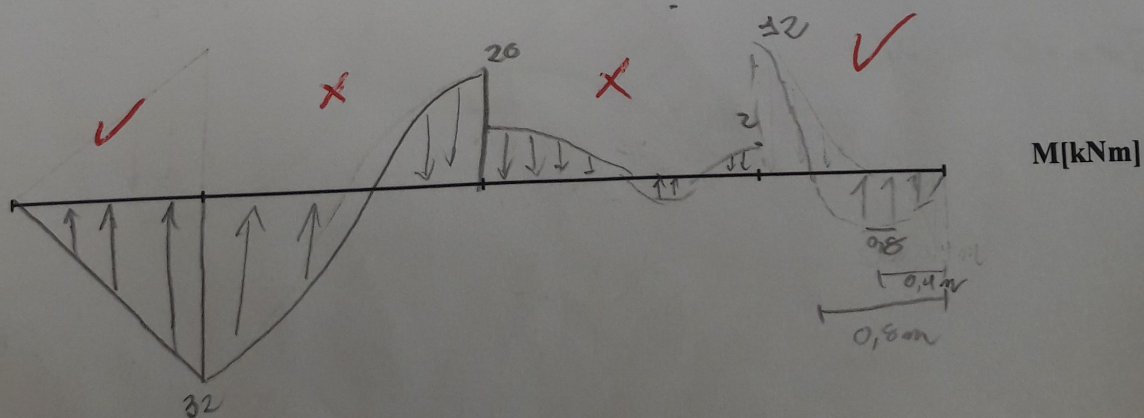
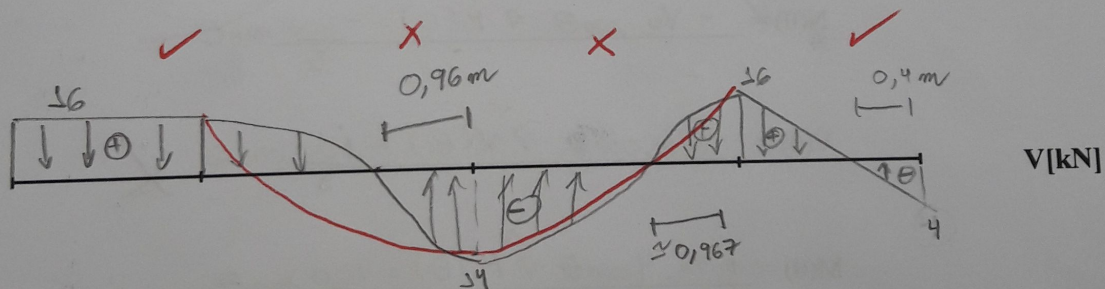
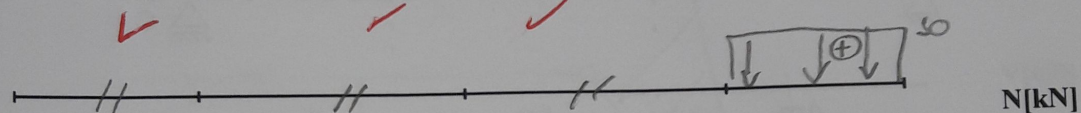
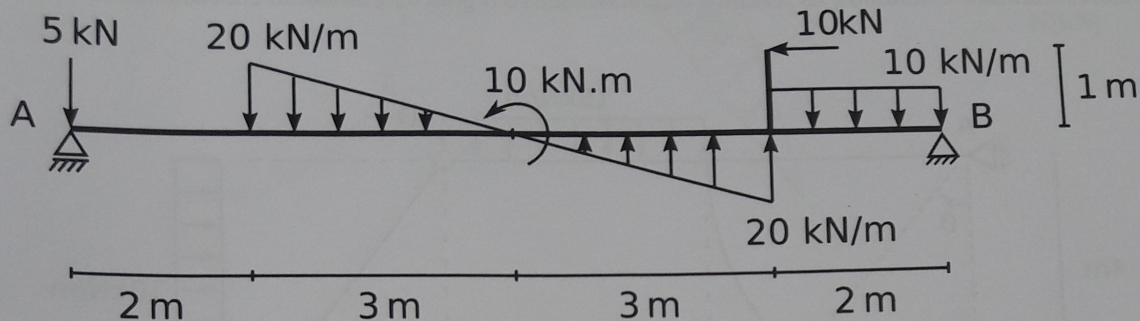
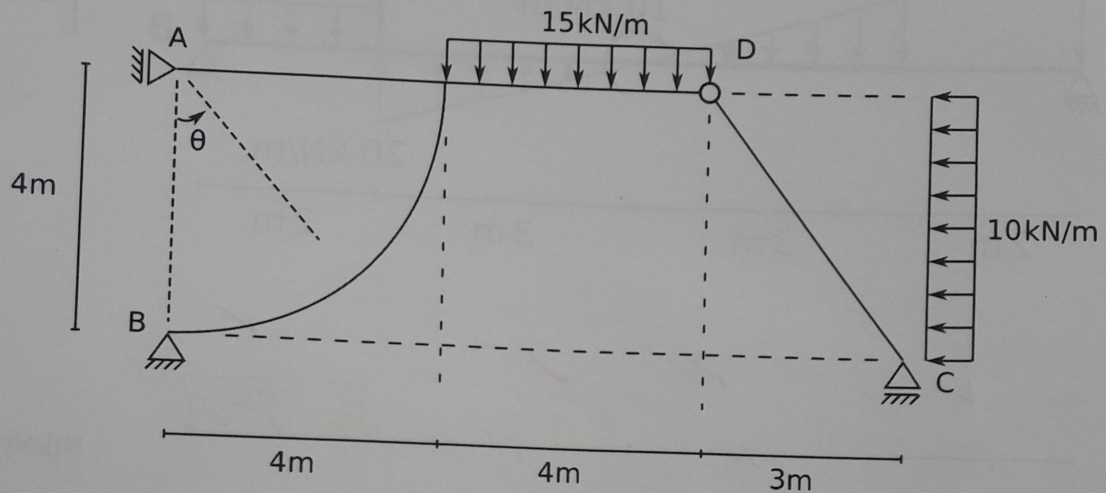


