

Nome: _____ Nº USP: _____
 (Colocar nome em todas as folhas!)

3^a Prova — 1^o semestre de 2017

1^a Questão (3,0 pontos)

Dada a estrutura da figura, engastada em A e articulada em B e C, determine:

- a posição x do apoio B que permitiria aplicar a máxima carga P ;
- a dimensão c da seção considerando $x = \frac{2}{3}\ell = 150$ cm e um coeficiente de segurança $\gamma = 3$.

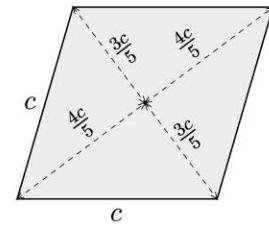
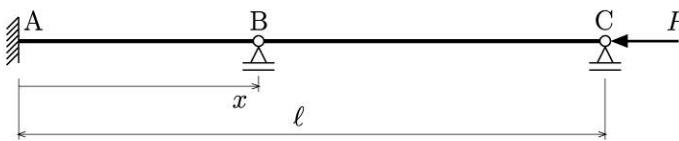
Admita que a vinculação indicada na figura é a mesma nos planos horizontal e vertical.

São dados:

$$\begin{aligned} P &= 100 \text{ kN}, \\ E &= 10^4 \text{ kN/cm}^2, \\ \sigma_y &= 20 \text{ kN/cm}^2, \\ \sigma_p &= 10 \text{ kN/cm}^2. \end{aligned}$$

$$\sigma_{\text{fl}} = \begin{cases} \sigma_y - (\sigma_y - \sigma_p) \left(\frac{\lambda}{\lambda_{\lim}} \right)^2 & \text{para } \lambda \leq \lambda_{\lim} \\ \frac{\pi^2 E}{\lambda^2} & \text{para } \lambda > \lambda_{\lim} \end{cases}$$

Seção transversal

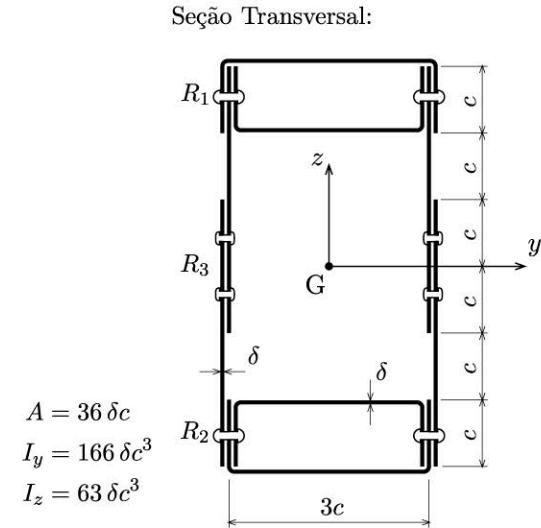
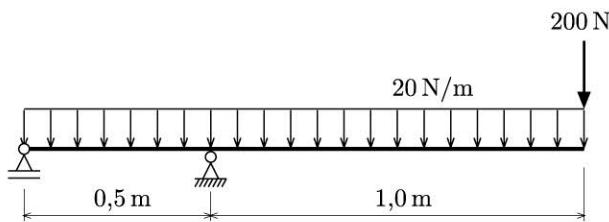


2^a Questão (3,5 pontos)

Determine a força cortante máxima e a seção em que ela atua para a viga da figura com a seção tranversal indicada. Com a força cortante obtida acima, e admitindo $\delta \ll a$ e espaçamento constante entre os rebites, calcule:

- as áreas mínimas A_1 e A_2 das seções dos rebites R_1 e R_2 de modo que se tenha espaçamento longitudinal $s_1 = s_2 = 50$ mm;
- o espaçamento longitudinal s_3 dos rebites R_3 com área $A_3 = 1$ mm².

São dados: $c = 10$ mm, $\delta = 1$ mm, a tensão tangencial admissível do rebite $\bar{\tau} = 100$ N/mm², e as propriedades geométricas da seção indicadas na figura.

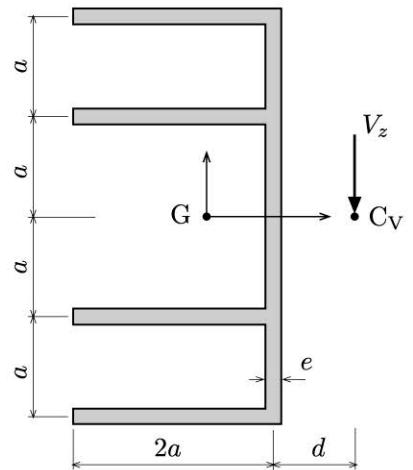


3^a Questão (3,5 pontos)

Para a seção transversal de paredes delgadas da figura, determine:

- a distribuição do fluxo de cisalhamento ao longo das paredes (não deixe de indicar os sentidos);
- as resultantes R_i das tensões de cisalhamento em cada trecho de parede;
- o valor da máxima tensão de cisalhamento no plano da seção;
- a distância d ao centro de cisalhamento.

