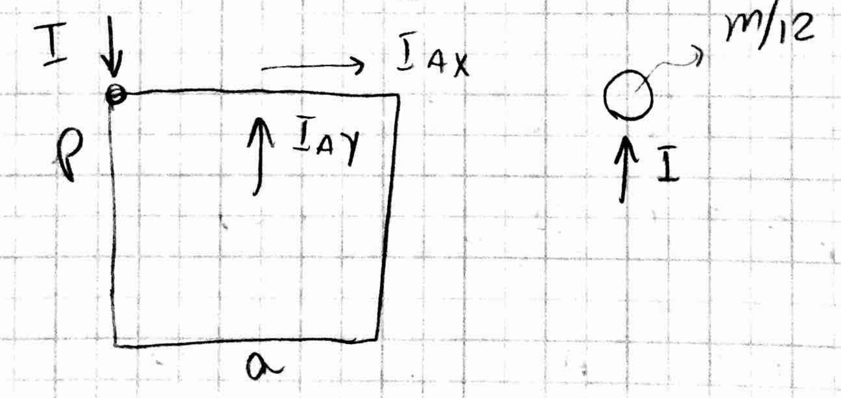
$$T_{sist}' = \frac{k}{2} y_{max}^2 \Rightarrow y_{max} = v_B' \cdot \sqrt{\frac{3m+M}{3k}}$$

$$\Rightarrow y_{max} = \frac{3m}{3m+M} \sqrt{2gh} \sqrt{\frac{3m+M}{3k}} \Rightarrow$$

$$y_{max} = 3m \sqrt{2gh} \cdot \sqrt{\frac{1}{3k(3m+M)}}$$

$$= m \sqrt{\frac{18gh}{3k(3m+M)}} = m \sqrt{\frac{6gh}{k(3m+M)}}$$

$\boxed{P2-2002}$ Q2



$$\vec{v}_P' = \vec{v}_A + \omega' \vec{k} \wedge (P-A) \Rightarrow \vec{v}_P' = -\frac{a}{2} \omega' \vec{j}$$

cons. de momento angular em rel. ao polo A:

$$\vec{K}_0 = + \frac{m}{12} v \frac{a}{2} \Rightarrow \vec{K}_0 = \boxed{\frac{mva}{24} \vec{k}}$$

$$\vec{K}' = \left(\frac{m}{12} v_P' \frac{a}{2} + K^A_{placa}\right)\vec{k} = \frac{m a v_P'}{24} + K^A_{placa} = \frac{ma^2}{48} \omega' + K^A_{placa};$$

$$\vec{K}^{A\,\prime}_{placa} = J^A_{33} \omega' \vec{k} = \frac{5}{12} m a^2 \omega'$$

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$\therefore \frac{\cancel{m}v\cancel{d}}{24} = \frac{\cancel{m}a^{\cancel{2}}}{48}\omega' + \frac{5}{12}\cancel{m}a^{\cancel{2}}\omega' \Rightarrow$

$\frac{V}{2} = \frac{a\omega'}{4} + 5a\omega' \Rightarrow 2V = a\omega' + 20a\omega' \Rightarrow$

$2V = 21a\omega' \Rightarrow \vec{\omega}' = \frac{2}{21}\frac{V}{a}\vec{k}$

$\therefore \vec{V_G} = \vec{V_A} + \omega'\vec{k} \wedge (G-A) \Rightarrow$

$\vec{V_G} = \omega'\vec{k} \wedge \left(-\frac{a}{2}\vec{j}\right) \Rightarrow \vec{V_G} = \omega'\frac{a}{2}\vec{i} \Rightarrow$

$\vec{V_G} = \frac{V}{21}\vec{i}$;

c) TRI na placa :

$\sum I^{ext}_{placa} = m\Delta\vec{V_G} = m\frac{V}{21}\vec{i} = I_{Ay}\vec{j} + I_{Ax}\vec{i} - I\vec{j}$

$\Rightarrow \vec{I_{Ax}} = \frac{mV}{21}\vec{i}$; $\vec{I_{Ay}} = I\vec{j}$

TRI na bola :

