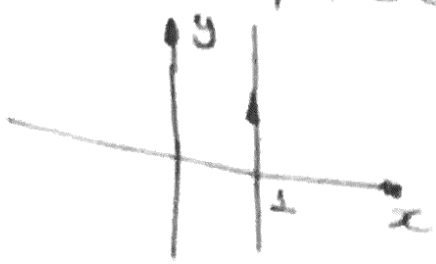
Olá!

Eu sou ALUNA de cálculo 2 de 2017
e Fiz a lista
Não tenho certeza de todas as
resoluções e respostas,
mas para os desesperados
melhor algo que nada!
Percebi que não tinha
Lista 1 - 2017 resolvida
aqui então aqui está
a minha ♡

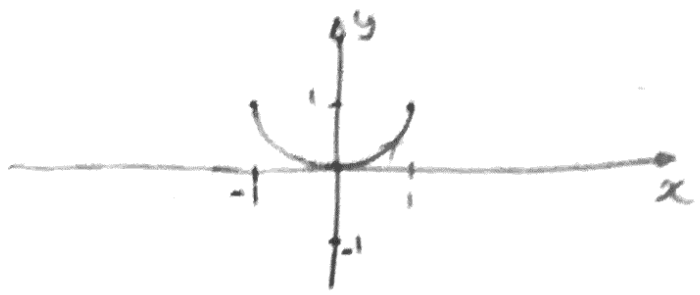
Ps. Desculpem a desorganização.
Alguns exercícios estão resolvidos
2 ou 3 vezes

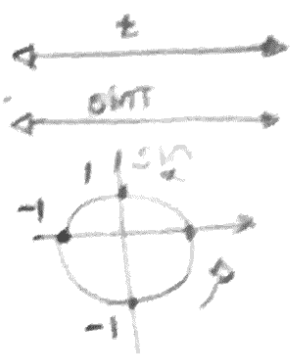
1.1

a) $\gamma(t) = (1, t)$ $t \in \mathbb{R}$



b) $\gamma(t) = (\sin t, \sin^2 t)$, $t \in \mathbb{R}$





c) $\gamma(t) = (\cos^2 t, \sin t)$ $0 \leq t \leq 2\pi$;

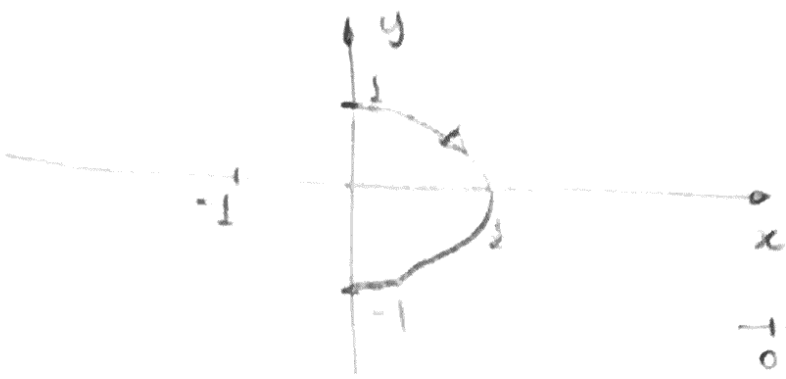
$x = \cos^2 t$

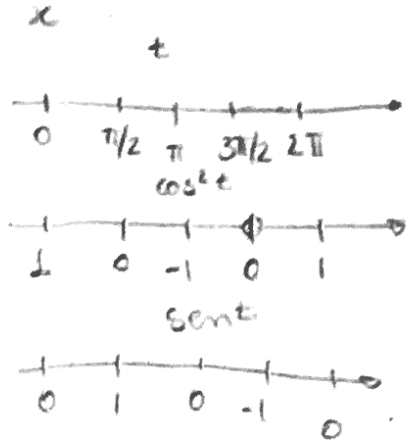
$y = \sin t$

$\cos^2 t + \sin^2 t = 1$

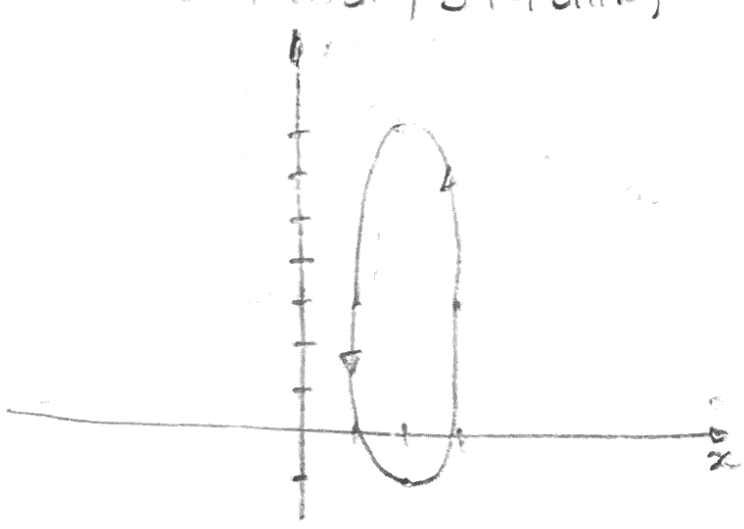
$x + y^2 = 1$

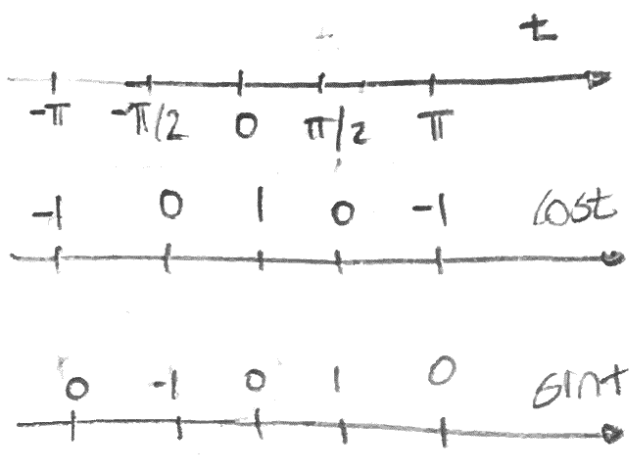
$x = 1 - y^2$

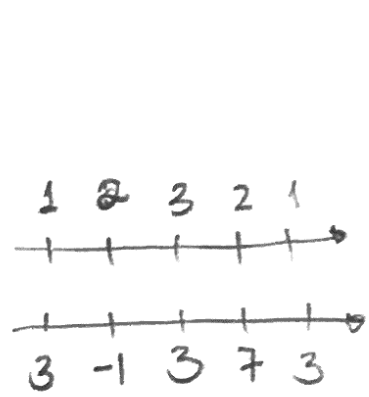




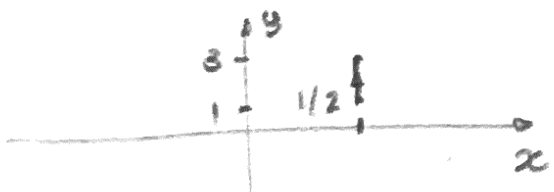
d) $\gamma(t) = (2 + \cos t, 3 + 4\sin t)$ $t \in [-\pi, \pi]$







e) $\gamma(t) = (\frac{1}{2}, 1 - t)$, $t \in [-2, 0]$



f) $\gamma(t) = (e^t \cos t, e^t \sin t)$ $t \geq 0$

$e^{2t}\cos^2 t + e^{2t}\sin^2 t = e^{2t}$

$x^2 + y^2 = e^{2t}$

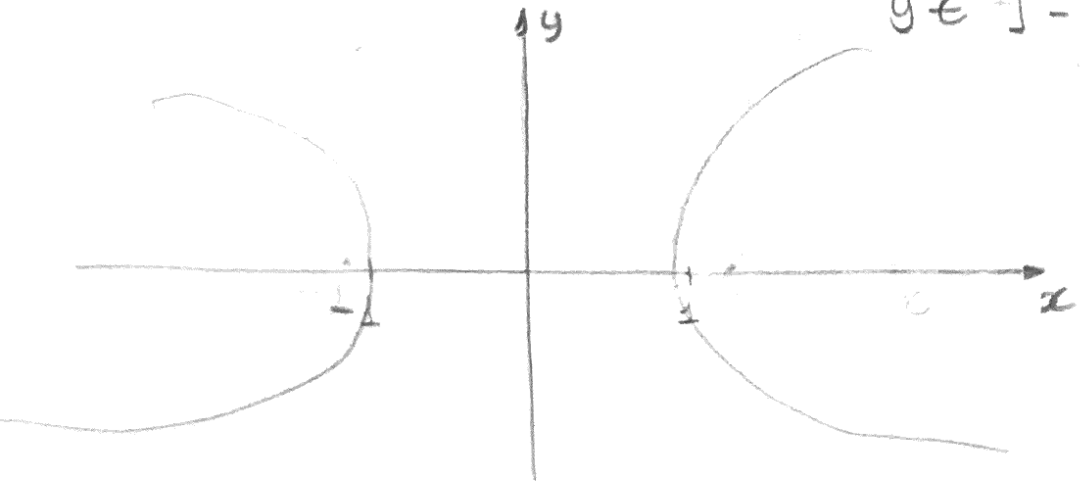
g) $\gamma(t) = (\sec t, \tan t)$, $t \in]-\pi/2, \pi/2[$