1ª Questão (5 pontos). Considere a estrutura representada na figura abaixo. Pede-se obter o diagrama de esforços normais (N em kN), de esforços cortantes (V em kN) e de momentos fletores (M em kNm). Devem ser obedecidos os critérios de sinal definidos em sala de aula. Indicar os valores máximos e mínimos, os ressaltos e concavidades e o grau do polinômio em cada trecho.



Questão (5 pontos). Considere a estrutura representada na figura abaixo. Pede-se obter o diagrama de esforços normais (N em kN), de esforços cortantes (V em kN) e de momentos fletores (M em kNm). Determine as equações para o trecho curvo (não é necessário desenhar os diagramas desse trecho). Devem ser obedecidos os critérios de sinal definidos em sala de aula. Indicar os valores máximos e mínimos, os ressaltos e concavidades e o grau do polinômio em cada trecho.



$$N(\theta) = -V_B \sin \theta \Rightarrow N(\theta) = -\frac{300}{3} \sin \theta$$

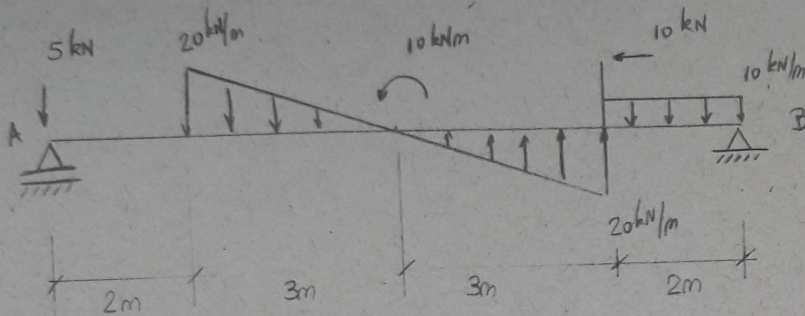
$$V(\theta) = +V_B \cos \theta \Rightarrow V(\theta) = \frac{300}{3} \cos \theta$$

$$M(\theta) = +V_B \cdot 4 \sin \theta \Rightarrow M(\theta) = \frac{400}{3} \sin \theta$$

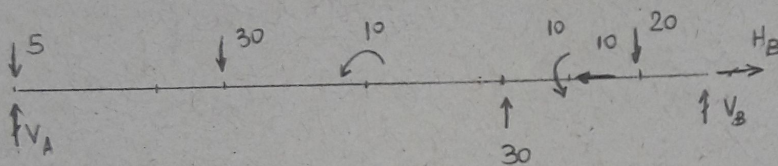
Confirmando...