$\sum I^{ext}_{bola} = \frac{m}{12}\Delta V = \frac{m}{12}(\vec{V_P}' - \vec{V}) = I\vec{j}$

$\vec{V_P}' = -\frac{\cancel{a}}{\cancel{2}}\frac{\cancel{2}}{21}\frac{V}{\cancel{a}}\vec{j} \Rightarrow \vec{V_P}' = \frac{-V}{21}\vec{j}$; $\vec{V} = -V\vec{j}$

$\Rightarrow \frac{m}{12}\left(-\frac{V}{21} + V\right)\vec{j} = I\vec{j} \Rightarrow \frac{m}{12}\cdot\frac{20}{21}V = I \Rightarrow$

$I = \frac{5}{63}mV \Rightarrow \vec{I_{Ay}} = \frac{5}{63}mV\vec{j}$

$\frac{1}{6}ma^2 + \frac{ma^2}{4} = \frac{4+6}{24}ma^2 = \frac{5}{12}ma^2$

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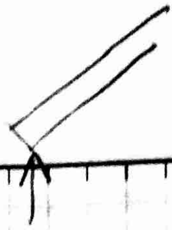


$\vec{V_A}' = V_A' \vec{i}$ __/__/__

$\vec{\omega}' = \omega' \vec{k}$

$\vec{V_G}' = V_G' \vec{j}$

Sub - 2014 | Q2 |

Hipótese de Newton : $u' = -e u \Rightarrow u' = 0 \Rightarrow$

$\vec{V_A}' \cdot \vec{j} = \vec{0} \Rightarrow V_{Ay}' = 0$; $\vec{V_A} = V_A' \vec{i}$;

$\vec{V_G}' = \vec{V_A}' + \omega' \vec{k} \wedge (G - A) \Rightarrow$

$\vec{V_G}' = \vec{V_A}' + \omega' \vec{k} \wedge (L\cos\alpha \vec{j} + L \,\text{sen}\alpha \vec{i})$?

(*) $\vec{V_G}' = V_A' \vec{i} + \omega' L \,\text{sen}\alpha \vec{j} - \omega' L \cos\alpha \vec{i}$

TRI na barra :

$\Sigma \vec{I}^{ext} = m \Delta \vec{V_G} \Rightarrow I \vec{j} = m(\vec{V_G}' + V\vec{j}) \rightarrow$

$I = m V_G' + m V \Rightarrow V_G' = \dfrac{I - mV}{m}$

$\Rightarrow$ TMI em G

$\Sigma M_G^{ext} = (A - G) \wedge I\vec{j} = (-L\cos\alpha \vec{j} - L\,\text{sen}\alpha \vec{i}) \wedge I\vec{j}$

$= -L\,\text{sen}\alpha\, I \vec{k} = \{\vec{i}\,\vec{j}\,\vec{k}\}\,[J_G]\,\Delta\underset{\sim}{\omega} \Rightarrow$

$-L\,\text{sen}\alpha\, I \vec{k} = \dfrac{m(2L)^2}{12}\omega' \vec{k} \Rightarrow -L\,\text{sen}\alpha\, I = \dfrac{mL^2}{3}\omega'$

$I = -\dfrac{mL}{3\,\text{sen}\alpha}\omega'$; (*) $V_G' = \omega' L\,\text{sen}\alpha$

$V_A' = \omega' L \cos\alpha$

$\Rightarrow -\dfrac{\cancel{m}L}{3\,\text{sen}\alpha}\omega' = \cancel{m}V_G' + \cancel{m}V \Rightarrow -\dfrac{L\omega'}{3\,\text{sen}\alpha} = \omega' L\,\text{sen}\alpha + V$

$\Rightarrow -\dfrac{\omega' L}{3\,\text{sen}\alpha} - \omega' L\,\text{sen}\alpha = V \Rightarrow -\omega' L - 3\omega' L\,\text{sen}^2\alpha = 3\,\text{sen}\alpha\, V$