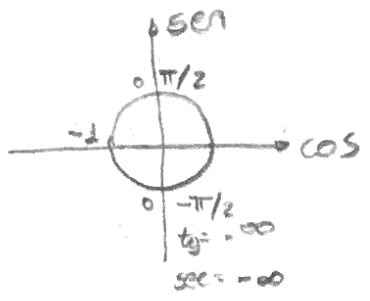
$x = \sec t$ $y = \tan t$

$x^2 - y^2 = 1$

$x \in]-\infty, \infty[$

$y \in]-\infty, \infty[$





$y = \sqrt{x^2 - 1}$

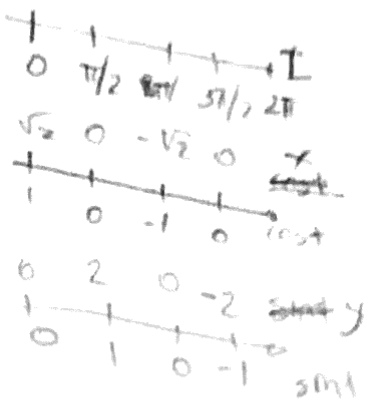
$y' = (x^2 - 1)^{-1/2} \cdot 2x = \frac{2x}{\sqrt{x^2 - 1}}$

$y'' = \frac{2}{\sqrt{x^2 - 1}} + 2x(x^2 - 1)^{-3/2} \cdot 2x$

$y'' = \frac{2}{\sqrt{x^2 - 1}} + \frac{4x^2}{\sqrt{(x^2 - 1)^3}}$

h) $\gamma(t) = (\sqrt{2}\cos t, 2\sin t), t \in \mathbb{R}$

$\sqrt{2}\cos t = x \quad 2\sin t = y$



$\frac{x^2}{2} + \frac{y^2}{4} = 1$

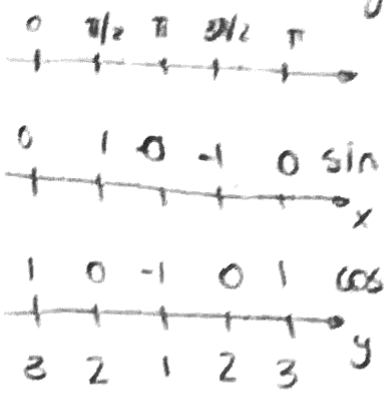
$x^2 + y^2/2 = 2$

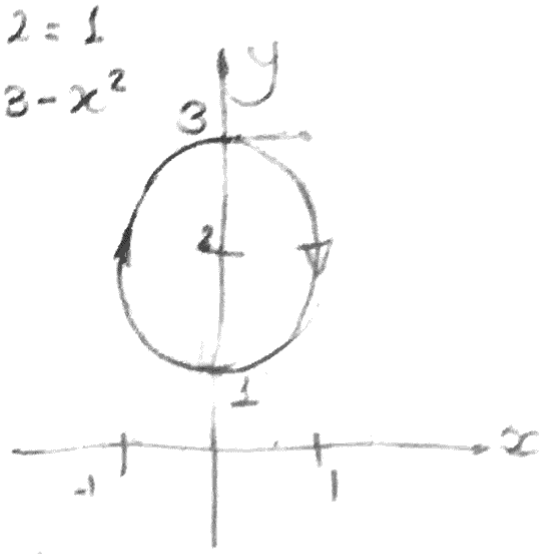
i) $\delta(t) = (\sin t, \cos 2t + 2), t \in \mathbb{R}$

$\sin t = x \quad \cos 2t + 2 = y$

$x^2 + y - 2 = 1$

$y = 3 - x^2$





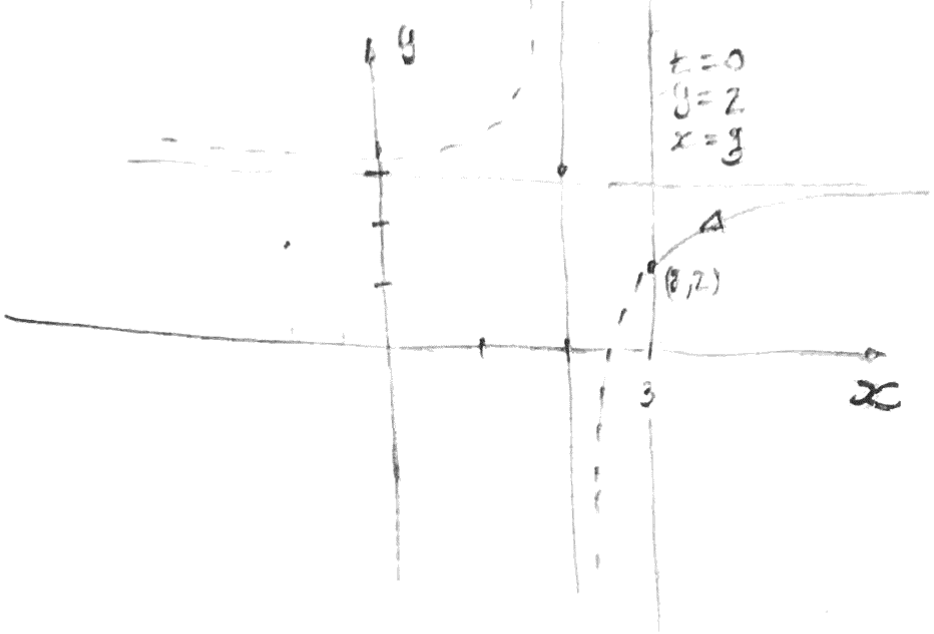
$\cos t, -2\sin t$

$t = 0$

$\cos t = 1, \quad -2 \cdot 0 = 0$

$1, 0$

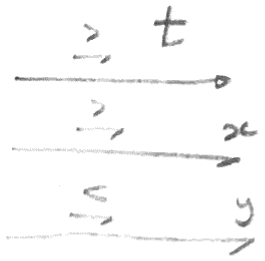
j) $\gamma(t) = (2 + e^{-t}, 3 - e^{t}), t \geq 0$



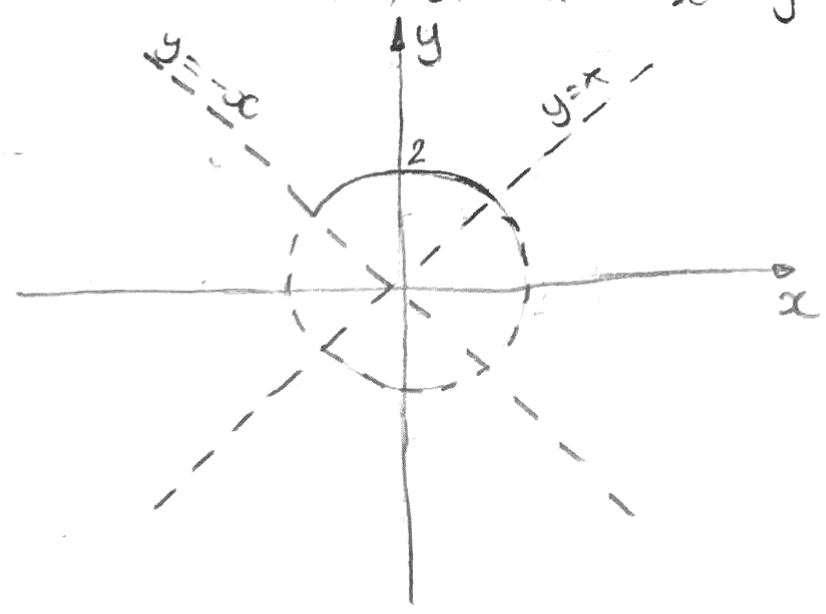
$x = 2 + e^{-t} \rightarrow x = 2 + \frac{1}{e^t} \rightarrow \frac{1}{x-2} = e^t$