As paredes têm espessura constante igual a $e \ll a$.

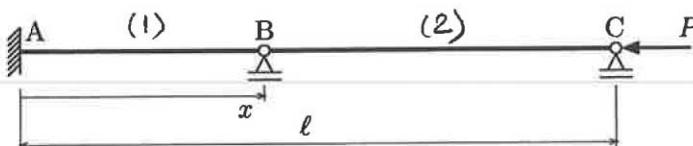


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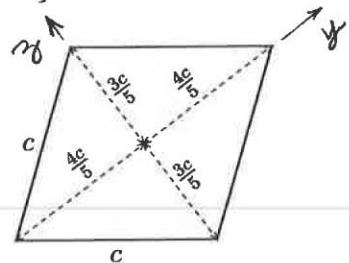
$$P = 100 \text{ kN}, \\ E = 10^4 \text{ kN/cm}^2, \\ \sigma_y = 20 \text{ kN/cm}^2, \\ \sigma_p = 10 \text{ kN/cm}^2.$$

$$\sigma_{fl} = \begin{cases} \sigma_y - (\sigma_y - \sigma_p) \left(\frac{\lambda}{\lambda_{lim}} \right)^2 & \text{para } \lambda \leq \lambda_{lim} \\ \frac{\pi^2 E}{\lambda^2} & \text{para } \lambda > \lambda_{lim} \end{cases}$$

$$\gamma = 3$$

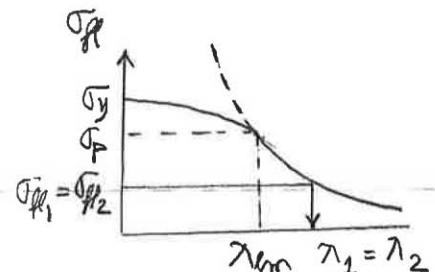


Seção transversal



- a) Simultaneidade da flambagem nos dois trechos

$$\ell_{fl_1} = \ell_{fl_2} \Rightarrow \lambda_{y_1} = \lambda_{y_2}$$



$$b) x = \frac{2}{3}l = 150 \text{ cm} \quad \begin{cases} l_1 = 150 \text{ cm} & \ell_{fl_1} = 0,7l_1 = 105 \text{ cm} \\ l_2 = 75 \text{ cm} & \ell_{fl_2} = l_2 = 75 \text{ cm} \end{cases} \Rightarrow \lambda_{y_1} > \lambda_{y_2}$$

$$A = 2 \left(\frac{8c}{5} \times \frac{3c}{5} / 2 \right) = \frac{24}{25} c^2 \quad \left. \begin{array}{l} I_y = 2 \left(\frac{8c}{5} \left(\frac{3c}{5} \right)^3 / 12 \right) = \frac{36}{625} c^4 \\ i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{36 \times 25}{625 \times 24} c^2} = \sqrt{\frac{3}{50}} c = 0,245 c \end{array} \right\}$$

$$\lambda_{y_1} = \frac{105}{0,245 c} = \frac{428,7}{c}$$

$$\lambda_{lim} = \pi \sqrt{\frac{E}{\sigma_p}} = \pi \sqrt{10^3} = 99,4$$

• Admitindo $\lambda_{y_1} \geq \lambda_{lim}$ $\sigma = \frac{P}{A} \leq \frac{\sigma_p}{\lambda_{y_1}} \Rightarrow \frac{P}{A} \leq \sigma_p$

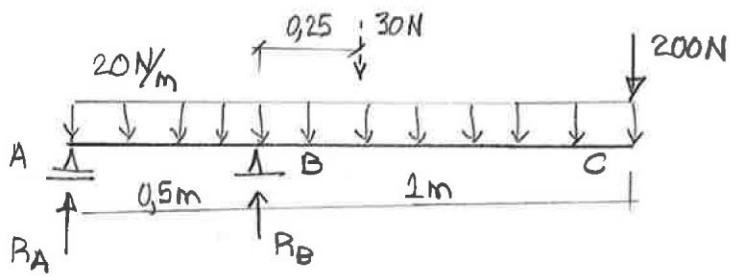
$$\frac{3 \times 100}{\frac{24}{25} c^2} \leq \frac{\pi^2 \cdot 10^4}{428,7^2} \Rightarrow \frac{300 \times 25 \times 428,7^2}{24 \times \pi^2 \times 10^4} \leq c^4 \Rightarrow c \geq 4,91 \text{ cm}$$

$$\lambda_y = \frac{428,7}{4,91} = 87,3 < \lambda_{lim} \text{ (hipótese inconcreta!)}$$