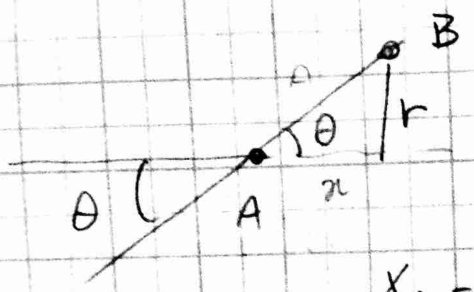
$\Rightarrow -\omega' L(1 + 3\,\text{sen}^2\alpha) = 3\,\text{sen}\alpha\, V \Rightarrow$

$\omega' = \dfrac{-3\,\text{sen}\alpha\, V}{L(1 + 3\,\text{sen}^2\alpha)}$ ✓

S T Q Q S S D

$$\vec{V_{G'}} = \frac{-3\,\text{sen}^2\alpha\, V}{1 + 3\,\text{sen}^2\alpha}\,\vec{j}\ ;\quad \underline{\text{OK}}$$

→ Sub - 2014 - $\boxed{Q3}$



$$tg\,\theta = \frac{r}{x} \Rightarrow x = \frac{r}{tg\,\theta}$$

$$\Rightarrow X_A = -\frac{r}{tg\,\theta}\,\vec{i} \Rightarrow$$

$$X_A = -\frac{r\cos\theta}{\text{sen}\,\theta}\,\vec{i} \Rightarrow \delta X_A = \frac{r}{\text{sen}^2\theta}\,\delta\theta\,\vec{i}$$

$$C_y = (L\,\text{sen}\,\theta - r)\,\vec{j} \Rightarrow \delta C_y = L\cos\theta\,\delta\theta\,\vec{y}$$

$$\Rightarrow \delta W_p = P\vec{i}\cdot\delta\vec{A_x} = \frac{Pr}{\text{sen}^2\theta}\,\delta\theta$$

$$\delta W_F = -F\vec{i}\cdot\delta C_y = -FL\cos\theta\,\delta\theta$$

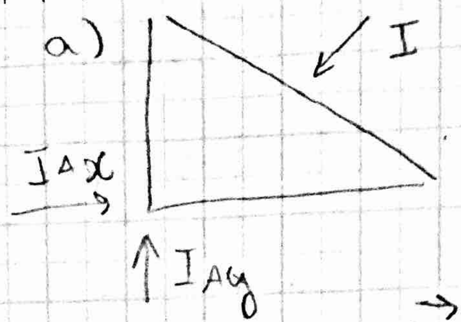
$$\Rightarrow \delta W_p + \delta W_F = 0 \Rightarrow \left(\frac{Pr}{\text{sen}^2\theta} - FL\cos\theta\right)\delta\theta = 0 \Leftrightarrow$$

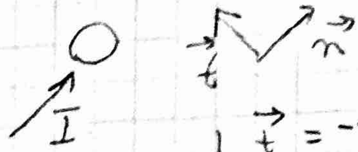
$$P = \frac{FL\cos\theta\,\text{sen}^2\theta}{r}$$

S T Q Q S S D

Sub 2011

Q2 a)





$$\begin{cases} \vec{t} = \dfrac{-4\vec{i} + 3\vec{j}}{5} \\ \vec{n} = \dfrac{3\vec{i} + 4\vec{j}}{5} \end{cases}$$

TRI na bala: $m\Delta\vec{V} = I\vec{n} \Rightarrow$

$m(v'\vec{i} + V\vec{j}) = I\left(\dfrac{3\vec{i}}{5} + \dfrac{4\vec{j}}{5}\right) \Rightarrow$

$$\begin{cases} mv' = \dfrac{3I}{5} \\ mV = \dfrac{4I}{5} \Rightarrow \underline{I = \dfrac{5mV}{4}} \end{cases} \Rightarrow \cancel{m}v' = \frac{3}{\cancel{5}}\cdot\frac{\cancel{5}}{4}\cancel{m}V \Rightarrow$$