$y = 3 - e^{t}$

$y = 3 - \frac{1}{x-2}$

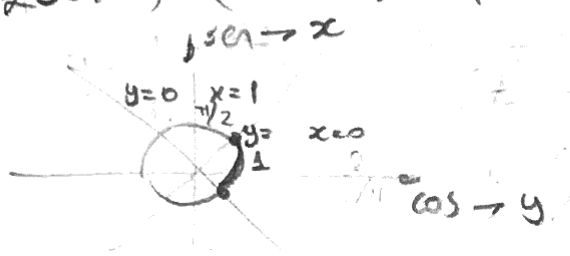


1.2 a) $C = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 4, \ y \geq -x \ e \ y \geq x\}$



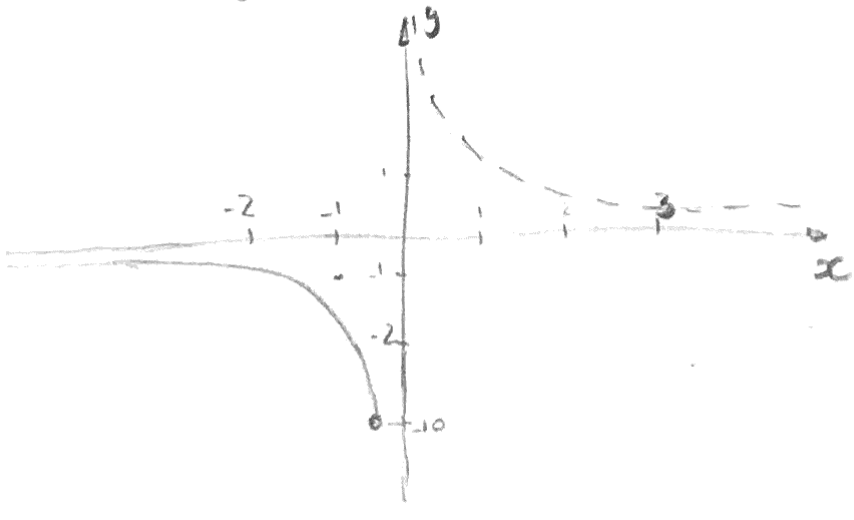
$\sin^2 t + \cos^2 t = 1$

$(2\sin t)^2 + (2\cos^2 t)^2 = 4$



$\gamma(t) = (2\sin t, 2\cos t), \ t \in [\pi/4, -\pi/4]$

b) $C = \{(x, y) \in \mathbb{R}^2 : xy = 1, \ x < 0 \ e \ y > -10\}$



$y = 1/x$

$x < 0 \quad y > -10$

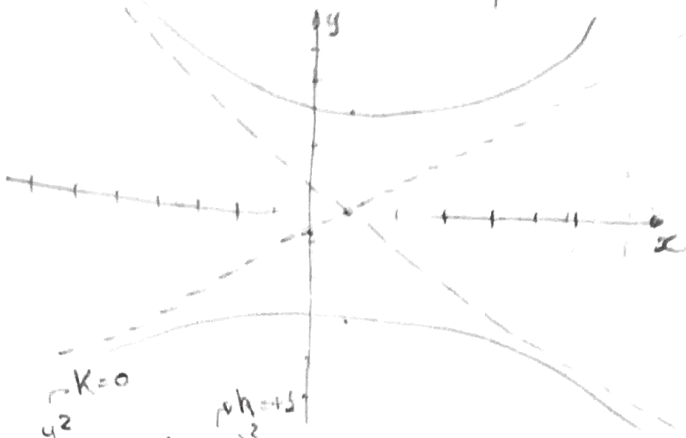
$y > -10 \qquad t < 0$

$\frac{1}{t} > -10$

$t < -1/10$

$\gamma(t) = (t, 1/t); \ t \in]-\infty, -1/10[$

c) $C = \{(x,y) \in \mathbb{R}^2 : \frac{y^2}{9} - \frac{(x-1)^2}{4} = 1 \text{ e } y < 0\}$



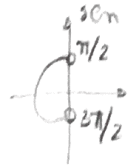
$\sec^2 t - \operatorname{tg}^2 t = 1$

$\left(\frac{y}{3}\right)^2 - \left(\frac{x-1}{2}\right)^2 = 1$

$\sec t = y/3 \qquad \operatorname{tg} t = \frac{x-1}{2}$

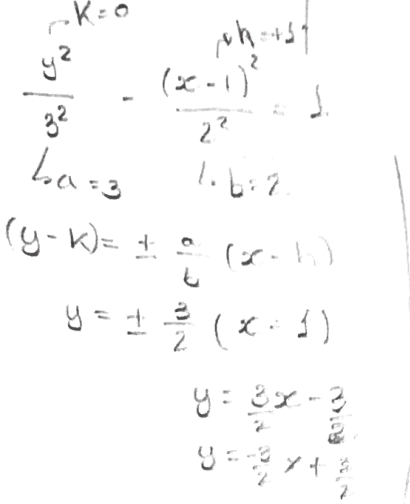
$y = 3\sec t \qquad x = 2\operatorname{tg} t + 1$

$3\sec t < 0$



$\sec t < 0 \rightarrow t \in]\pi/2, 3\pi/2[$

$\gamma(t) = (2\operatorname{tg} t + 1, 3\sec t), \; t \in]\pi/2, 3\pi/2[$



d) $C = \{(x,y) \in \mathbb{R}^2 : d((x,y), r) = d((x,y), P)\}$, sendo $P = (0,3)$ e r a reta $y = 4$

$C = \{(x,y) \in \mathbb{R}^2 : d((x,y),(x,4)) = d((x,y),(0,3))\}$

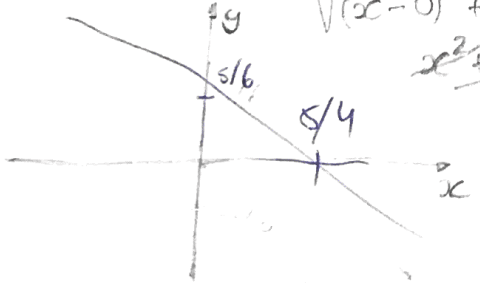
$\sqrt{0^2 + (y-4)^2} = \sqrt{x^2 + (y-3)^2}$

$x = y^2 - 8y + 16 - y^2 + 6y - 9$

$x = -2y + 7 \rightarrow y = -\frac{x}{2} + \frac{7}{2}$

$\gamma(t) = (t, -2t + 7), \; t \in \mathbb{R}$

e) $C = \{(x,y) \in \mathbb{R}^2 : d((x,y),P) + d((x,y),Q) = 10\}$, $P = (0,3)$, $Q = (-2,0)$



$\sqrt{(x-0)^2 + (y-3)^2} = \sqrt{(x+2)^2 + (y-0)^2}$

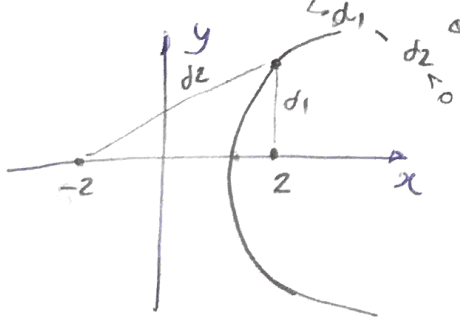
$\cancel{x^2} + \cancel{y^2} - 6y + 9 = \cancel{x^2} + 4x + 4 + \cancel{y^2}$

$-6y = 4x - 5$

$y = -4/6\, x + 5/6$

$\gamma(t) = (-3/2\, t + 5/6, t), \; t \in \mathbb{R}$

f) $C = \{(x,y) \in \mathbb{R}^2 : |d((x,y),P) - d((x,y),Q)| = 1, \; x > 0\}$, com $P = (2,0)$ e $Q = (-2,0)$



$|d((x,y),(2,0)) - d((x,y),(-2,0))| = 1$

$\sqrt{(x-2)^2 + (y)^2} - \sqrt{(x+2)^2 + y^2} = \pm 1$

$d((x,y),(2,0)) - d((x,y),(-2,0)) = -1$

$2x(t) = \cosh t$

$2y(t) = \operatorname{senh} t$

$\sqrt{5}$

$\gamma(t) = \left(\frac{\cosh t}{2}, \frac{\sqrt{5}\operatorname{senh} t}{2}\right), \; t \in \mathbb{R}$

$\sqrt{x^2 - 2)^2 + y^2} - \sqrt{(x+2)^2 + y^2} = -1$

$\sqrt{(x-2)^2 + y^2} = -1 + \sqrt{(x+2)^2 + y^2}$

$(x-2)^2 + y^2 = 1 - 2\sqrt{(x+2)^2 + y^2} + (x+2)^2 + y^2$

$2\sqrt{\;\;} = 8x + 1 \rightarrow 4(x+2)^2 + y^2 = 64x^2 + 16x + 1$

$\rightarrow 4x^2 + 16x + 16 + 4y^2 = 64x^2 + 16x + 1$

$60x^2 - 4y^2 = +15$

hipérbole!

