

Nome: _____ N^o USP: _____
 (Colocar nome em todas as folhas!)

3^a Prova — 1^o semestre de 2016

1^a Questão (3,5 pontos)

Dada a estrutura da figura, determine:

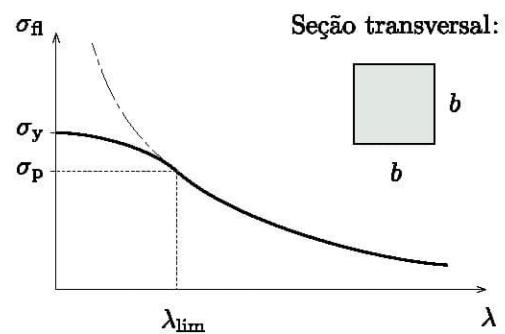
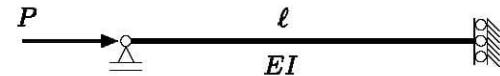
- a) o comprimento de flambagem ℓ_{fl} , mediante a integração da equação diferencial da linha elástica;
- b) o valor de b de modo que se tenha coeficiente de segurança $\gamma = 3$.

São dados:

$$\begin{aligned} P &= 5000 \text{ kgf} & E &= 700 \times 10^3 \text{ kgf/cm}^2 \\ \ell &= 100 \text{ cm} & \sigma_p &= 400 \text{ kgf/cm}^2 \\ \gamma &= 3 & \sigma_y &= 500 \text{ kgf/cm}^2 \end{aligned}$$

Considere:

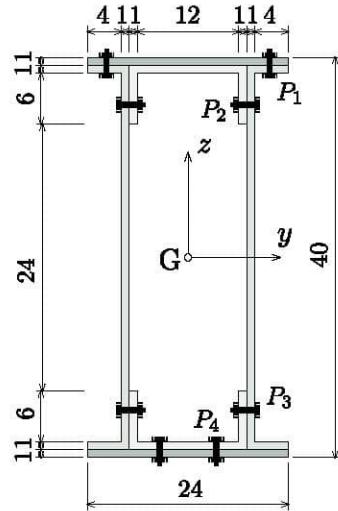
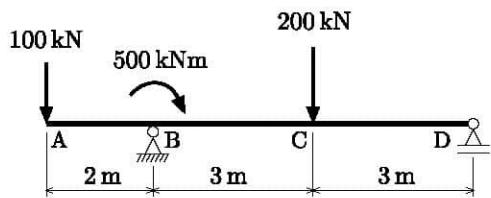
$$\sigma_{fl} = \begin{cases} \sigma_y - (\sigma_y - \sigma_p) \left(\frac{\lambda}{\lambda_{lim}} \right)^2, & \text{para } 0 \leq \lambda \leq \lambda_{lim}, \\ \frac{\pi^2 E}{\lambda^2}, & \text{para } \lambda_{lim} < \lambda. \end{cases}$$



2^a Questão (3,0 pontos)

Quatro perfis U e duas chapas retangulares são unidos por parafusos formando a viga com a seção indicada na figura. Determine os espaçamentos dos parafusos P_1 , P_2 , P_3 e P_4 no trecho mais solicitado da viga. Todos os parafusos têm diâmetro $d = 1,0$ cm e, seu material, tensão tangencial admissível $\tau = 12,0 \text{ kN/cm}^2$. Dado: $I_y = 47\,936 \text{ cm}^4$.

Seção Transversal: (cm)



3^a Questão (3,5 pontos)