$\underline{\vec{V}' = \dfrac{3}{4}V\vec{i}}$ //

TMI na cunha (polo A)

$\sum \vec{M}_A^{ext} = (D-A)\wedge(-I\vec{n}) = \left(2a\vec{i} + \dfrac{3}{2}a\vec{j}\right)\wedge$

$\left(2a\vec{i} + \dfrac{3}{2}a\vec{j}\right)\wedge(-I)\left(\dfrac{3\vec{i}}{5} + \dfrac{4\vec{j}}{5}\right) =$

$-I\cdot a\dfrac{8}{5}\vec{k} + Ia\dfrac{9}{10}\vec{k} = aI\dfrac{+9-16}{10} = aI\dfrac{-7}{10}\vec{k}$

$= J_{Az}\omega'\vec{k} \Rightarrow J_{Az}\omega' = aI\dfrac{-7}{10} \Rightarrow$

$\omega' = \dfrac{-7}{\cancel{10}_{2}}a\cdot\dfrac{\cancel{5}mV}{4J_{Az}} = \underline{\dfrac{-7amV}{8J_{Az}}}$ //

$\vec{V}_D' = \vec{V}_A + \omega'\vec{k}\wedge(D-A) \Rightarrow \vec{V}_D = \omega'\vec{k}\wedge\left(2a\vec{i} + \dfrac{3}{2}a\vec{j}\right)$

$\Rightarrow \vec{V}_D' = \omega' 2a\vec{j} - \omega'\dfrac{3}{2}a\vec{i}$;

$\vec{V}_D'\cdot\vec{n} = \omega' a\left(2\vec{j} - \dfrac{3}{2}\vec{i}\right)\cdot\left(\dfrac{3\vec{i}}{5} + \dfrac{4\vec{j}}{5}\right) \Rightarrow$

$V_{Dn}' = \omega' a\left(\dfrac{8}{5} - \dfrac{9}{10}\right) \Rightarrow V_{Dn}' = \omega' a\left(\dfrac{16-9}{10}\right) = \dfrac{7}{10}\omega' a$

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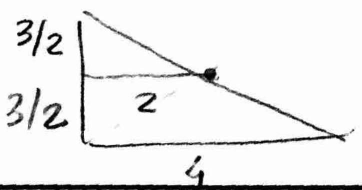

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$$V_E \cdot \vec{n} = -V\vec{j} \cdot \left(\frac{3\vec{i}}{5} + \frac{4}{5}\vec{j}\right) \Rightarrow V_{En} = -\frac{4}{5}V$$

$$V_E' \cdot \vec{n} = \frac{3}{4}V\vec{i} \cdot \left(\frac{3\vec{i}}{5} + \frac{4}{5}\vec{j}\right) \Rightarrow V_{En}' = \frac{9}{20}V$$

$\Rightarrow$ Hipótese de Newton $u' = -eu$

$$u' = (\vec{V_E}' - \vec{V_D}') \cdot \vec{n} = \left(\frac{9}{20}V - \frac{7}{10}\omega' a\right)$$

$$u = (\vec{V_E} - \vec{0}) \cdot \vec{n} = \frac{4}{5}V \Rightarrow$$

$$+\frac{4}{5}e\cancel{V} = \frac{9}{20}\cancel{V} - \frac{7}{10}a\left(\frac{-7am\cancel{V}}{8J_{Az}}\right) \Rightarrow$$

$$\frac{4}{5}e = \frac{9}{20} + \frac{49}{80}\frac{ma^2}{J_{Az}} \Rightarrow e = \frac{9}{20}\cdot\frac{5}{4} + \frac{49}{80}\cdot\frac{5}{4}\frac{ma^2}{J_{Az}}$$

$$e = \frac{9}{16} + \frac{49}{64}\frac{ma^2}{J_{Az}} \Rightarrow e = \frac{36J_{Az} + 49ma^2}{64J_{Az}}$$

$$e = \frac{1}{16}\left(9 + \frac{49ma^2}{4J_{Az}}\right)$$

$$\vec{V_G}' = \vec{V_A} + \ldots$$