1.3 $\gamma(t) = (\cos t, \sin t(\cos t)), t \in \mathbb{R}$

Não injetora p/ 2 valores de T → coordenadas iguais

$t=0$ e $T=2\pi$

Não é injetora → (0,0) por exemplo

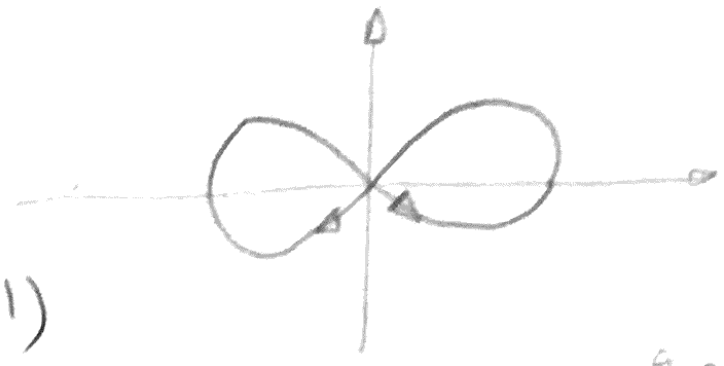
Mostre que tem 2 tg (0,0) e encontre suas equações

Faça esboço da curva

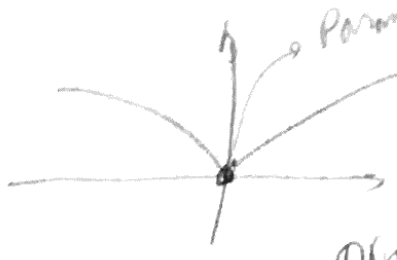
$\gamma(\pi/2) = (0,0) = \gamma(3\pi/2)$

$\gamma'(t) = (-\sin t, \cos 2t)$

$\gamma'(\pi/2) = (-1,-1)$ e $\gamma'(3\pi/2) = (1,-1)$



1.4 $f(x) = (\sqrt[3]{x})^2$



a) Mostre que f não é derivável em x=0

$$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{\sqrt[3]{x^2}}{x} = \lim_{x\to 0} \frac{1}{\sqrt[3]{x}}$$

Direita e esquerda não convergem } Não existe derivada

b) $\gamma(t) = (t, \sqrt[3]{t^2})$, $\gamma'(0) = \nexists$ → Não é derivável

$y = (\sqrt[3]{x})^2 \rightarrow y^3 = x^2$

$\gamma(t) = (t^2, t^3)$, $t \in \mathbb{R}$ Parametrização derivável

$\gamma'(t) = (2t, 3t^2)$

$\gamma'(0) = (0,0)$

↳ vetor velocidade nulo

1.3 $\gamma(t) = (\cos t, \sin t \cos t)$

2 tangentes em (0,0)

Equações? Esboço!

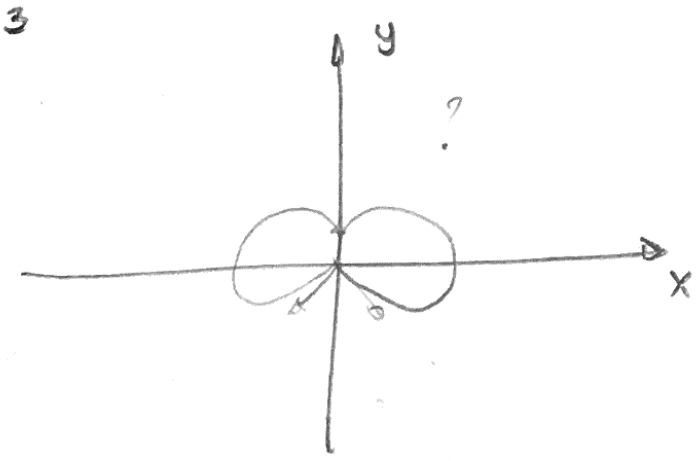
Não é injetora → $t=0$ e $t=2\pi$,

$\gamma(t) = (0,0)$

$x = \cos t \rightarrow \sin t = \sqrt{1-\cos^2 t}$

$y = \sin t \cos t = \sqrt{1-x^2}\, x$

$y^2 = x - x^3$



$\gamma'(t) = (-\sin t, \cos^2 t - \sin^2 t) = (-\sin t, \cos 2t)$

$\gamma'(0) = (-0, 1)$ $\gamma'(2\pi) = (0, 1)$

$\gamma'(\pi/2) = (-1,-1)$ $\gamma'(3\pi/2) = (+1,-1)$

$\gamma(\pi/2) = (0,0)$ $\gamma(3\pi/2) = (0,0)$

1.4 $f(x) = (\sqrt[3]{x})^2$

a) $$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{\sqrt[3]{x^2}-0}{x} = \lim_{x\to 0} \frac{\sqrt[3]{x^2}}{x}$$

direita $\lim_{x\to 0^+} \frac{\sqrt[3]{x^2}}{x} > 0$

esquerda $\lim_{x\to 0^-} \frac{\sqrt[3]{x^2}}{x} < 0$

b) ~~$\gamma: \mathbb{R} \to \mathbb{R}^2$ $\gamma = (x, \sqrt[3]{x^2})$ → $\gamma'(0)$ não é derivável~~

$\gamma: \mathbb{R} \to \mathbb{R}^2$ $\gamma = (x^3, x^2)$, $x \in \mathbb{R}$

$\gamma(0) = (0,0)$ → velocidade é nula em 0

1.5 $\|\gamma(t)\| = C$ | Limites

$t \in I$

$\gamma : I \to \mathbb{R}^2$

$\gamma(k) = (f(x), g(x))$

$\sqrt{(f(x))^2 + (g(x))^2} =$ constante

$x(t)^2 + y^2(t) = C^2$

$2x(t)x'(t) + 2y(t)y'(t) = 0$

$x(t)x'(t) + y(t)y'(t) = 0$

Produto escalar $\langle \gamma, \gamma' \rangle = 0$ → vetor velocidade é sempre $\perp$ à posição

$\gamma'(t) = (x'(t), y'(t))$

$\gamma(t) = (x(t), y(t))$

Velocidade é constante pois a distância é constante

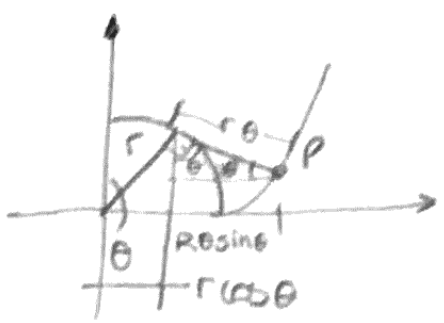
$\gamma(k)$ representa um círculo. → Em um círculo a trajetória é sempre $\perp$ à velocidade

1.6

$x = r(\cos\theta + \theta\sin\theta)$

$y = r(\sin\theta - \theta\cos\theta)$

$\gamma(\theta, r) = (r(\cos\theta + \theta\sin\theta), r(\sin\theta - \theta\cos\theta))$



$P(x) = r\cos\theta + r\theta\sin\theta$

$P(y) = r\sin\theta + r\theta\cos\theta$

1.7. Eq paramétrica para a cardioide $x(\theta) = r\theta - r\sin\theta$ $y(\theta) = r - r\cos\theta$

$P(x) = r\cos\theta + \theta r$

$P(y) = r\cos(\theta + \pi) + r$

$\gamma(\theta) = (r(\sin\theta + \theta), r(\cos(\theta + \pi) + 1))$

2 |g| $\lim_{(x,y)\to(0,0)} \frac{xy}{x^3 - y}$

$\gamma_1(t) = (t, 0) \to f(\gamma_1(t)) = 0$

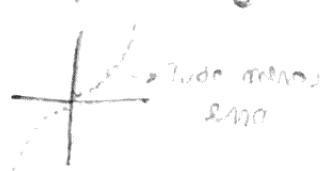
$\to \lim_{t\to 0} f(\gamma_1(t)) = 0$

$\gamma_2(t) = (0, t) \to \lim_{t=0} f(\gamma_2(t)) = 0$

$\gamma_3(t) = (at, bt) \to f(\gamma(t)) = \frac{abt^2}{a^3t^3 - bt}$

$f(\gamma(t)) = \frac{abt}{a^3t^2 - b}$

$\lim_{t\to 0} f(\gamma(t)) = 0$

Df = 

Curvas de nível

$f(x,y) = C \longleftrightarrow \frac{xy}{x^3 - y} = C \longrightarrow y = \frac{cx^3}{c + x}$