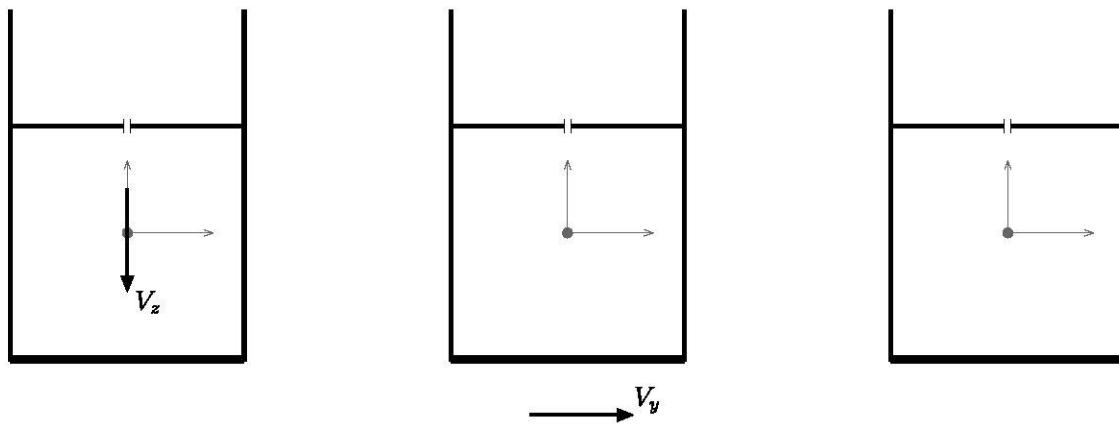
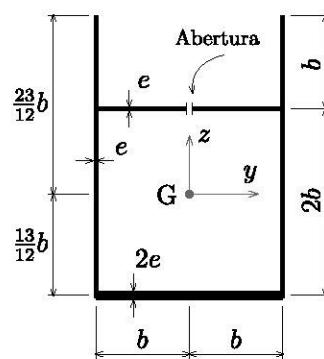
- a) Trace os diagramas dos fluxos de cisalhamento q_1 e q_2 gerados por forças cortantes nas direções dos eixos y e z atuando na seção da figura.
- b) Determine o centro de cisalhamento da seção e indique a sua posição.

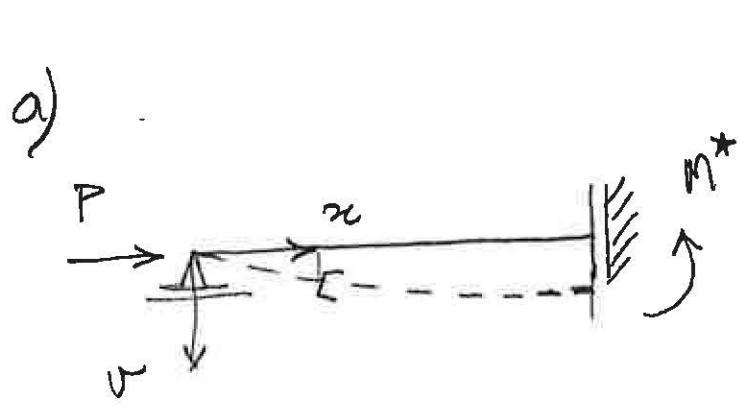
Seção transversal

$$A = 12eb$$

$$I_y = \frac{143}{12}eb^3$$

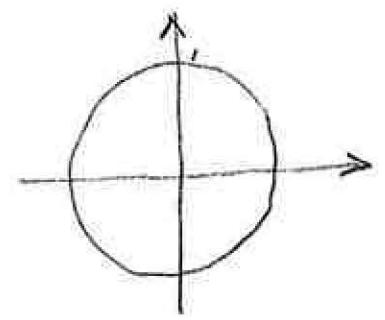
$$I_z = 8eb^3$$





$$V''(x) = -\frac{M(x)}{EI} = -\frac{P_1 V}{EI}$$

$$V'' + k^2 V = 0$$



$$V(x) = A_{\text{amplitude}} \sin \omega x + B_{\text{constant}}$$

C.C. $V(0) = 0 \Rightarrow B = 0$

$$V'(l) = 0 \Rightarrow A_k \cos k l = 0 \Rightarrow$$

$$k l = \frac{\pi}{2}$$

$$P_{fl} = \frac{\pi^2 EI}{(2l)^2}$$

$$J_{fl} = 2l$$

b) Admittendo $\lambda > \lambda_{\text{lim}} = \pi \sqrt{\frac{E}{J_F}} = 131,4$

$$J = \frac{\pi^2 E}{\lambda^2}$$

$$-\sigma = \frac{5000}{b^2} = \frac{1}{3} \frac{\pi^2 700000}{\frac{48 \times 100^2}{b^2}} \Rightarrow b^4 = 1042,16$$

$$b = 5,68 \text{ cm}$$

$$\lambda = 122 < \lambda_{\text{lim}}$$

$$\lambda = \frac{2l}{\sqrt{\frac{b^4}{12b^2}}} = \frac{4\sqrt{3}l}{b}$$

$$\frac{15000}{b^2} = 500 - 100 \cdot \left(\frac{400\sqrt{3}}{b \cdot 131,4} \right)^2 \Rightarrow 500b^2 = 15000 + 2780$$

$$b = 5,96 \text{ cm}$$

$$\lambda = 116 - \text{OK}$$

96P3Q2 (Ex. 2)

Sugestão para redução:

$$\uparrow \quad \left\{ \begin{array}{l} R_B + R_D = 500 \end{array} \right.$$

$$\text{CB} \quad \left\{ \begin{array}{l} -100 \times 2 + 500 + 200 \times 3 - R_D \times 6 = 0 \end{array} \right.$$