$$\sum M_A = 0 \Rightarrow M_B$$

Q1



DCL

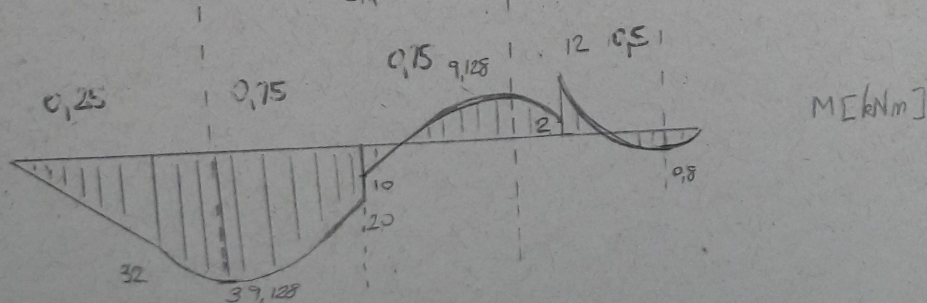
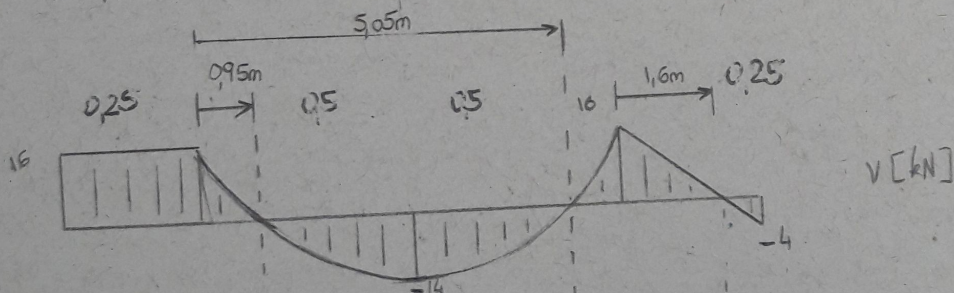
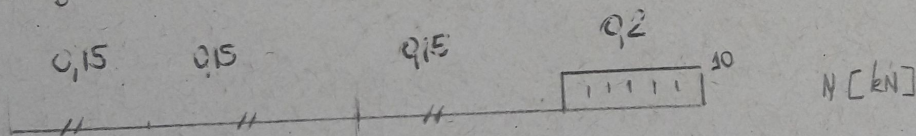


$$\sum F_H = 0 \quad -10 + H_B = 0 \Rightarrow \boxed{H_B = 10 \text{ kN}} \quad 0,2$$

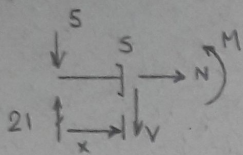
$$\sum F_V = 0 \quad V_A - 5 - 30 + 30 - 20 + V_B = 0 \Rightarrow V_A + V_B = 25$$

$$\sum M_A = 0 \quad -30 \cdot 3 + 10 + 30 \cdot 7 + 10 - 20 \cdot 9 + V_B \cdot 10 = 0$$

$$10V_B - 90 + 10 + 210 + 10 - 180 = 0 \Rightarrow 10V_B = 40 \Rightarrow \boxed{V_B = 4 \text{ kN}} \Rightarrow \boxed{V_A = 21 \text{ kN}} \quad 0,2 \quad 0,2$$



Trecho 1 ($0 < x < 2m$)

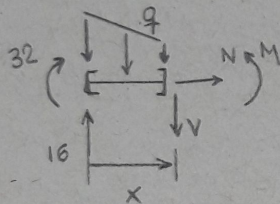


$$\sum F_H = 0 \quad N = 0$$

$$\sum F_V = 0 \quad V = 21 - 5 \Rightarrow V = 16 \text{ kN}$$

$$\sum M_S = 0 \quad M + 5x - 21x = 0 \Rightarrow M = 16x$$

Trecho 2 ($0 < x < 3m$)



$$q(x) = 20 - \frac{20}{3}x$$

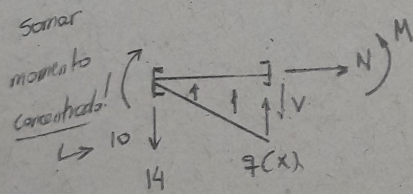
$$N = 0$$

$$\frac{dV}{dx} = -q(x) \Rightarrow \frac{dV}{dx} = \frac{20}{3}x - 20 \Rightarrow V(x) = \frac{10x^2}{3} - 20x + C_1 \Rightarrow \boxed{V(x) = \frac{10x^2}{3} - 20x + 16}$$