$C = 1 \to y = \frac{x^3}{1+x} \to \eta_1(t) = (t, \frac{t^3}{1+t}),\ t > -1$

$f(\eta_1(t)) = C = 1 \longrightarrow \lim_{t\to 0} f(\eta_1(t)) = 1$

↳ Pedaço da curva de nível 1 Constante

$C = 2 \to y = \frac{2x^3}{x+2} \quad \eta_2(t) = \left(t, \frac{2t^3}{2t}\right) t > 2$

$f(\eta_2(t)) = 2 \to \lim_{t=0} f(\eta_2(t)) = 2$

$\neq$ Valores p/ mesmo lim

Um não existe

1.5. $\|\gamma(t)\| = C$, $t \in I$, $\gamma: I \to \mathbb{R}^2$ — C/ mesmo

$\gamma(k) = (f(x), g(x))$

$\sqrt{(f(x))^2 + (g(x))^2}$ = constante

$x(t)^2 + y^2(t) = C^2$

$2x(t)x'(t) + 2y(t)y'(t) = 0$

$x(t)x'(t) + y(t)y'(t) = 0$

Produto escalar $\langle \gamma, \gamma' \rangle = 0$ → velocidade é sempre ⊥ à posição

$\gamma'(t) = (x'(t), y'(t))$

$\gamma(t) = (x(t), y(t))$

Velocidade é constante pois a distância é constante

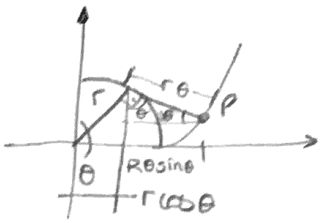
$\gamma(k)$ representa um círculo. → Em um círculo a trajetória é sempre ⊥ à velocidade

1.6

$x = r(\cos\theta + \theta\sin\theta)$

$y = r(\sin\theta - \theta\cos\theta)$

$\gamma(\theta, r) = (r(\cos\theta + \theta\sin\theta), r(\sin\theta - \theta\cos\theta))$



$P(x) = r\cos\theta + r\theta\sin\theta$

$P(y) = r\sin\theta + r\theta\cos\theta$

1.7. Eq paramétrica para a curva $x(\theta) = r\theta - r\sin\theta$ $\quad y(\theta) = r - r\cos\theta$

$P(x) = r\sin\theta + \theta r$

$P(y) = r\cos(\theta + \pi) + r$

$\gamma(\theta) = (r(\sin\theta + \theta), r(\cos(\theta + \pi) + 1))$

2. (g) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^3 - y}$

Df = $\mathbb{R}^2$ tudo menos zero

$\gamma_1(t) = (t, 0) \to f(\gamma_1(t)) = 0 \to \lim_{t\to 0} f(\gamma_1(t)) = 0$

$\gamma_2(t) = (0, t) \to \lim_{t=0} f(\gamma_2(t)) = 0$

$\gamma_3(t) = (at, bt) \to f(\gamma(t)) = \frac{abt^2}{a^3t^3 - bt}$

$f(\gamma(t)) = \frac{abt}{a^3t^2 - b}$

$\lim_{t\to 0} f(\gamma(t)) = 0$

Curvas de nível

$f(x,y) = c \longleftrightarrow \frac{xy}{x^3 - y} = c \longrightarrow y = cx^3/(c + x)$

$c = 1 \to y = x^3/1 + x \to \eta_1(t) = (t, t^3/1 + t)$, $t > -1$

$f(\eta_1(t)) = c = 1$ ⟶ $\lim_{t\to 0} f(\eta_1(t)) = 1$

↳ pedaço da curva de nível 1 constante

$c = 2 \to y = 2x^3/x + 2 \quad \eta_2(t) = \left(t, 2t^3/2t\right)$ $t > 2$

$f(\eta_2(t)) = 2 \to \lim_{t=0} f(\eta_2(t)) = 2$

≠ Valores p/ mesmo lim

Um não existe

2.1 l) $\lim_{(x,y)\to(0,0)} \frac{x^3+\operatorname{sen}(x^2+y^2)}{y^4+\operatorname{sen}(x^2+y^2)} = \lim_{(x,y)\to(0,0)} \frac{\frac{x^3}{x^2+y^2}+\frac{\operatorname{sen}(x^2+y^2)}{x^2+y^2}}{\frac{y^4}{x^2+y^2}+\frac{\operatorname{sen}(x^2+y^2)}{x^2+y^2}} = 1$

($\frac{x^3}{x^2+y^2} \to 0$; $\frac{\operatorname{sen}(x^2+y^2)}{x^2+y^2} \to 1$; $\frac{y^4}{x^2+y^2} \to 0$; $\frac{\operatorname{sen}(x^2+y^2)}{x^2+y^2} \to 1$)

$\lim_{(x,y)\to(0,0)}$ x: $\frac{x^2}{x^2+y^2} = 0$

$\frac{x^2}{x^2+y^2}$ limitado ≤ 1 pois: $0 \leq x^2 \leq x^2+y^2$

$\to 0$

$\lim_{(x,y)\to(0,0)}$ y^2: $\frac{y^2}{x^2+y^2} = 0$

$\to 0$

$\frac{y^2}{x^2+y^2}$ limitado ≤ 1

2.2 c) $\lim_{(x,y)\to(0,0)} x^2 \ln(3x^2+y^2)\operatorname{arctg}\left(\frac{1}{y^2-x^2}\right)$ — entre $-\pi/2$ e $\pi/2$ → Limitada

$\lim_{(x,y)} \frac{x^2}{(3x^2+y^2)} \cdot \underbrace{(3x^2+y^2)\ln(3x^2+y^2)}_{0^*} \operatorname{arctg}\left(\frac{1}{y^2-x^2}\right) = 0$

↳ Limitado

$0 \leq x^2 \leq 3x^2 \leq 3x^2+y^2$

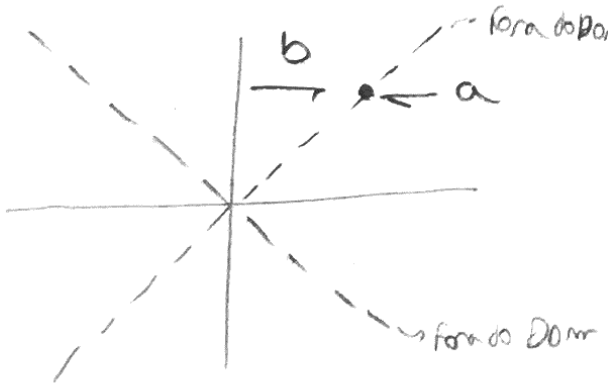
→ $g(t) = t\ln(t)$

$\lim_{t\to 0} g(t) = \lim_{t\to 0^+} \frac{1}{t}\cdot\left(\frac{-1}{t^2}\right) = \lim_{t\to 0^+} -t = 0^*$

$\lim_{(x,y)\to(0,0)} g(3x^2+y^2) = 0$

→ $\lim_{(x,y)\to(0,0)} 3x^2+y^2 = 0^*$

d) $\lim_{(x,y)\to(1,1)} x^2\ln(3x^2+y^2)\operatorname{arctg}\left(\frac{1}{y^2-x^2}\right)$



ⓐ $\begin{cases} \gamma_1(t) = (t,1)\ t>1 \\ \lim_{t\to 1^+} f(\gamma_1(t)) = \ln(4)\cdot\pi/2 \end{cases}$

ⓑ $\begin{cases} \gamma_2(t) = (t,1),\ t<1 \\ \lim_{t\to 1^-} f(\gamma_2(t)) = (\ln 4)\,\pi/2 \end{cases}$

ⓐ $x>y$ → arctg vai p/ $+\infty$ e → $\pi/2$

ⓑ $x<y$ → $-\infty \to -\pi/2$

e deram valores ≠ limite não existe

1.8e) $f(x,y) = \frac{x}{y^2}$

$\operatorname{Dom} f = \{(x,y)\in\mathbb{R}^2;\ y>0\}$ → poucos pontos onde é possível calcular

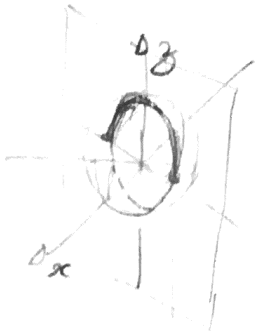
$\gamma(t) = (\cos t, \cos t, \sqrt{2}\operatorname{sen} t)$, $t \in [0, \pi/2)$