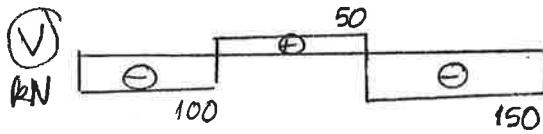
$$6R_D = 900 \Rightarrow R_D = 150 \quad | \\ R_B = 150$$

verif

$$\text{CD} \quad \left\{ \begin{array}{l} -800 \quad 900 \quad -600 \\ -100 \times 8 + 150 \times 6 - 200 \times 3 + 500 = 0 \end{array} \right. \checkmark$$

Força constante admisível no parafuso.

$$\bar{x} = \bar{z}_{\text{par}} \times A_{\text{par}} = 12 \times \pi \times 0,5^2 = 9,425 \text{ kN}$$

Parafusos

$$P_1) \quad \bar{s}_1 = (24 \times 1) \times 19,5 = 468 \text{ m}^3$$

$$\frac{24}{\text{---}} \quad q_1 = \frac{\sqrt{\bar{s}_1}}{I_y} = \frac{150 \times 468}{47936} = 1,464 \frac{\text{kN}}{\text{m}}$$

$$P_2) \quad \bar{s}_2 = (14 \times 1) \times 18,5 + 2[(1 \times 6) \times 15] \\ = 439 \text{ cm}^3$$

$$\frac{14}{\text{---}} \quad q_2 = \frac{\sqrt{\bar{s}_2}}{I_y} = \frac{150 \times 439}{47936} = 1374 \frac{\text{kN}}{\text{m}}$$

$$P_4) \quad \bar{s}_4 = \bar{s}_1$$

$$P_3) \quad \bar{s}_3 = \bar{s}_1 + \bar{s}_2 = 468 + 439 = 907 \text{ cm}^3$$

$$\frac{24}{\text{---}} \quad q_3 = \frac{150 \times 907}{47936} = 2,838 \frac{\text{kN}}{\text{m}}$$

m. de filérias
de parafuso

$$q_1 a_1 = 2\bar{x} \Rightarrow a_1 = \frac{2 \times 9,425}{1,464} = 12,9 \text{ cm}$$

$$nf = \frac{300}{12,9} = 23,3 \quad \text{admitindo } 2 \times 24 \text{ parafuso} \\ a_1^* = 12,5 \text{ cm}$$

$$q_2 a_2 = 2\bar{x} \Rightarrow a_2 = 13,7 \text{ cm}$$

$$nf = \frac{300}{13,7} = 21,9 \quad \text{admitindo } 2 \times 22 \text{ parafuso} \\ a_2^* = 13,6 \text{ cm}$$

$$a_1^* = a_2^* = 12,5 \text{ cm}$$

$$q_3 a_3 = 2\bar{x} \Rightarrow a_3 = 6,64 \text{ cm}$$

$$nf = \frac{300}{6,64} = 45,2 \quad \text{admitindo } 2 \times 46 \text{ parafuso} \\ a_3^* = 6,5 \text{ cm}$$

Soluções práticas considerando a fabricação do perfil.

$$P_1 : 48 \text{ parafusos} \quad a = 12,5 \text{ cm}$$

$$P_2 : 48 \quad " \quad a = 12,5 \text{ cm}$$

$$P_3 : 96 \quad " \quad a = 6,25 \text{ cm}$$

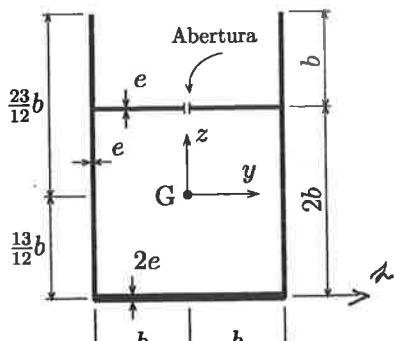
$$P_4 : 48 \quad " \quad a = 12,5 \text{ cm}$$