$$\frac{dM}{dx} = V(x) \Rightarrow \frac{dM}{dx} = \frac{10}{3}x^2 - 20x + 16 \Rightarrow M(x) = \frac{10x^3}{9} - 10x^2 + 16x + C_2 \Rightarrow \boxed{M(x) = \frac{10x^3}{9} - 10x^2 + 16x + 32}$$

$$\text{Zeros: } V(x) = 0 \Rightarrow x = \frac{20 \pm \sqrt{400 - 4(10/3)(16)}}{20/3} \Rightarrow \begin{cases} x = 0,95m \\ x = 5,05m \end{cases} \Rightarrow \begin{cases} M = 39,128 \text{ kNm} \\ M \text{ fora do intervalo} \end{cases}$$

Trecho 3 ($0 < x < 3m$)



$$N = 0$$

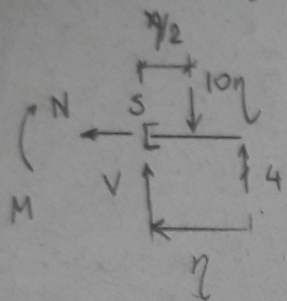
$$q(x) = -\frac{20}{3}x \quad (\text{positivo } \downarrow \text{ para baixo!})$$

$$\frac{dV}{dx} = -q(x) \Rightarrow \frac{dV}{dx} = \frac{20}{3}x \Rightarrow V(x) = \frac{10x^2}{3} + C_1 \Rightarrow \boxed{V(x) = \frac{10x^2}{3} - 14}$$

$$\frac{dM}{dx} = V(x) \Rightarrow \frac{dM}{dx} = \frac{10}{3}x^2 - 14 \Rightarrow M(x) = \frac{10x^3}{9} - 14x + C_2 \Rightarrow \boxed{M(x) = \frac{10x^3}{9} - 14x + 10}$$

$$\text{Zeros: } V(x) = 0 \quad x = \pm \sqrt{\frac{3 \cdot 14}{10}} \Rightarrow \begin{cases} x = -2,05m \\ x = 2,05m \end{cases} \Rightarrow \begin{cases} M \text{ fora do intervalo} \\ M = -9,128 \text{ kNm} \end{cases}$$

Trecho 4 ($0 < \eta < 2\text{m}$)



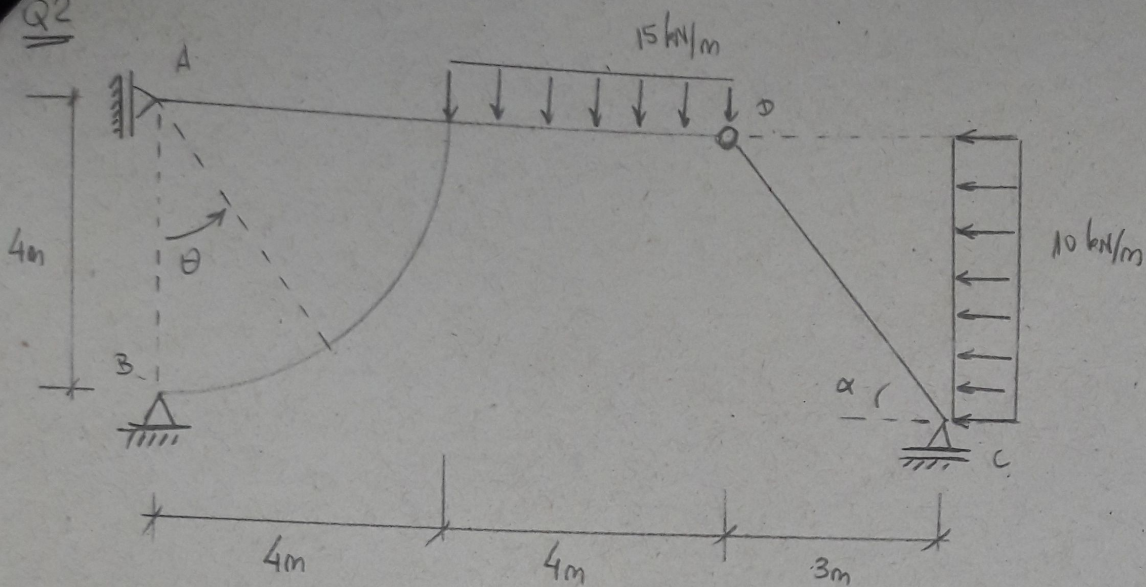
$$\sum F_H = 0 \quad \underline{N = 0}$$