$x^2(t) + y^2(t) + z^2(t) = 2\cos^2 t + 2\operatorname{sen}^2 t = 2$

$\operatorname{Im}\gamma \subset S^2(\sqrt{2})$

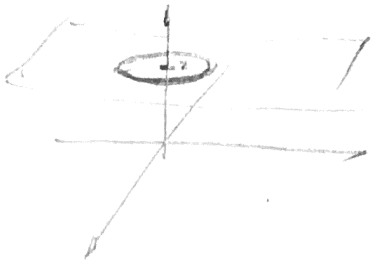
esfera ↳ raio

$x(t) = y(t) \rightarrow \operatorname{Im}\gamma \subset \pi_1$ (plano) $\pi_1: x - y = 0 \rightarrow x = y$



z sempre ≥ 0

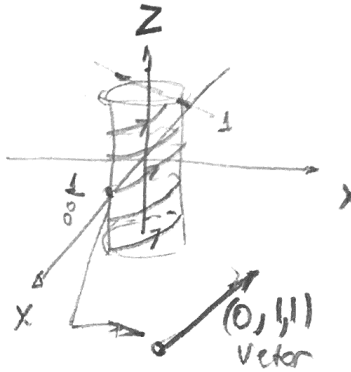
1.17. b) $\gamma(t) = (\cos t, \operatorname{sen} t, 2)$



b') $\gamma(t) = (\cos t, \operatorname{sen} t, t) \rightarrow \gamma'(t) = (-\operatorname{sen} t, \cos t, 1)$

↳ z sobe sempre

$x^2(t) + y^2(t) = 1$

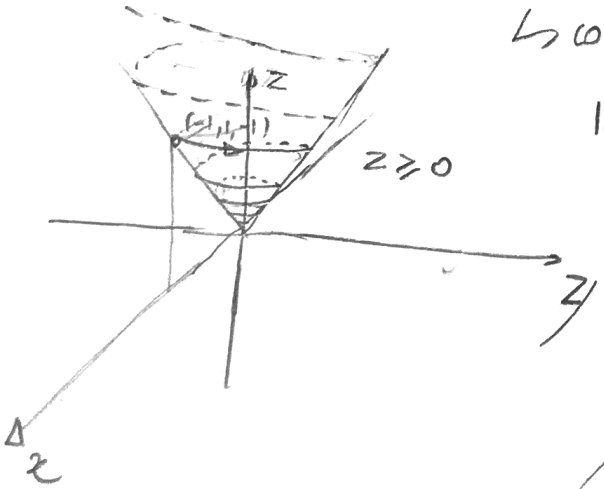


c) $\gamma(t) = (e^{-t}\cos t, e^{-t}\operatorname{sen} t, e^{-t})$, $t \geq 0$

$x(t)^2 + y(t)^2 = e^{-2t} = z(t)^2$

$\operatorname{Im}\gamma \subset S = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 - z^2 = 0\}$

↳ cone



$\|\gamma(t)\| = \sqrt{2e^{-2t}}$

↳ valor de t, mais próximo da origem

$e^{-t}\cos t, e^{-t}\operatorname{sen} t$

↳ sentido anti-horário se aproxima da origem

$\gamma'(t) = (-e^{-t}(\cos t + \operatorname{sen} t), e^{-t}(-\operatorname{sen} t + \cos t))$

$(-1, 1, -1)$

a) $c=1$, $c=2$, $c=3$ $\Big\}$ $f^{-1}(c) = \{(x,y) \in Df : f(x,y) = c\}$

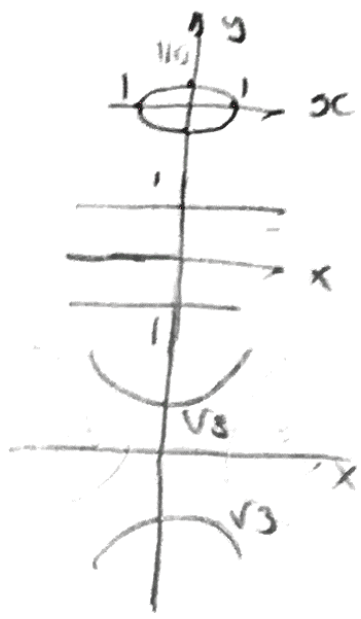
↳ inversa de f

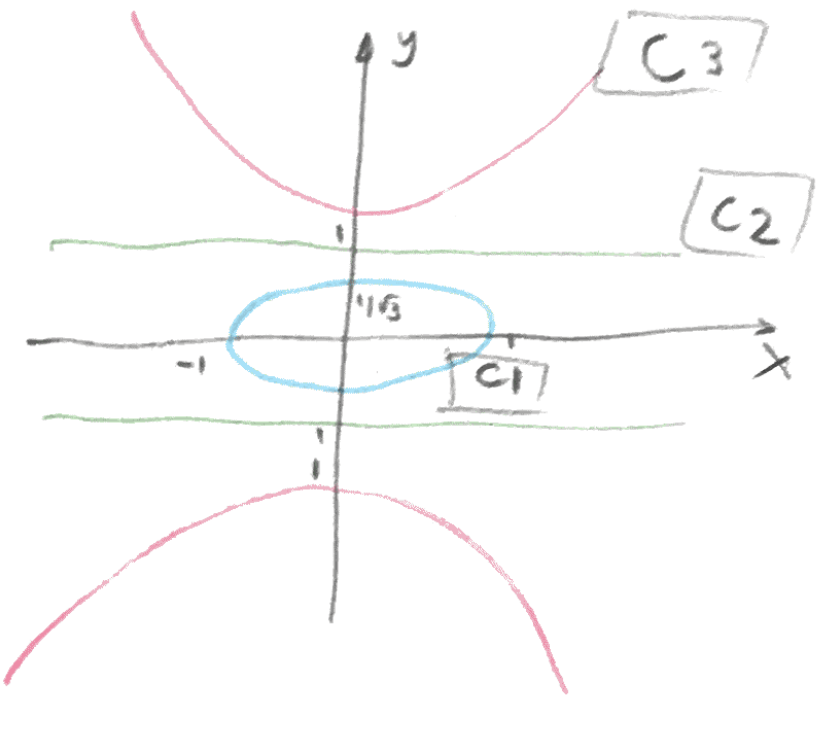
$$\frac{2x^2+4y^2}{x^2+y^2+1} = c \rightarrow (2-c)x^2 + (4-c)y^2 = c$$

$c=1 \rightarrow x^2+3y^2=1$

$c=2 \rightarrow y^2=1$

$c=3 \rightarrow -x^2+y^2=3$





$\rightarrow c \in [0,1]$

$c=1 : x^2+3y^2=1 \rightarrow (x^2)+(\sqrt{3}y)^2 = 1$

$x(t) = \cos t$

$\sqrt{3}\,y(t) = \operatorname{sen} t \rightarrow y(t) = \operatorname{sen} t/\sqrt{3}$

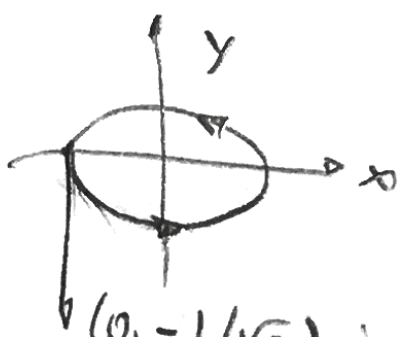
$\gamma_1(t) = (\cos t, \operatorname{sen} t/\sqrt{3})$

$T \in [0, 2\pi]$

$(-1,0)$, $\gamma_1(t) = (-1,0) \rightarrow t_0 = \pi$

$\gamma_1'(t) = (-\operatorname{sen} t, \cos t/\sqrt{3})$

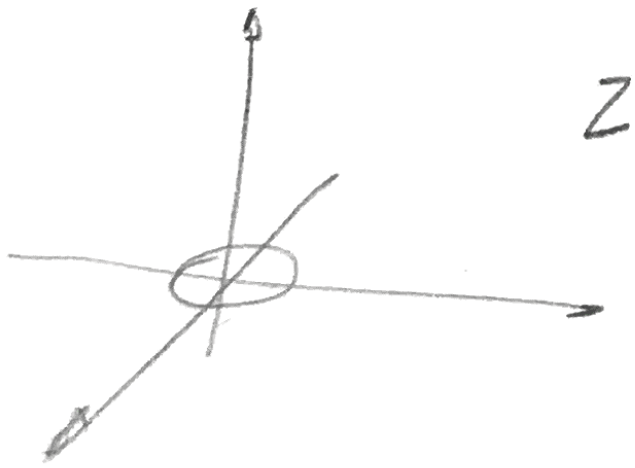
$\gamma_1'(\pi) = (0, -1/\sqrt{3})$



$: [0, 2\pi] \rightarrow \mathbb{R}^3$

$T(t) = (\operatorname{sen} t, \cos t, z(t))$ tg Im f C Gr f

Encontrar vetor tg a T em $T(\pi/3)$



$Z(t) = f(x(t), y(t))$

$= \dfrac{2\operatorname{sen}^2 t + 4\cos^2 t}{2} = \operatorname{sen}^2 + 2\cos^2 t = \boxed{1+\cos^2}$