Seção transversal

$$A = 12eb$$

$$I_y = \frac{143}{12} eb^3$$

$$I_z = 8eb^3$$



$$A = 2(3eb + eb) + 4eb = 12eb$$

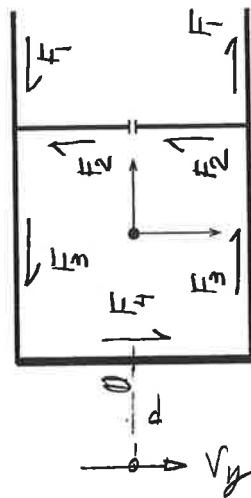
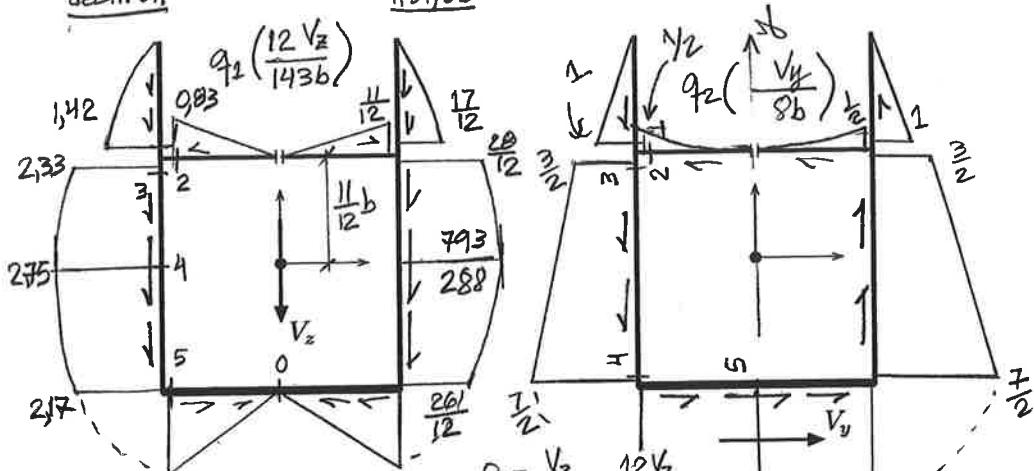
$$S_{\text{z}} = 2(3eb) \times \frac{3b}{2} + 2eb \times 2b = 13eb^2$$

$$t_G = \frac{S_{\text{z}}}{A} = \frac{13}{12} b$$

$$I_{\text{z}} = 2\left[\frac{e(3b)^3}{3} + eb(2b)^2\right] = 26eb^3$$

$$I_y = I_{\text{z}} - At_G^2 = (26 - \frac{169}{12})eb^3 = \frac{143}{12}eb^3$$

$$I_z = 2[3eb \times b^2] + \frac{3e(2b)^3}{12} = 8eb^3$$

decimalfração

$$\frac{V_z}{q_1} = \beta_z eb \left(\frac{b}{2} + \frac{11}{12}b \right) = \frac{17}{12} \beta_z eb^2 = 1.42$$

$$q_2 = \beta_z eb \frac{11}{12}b = \frac{11}{12} \beta_z eb^2 = 0.83$$

$$q_3 = q_1 + q_2 = \frac{28}{12} \beta_z eb^2 = 2.33$$

$$q_4 = q_3 + \beta_z \frac{e}{2} \left(\frac{11}{12}b \right)^2 = \frac{793}{288} \beta_z eb^2 = 2.75$$

$$q_5 = 2be \times \frac{13b}{12} = \frac{13}{6} \beta_z eb^2 = 2.17$$

$$\beta_y = \frac{V_y}{q_2} = \frac{V_y}{0.83} = \frac{V_y}{8eb^3}$$

$$q_1 = \beta_y eb^2$$

$$q_2 = \beta_y eb^2/2$$

$$q_3 = q_1 + q_2 = \frac{3}{2} \beta_y eb^2$$

$$q_4 = q_3 + 2eb^2 = \frac{7}{2} \beta_y eb^2$$

$$q_5 = q_4 + 2eb^2/2 = \frac{9}{2} \beta_y eb^2$$

Corre C_V está sobre o eixo de simetria barata lidar com $q_2 (V_y)$

$$F_1 = \frac{1}{2} \beta_y eb^3$$

$$F_2 = \frac{1}{3} \frac{1}{2} \beta_y eb^3 = \frac{1}{6} \beta_y eb^3$$

$$F_3 = \left(\frac{3}{2} + \frac{1}{2} \right) \frac{1}{2} \beta_y eb^3 = 5 \beta_y eb^3$$

$$F_4 = \frac{7}{2} \beta_y eb^3 \times \frac{1}{2} + \frac{2}{3} \times 1 \beta_y eb^3 \times 2 = \frac{25}{3} \beta_y eb^3$$

$$V_y \cdot d \equiv 2(F_1 + F_3)b + 2F_2 \times 2b$$

$$= \left(11 + \frac{2}{3} \right) \beta_y eb^4$$

$$= \frac{35}{3} \frac{V_y}{8eb^3} eb^4$$

$$d = \frac{35}{24} b = 1.458 b$$