$$\sum F_V = 0: V - 10\eta + 4 = 0 \Rightarrow \underline{V = 10\eta - 4}$$

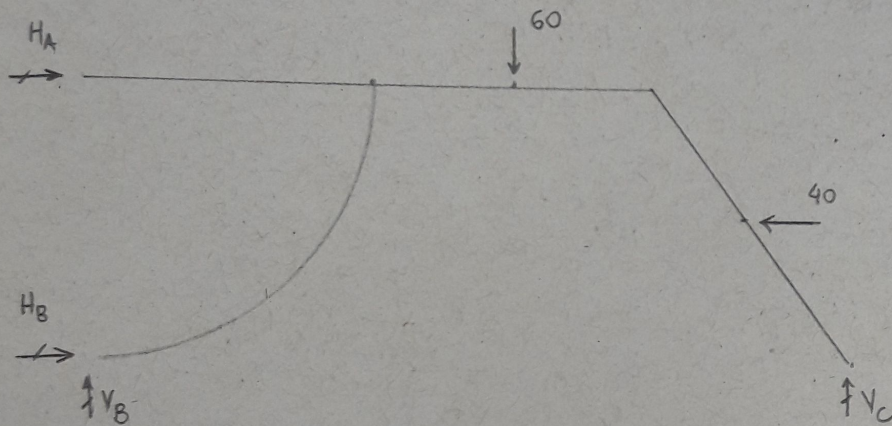
$$\sum M_S = 0: -M - 10\eta \cdot \eta/2 + 4\eta = 0 \Rightarrow \underline{M = -5\eta^2 + 4\eta}$$

$$\text{Zero: } V(\eta) = 0 \Rightarrow \eta = 0,4\text{m} \Rightarrow \underline{M = 0,8 \text{ kNm}}$$

Q2



$$\begin{aligned} \sin \alpha &= 0,8 \\ \cos \alpha &= 0,6 \end{aligned}$$

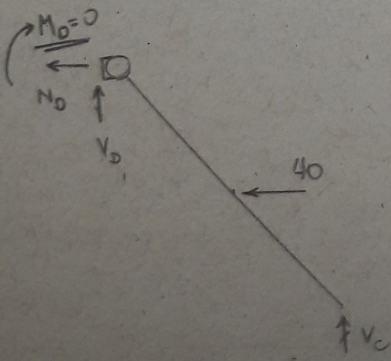


$$\sum F_H = 0 \quad H_A + H_B = 40$$

$$\sum F_V = 0 \quad V_B + V_C = 60$$

$$\sum M_B = 0 \quad -H_A \cdot 4 - 60 \cdot 6 + 40 \cdot 2 \quad V_C \cdot 11 = 0 \Rightarrow 11V_C - 4H_A = 280$$

- corte em D



$$\sum M_D = 0 \Rightarrow -40 \cdot 2 + V_C \cdot 3 = 0 \Rightarrow V_C = 80/3 \text{ kN} = \underline{\underline{26,67 \text{ kN}}}$$

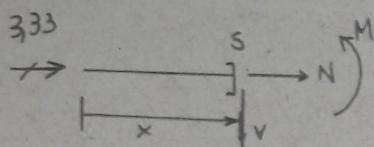
$$H_A = 3,33 \text{ kN} = 10/3$$

$$H_B = 36,67 \text{ kN} = 110/3$$

$$V_B = 33,33 \text{ kN} = 100/3$$

0,2 por arredond

Trecho 1 ($0 < x < 4\text{m}$)