$T(t) = (\operatorname{sen} t, \cos t, 1+\cos^2 t)$

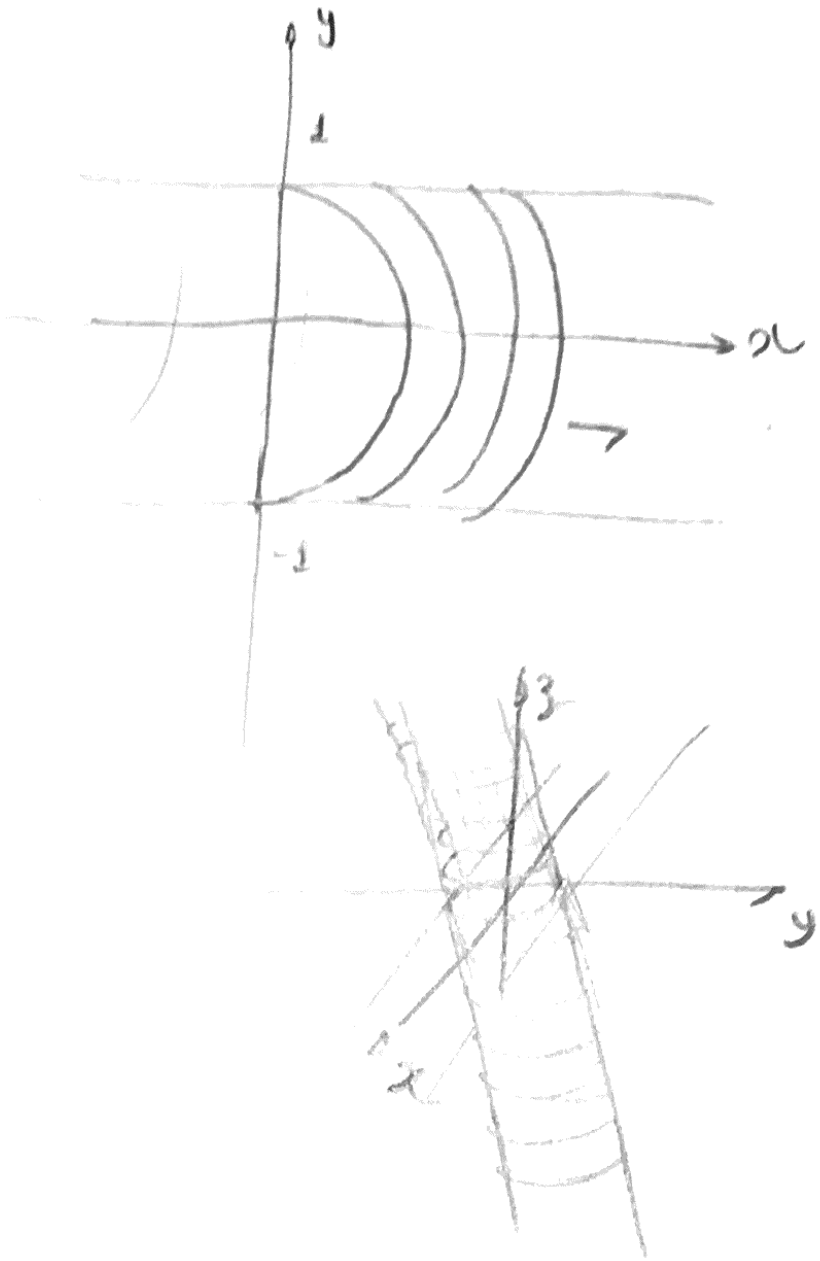
$T'(t) = (\cos t, -\operatorname{sen} t, -2\operatorname{sen} t \cos t)$

$t = \pi/3$

$(1/2, -\sqrt{3}/2, -2\sqrt{3}/4)$

1. 9 b) $f(x) = x - \sqrt{1-y^2}$

Df $|y| \leq 1 \rightarrow \{(x,y) \in \mathbb{R}^2 : |y| \leq 1\}$



$c = x - \sqrt{1-y^2}$

$1-y^2 = (x-c)^2$

$1 = (x-c)^2 + y^2$

$R = 1$; centro $(c, 0)$

$x - c \geq 0$

pois $\sqrt{1-y^2} \geq 0$

$x \geq c$

2. 1 j) $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2} \lim \left(\frac{xy}{\sqrt{x^2+y^2}}\right)$

↳ Limitado ↳ Limitado

N Existe 2 curvas ≠ mesmo ponto

Existe Limitada · tende a 0

$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$ → $y = 0$, função vale 1 ; $\gamma_1(t) = (t, 0)$

$x = 0$, função vale 0 ; $\gamma_2(t) = (0, t)$ } Não existe

$y \leq |y| = \sqrt{y^2}$

$y \leq \sqrt{x^2+y^2}$

$1 \leq \frac{y}{\sqrt{x^2+y^2}} \leq 1$

$\lim_{(x,y)\to(0,0)} \operatorname{sen}\left(\frac{xy}{\sqrt{x^2+y^2}}\right) \rightarrow \lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} \Rightarrow \lim_{(x,y)\to(0,0)} x \cdot \frac{y}{\sqrt{x^2+y^2}}$

↳ 0 ↳ Limitado

$\lim_{(x,y)\to(0,0)} \frac{\operatorname{sen} xy}{\sqrt{x^2+y^2}} = \operatorname{sen}\left(\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}\right) = 0$

1.8

a) $f(x,y) = \sqrt{x-y}$

$x - y \geq 0$

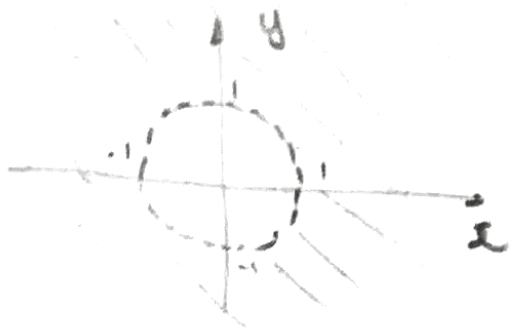
$x \geq y$

b) $f(x,y) = \operatorname{arctg} \frac{y}{x}$

$x \neq 0$

c) $f(x,y) = \frac{1}{\sqrt{x^2+y^2-1}}$

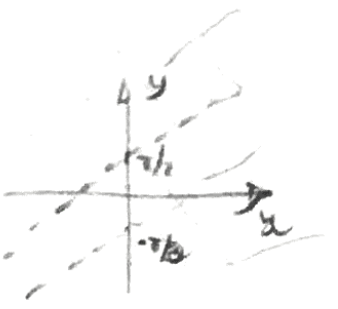
$\left.\begin{array}{l} x^2+y^2-1 \geq 0 \\ \sqrt{x^2+y^2-1} \neq 0 \\ x^2+y^2-1 \neq 0 \end{array}\right\} \; x^2+y^2 > 1$



$S = \{x, y \in \mathbb{R}, x+y>1\}$

d) $f(x,y) = \tan(x-y) \rightarrow |x-y| \neq \pi/2\,k$

$y \neq x + \frac{\pi}{2} + k\pi \;, k \in \mathbb{Z}$



e) $f(x,y) = \frac{x}{y^x} \rightarrow y \neq 0$, $y > 0$

$\frac{x}{|y|^x}$ $y<0$ para x par → + para x impar → −

→ 2 valores de y → 1 ponto na curva (mesmo x)