• Admitindo $\lambda_{y_1} \leq \lambda_{lim}$

$$\frac{300 \times 25}{24 c^2} \leq 20 - 10 \left(\frac{428,7}{c \times 99,4} \right)^2 = 20 - \frac{186}{c^2} \Rightarrow \frac{498,5}{c^2} \leq 20 \Rightarrow c = 4,99 \text{ cm}$$

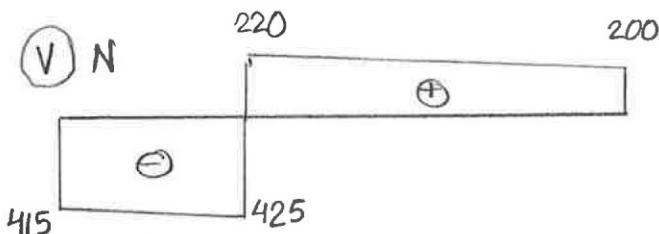
$$\lambda_y = \frac{428,7}{4,99} = 85,9 < \lambda_{lim} \text{ (OK!)}$$



$$\uparrow \left\{ \begin{array}{l} R_A + R_B = 230 \end{array} \right.$$

$$\textcircled{B} \quad \left\{ \begin{array}{l} R_A \cdot 0,5 + 30 \cdot 0,25 + 200 \cdot 1 = 0 \end{array} \right.$$

$$R_A = -415 \text{ N} \quad R_B = 645 \text{ N}$$



verif.

$$\textcircled{A} \quad \left\{ \begin{array}{l} 200 \cdot 1,5 + 30 \cdot 0,75 - 645 \cdot 0,5 = 0 \end{array} \right.$$

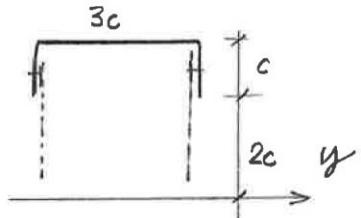
a) $V_{\text{máx.}} = 425 \text{ N}$ na seção à esquerda do apoio fixo B.

b) Rebite R₁ ($a_1 = 50 \text{ mm}$)

$$\bar{s}_1 = 3bc \times 3c + 2(bc \times \frac{5}{2}c) = 14bc^2$$

$$q_1 = \frac{\sqrt{\bar{s}_1}}{I_y} = \frac{425 \cdot 14bc^2}{166bc^3} = \frac{35,84}{c}$$

$$q_1 a_1 = \bar{c}(2A_R) \Rightarrow \frac{35,84}{10} \times 50 = 200A_R \Rightarrow \underline{A_R^{(1)} = 0,896 \text{ mm}^2}$$

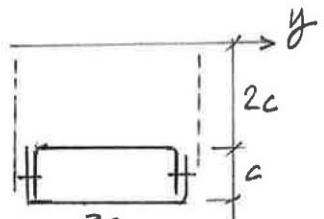


c) Rebite R₂ ($a_2 = 50 \text{ mm}$)

$$\bar{s}_2 = \bar{s}_1 + 3bc \times 2c + 2(bc \times \frac{5}{2}c) = 25bc^2$$

$$q_2 = \frac{425 \cdot 25bc^2}{166bc^3} = \frac{64,00}{c}$$

$$q_2 a_2 = \bar{c}(2A_R) \Rightarrow \frac{64,0}{10} \times 50 = 200A_R \Rightarrow \underline{A_R^{(2)} = 1,60 \text{ mm}^2}$$