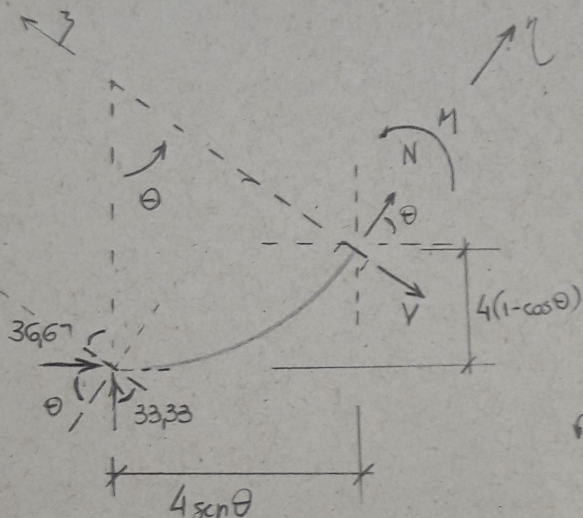


$$\sum F_x = 0 \quad N = -33,33 \text{ kN}$$

$$\sum F_y = 0 \quad V = 0$$

$$\sum M_S = 0 \quad M = 0$$

Trecho 2 ($0 < \theta < 90^\circ$)



$$\sum F_x = 0 \quad N + 33,33 \sin \theta + 36,67 \cos \theta = 0$$

$$N = -33,33 \sin \theta - 36,67 \cos \theta$$

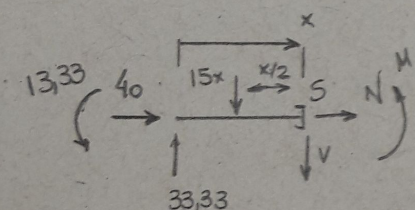
$$\sum F_y = 0 \quad V - 36,67 \sin \theta + 33,33 \cos \theta = 0$$

$$V = 33,33 \cos \theta - 36,67 \sin \theta$$

$$\sum M_S = 0 \quad M - 33,33 \cdot 4 \sin \theta + 36,67 \cdot 4 \cdot (1 - \cos \theta) = 0$$

$$M = 133,33 \sin \theta + 146,67 \cos \theta - 146,67$$

Trecho 3 ($0 < x < 4\text{m}$)



$$M^* = 33,33 \cdot 4 - 36,67 \cdot 4 = -13,33 \text{ kNm}$$

$$\sum F_x = 0 \quad N = -40 \text{ kN}$$

$$\sum F_y = 0 \quad 33,33 - 15x - V = 0 \Rightarrow V = 33,33 - 15x$$

$$\sum M_S = 0 \quad 13,33 + M - 33,33x + 15x \cdot \frac{x}{2} = 0 \Rightarrow M = -7,5x^2 + 33,33x - 13,33$$

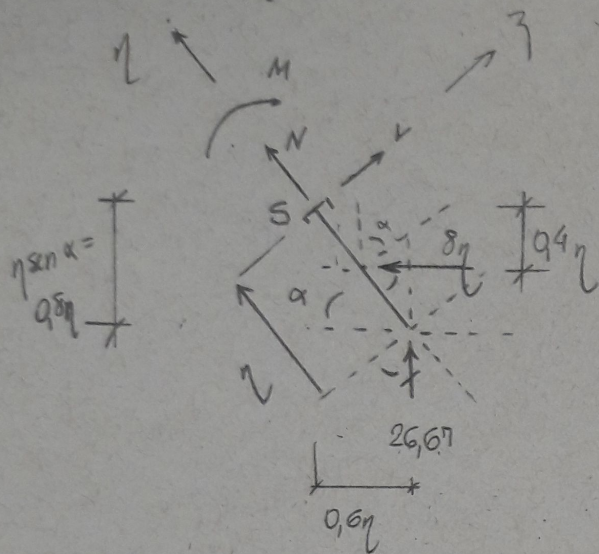
$$* N = -\frac{100}{3} \sin \theta - \frac{110}{3} \cos \theta$$

$$V = \frac{100}{3} \cos \theta - \frac{110}{3} \sin \theta$$

(em frações)

$$M = \frac{400}{3} \sin \theta + \frac{440}{3} \cos \theta - \frac{440}{3}$$

Trecho 4 ($0 < \eta < 5m$)



$$\sum F_y = 0: N + 26,67 \sin \alpha + 8\eta \cdot \cos \alpha = 0$$

$$N = -4,8\eta - 21,336$$

$$\sum F_x = 0: V + 26,67 \cos \alpha - 8\eta \sin \alpha = 0$$

$$V = 6,4\eta - 16$$

$$\sum M_B = 0: -M - 8\eta \cdot 0,4\eta + 26,67 \cdot 0,6\eta = 0$$

$$M = -3,2\eta^2 + 16\eta$$