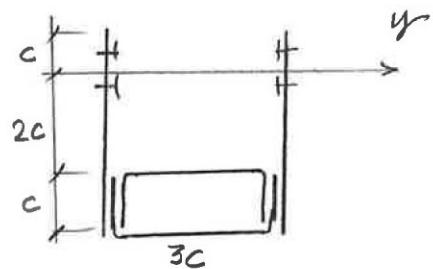


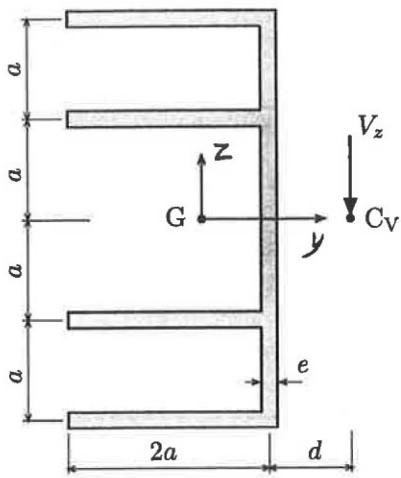
d) Rebite R₃ ($A_R^{(3)} = 1 \text{ mm}^2$)

$$\bar{s}_3 = \bar{s}_2 + 2(4bc \times c) = 33bc^2$$

$$q_3 = \frac{425 \cdot 33bc^2}{166bc^3} = \frac{84,49}{c}$$

$$q_3 a_3 = \bar{c}(4 \times 1) \Rightarrow \frac{84,49}{10} \times 1 = 400 \Rightarrow \underline{a_3 = 47,3 \text{ mm}}$$





$$I_y = 2(2ea \times (2a)^2 + 2ea \times a^2) + \frac{e(4a)^3}{12} = \frac{76}{3}ea^3$$

a) Fluxo de cisalhagem.

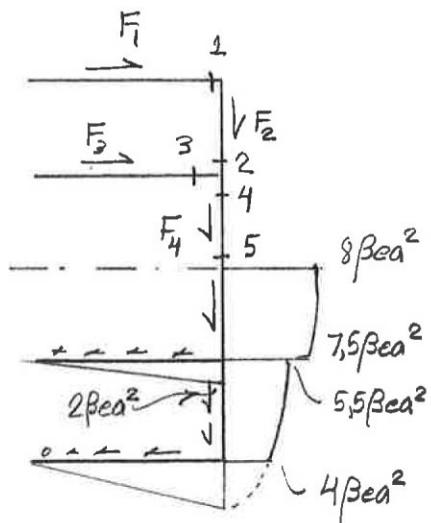
$$\bar{s}_1 = 2ea \times 2a = 4ea^2 \Rightarrow q_1 = 4\beta ea^2$$

$$\bar{s}_2 = \bar{s}_1 + ea \times 1,5a = 5,5ea^2 \Rightarrow q_2 = 5,5\beta ea^2$$

$$\bar{s}_3 = 2ea \times a = 2ea^2 \Rightarrow q_3 = 2\beta ea^2$$

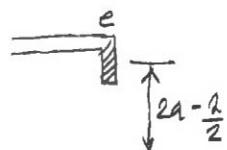
$$\bar{s}_4 = \bar{s}_2 + \bar{s}_3 = 7,5ea^2 \Rightarrow q_4 = 7,5\beta ea^2$$

$$\bar{s}_5 = \bar{s}_4 + ea \times 0,5a = 8ea^2 \Rightarrow q_5 = 8\beta ea^2$$



b) Resultantes

$$R_1 = \frac{1}{2}q_1 2a = \frac{1}{2}4\beta ea^2 \cdot 2a = \underline{4\beta ea^3}$$



$$\bar{s}(a) = 4ea^2 + ea(2a - \frac{1}{2}) = 4ea^2 + 2ea^2 - \frac{ea^2}{2}$$

$$R_2 = \beta \int_0^a (4ea^2 + 2ea^2 - \frac{ea^2}{2}) da = \beta [4ea^2a + 2ea^2a - \frac{ea^3}{6}]_0^a = \underline{\frac{29}{6}\beta ea^3}$$

$$R_3 = \underline{\frac{2\beta ea^2 \times 2a}{2} = 2\beta ea^3}$$

$$R_4 = \underline{[7,5\beta ea^2 + \frac{2}{3} \times 0,5\beta ea^2]a = \frac{47}{6}\beta ea^3} \quad \text{Verif. } 2(R_2 + R_4) = 2(\frac{29}{6} + \frac{47}{6})\beta ea^3 = V$$

$$c) \quad \underline{T_5 = \frac{q_5}{e} = \frac{8\beta ea^2}{\frac{76}{3}ea^3} = \frac{6}{19} \frac{V}{ea}} \quad (\text{comparar com } T_m = \frac{V}{A_{\text{área}}} = \frac{V}{4ea})$$

$$d) \quad \underline{C_V : \quad V \times d = F_1 \times 4a + F_3 \times 2a = 16\beta ea^4 + 4\beta ea^4 \Rightarrow d = \frac{15}{19}a}$$

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