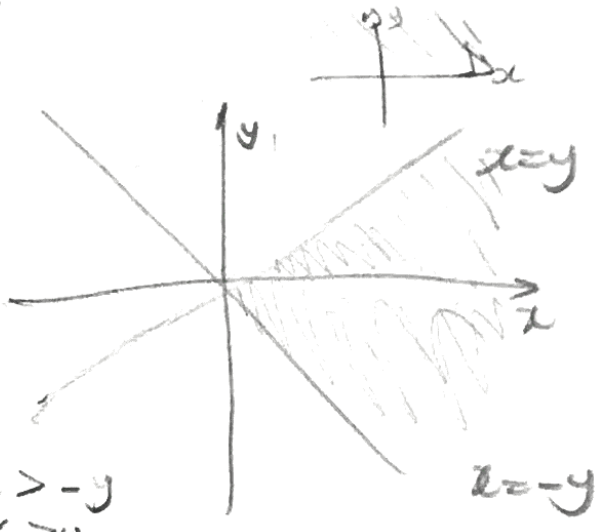
f) $f(x,y) = \ln(xy^2 - x^3)$

$xy^2 - x^3 > 0$

$x > 0 \rightarrow |y| \geq |x|$

$x < 0 \rightarrow |y| \leq |x|$

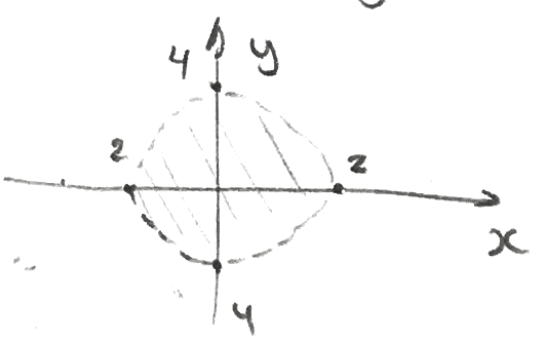
$x(x+y(x-y)) > 0$



g) $f(x,y) = \ln(16 - 4x^2 - y^2)$

$16 - 4x^2 - y^2 > 0$

$4x^2 + y^2 < 16$



1.9 a) $f(x,y) = \frac{x+y}{x-y} \rightarrow$ Dom → $x \neq y$

$\angle\, x - y \neq 0$

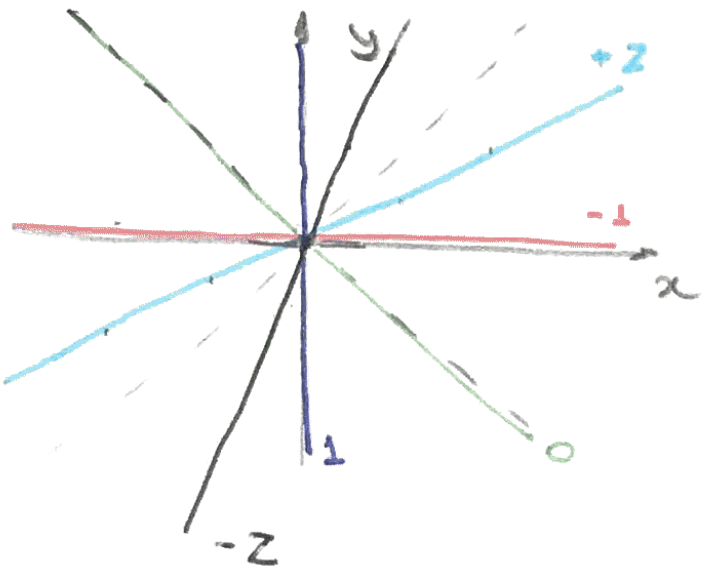
$-2 \rightarrow 3x = y$

$-1 \rightarrow x = 0$

$0 \rightarrow x = -y$

$1 \rightarrow y = 0$

$2 \rightarrow 3y = x$



$\sqrt{2}/2 - \sqrt{1-\frac{1}{2}} = \sqrt{\frac{2}{2}} - \sqrt{\frac{1}{2}} = 1 - \sqrt{1} = 0$

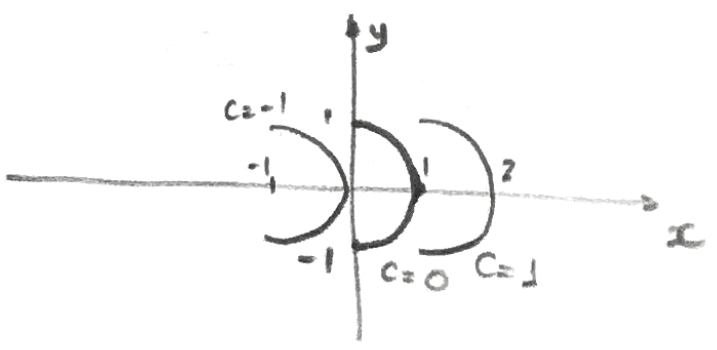
b) $f(x,y) = x - \sqrt{1-y^2} \rightarrow$ Dom → $|y| \leq 1$

$c = x - \sqrt{1-y^2}$

$1 - y^2 = (x-c)^2$

$x - c^2 \geq 0$

$x \geq c$



c) $f(x,y) = \frac{x^2}{x^2-y^2}$

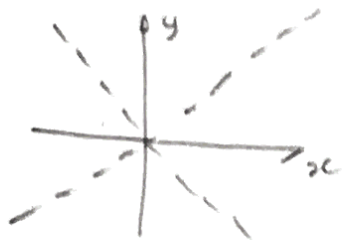
$x^2 - y^2 \neq 0$

$x^2 \neq y^2$

sempre positivo

$|x| \neq |y| \begin{cases} x \neq y \\ x \neq -y \end{cases}$

Dom:

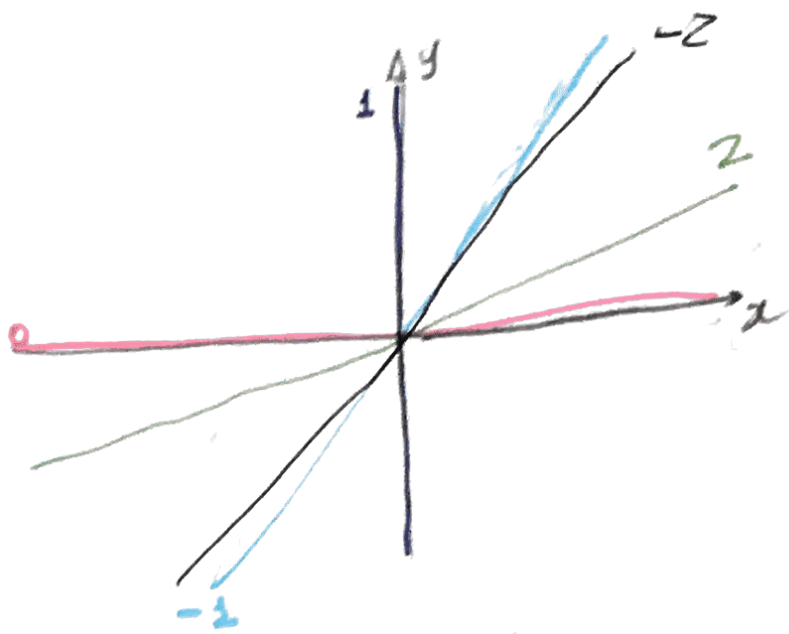


$c(x^2 - y^2) = x^2$

$(c-1)x^2 - cy^2 = 0$

$y^2 = x^2 \frac{(c-1)}{c}$

$y = \pm x \sqrt{\frac{c-1}{c}}$



$f(x,y) = 0 \rightarrow x = 0$

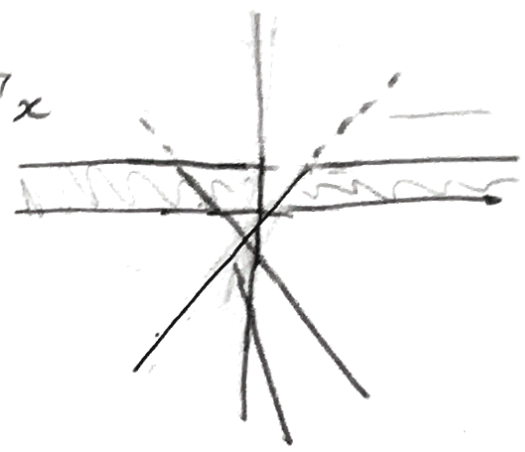
$f(x,y) = 1 \rightarrow y = 0$

$f(x,y) = -1 \rightarrow y^2 = x^2 \cdot 2 \rightarrow y = \pm\sqrt{2}x$

$f(x,y) = 2 \rightarrow y = \pm \frac{x}{\sqrt{2}}$

$f(x,y) = -2 \rightarrow y = \pm x\sqrt{3/2}$

$f(x,y) = \frac{1}{2} \rightarrow y = \pm x\sqrt{-1}$



$\frac{1}{2}x^2 - \frac{1}{2}y^2 = x^2$ ↳ NÃO

$-\frac{1}{2}y^2 = -\frac{1}{2}x^2$

$y^2 = x^2 \rightarrow$ NÃO

d) $f(x,y) = \frac{2xy^2}{x^2+y^4}$

$x \neq y$

$x \neq -y$

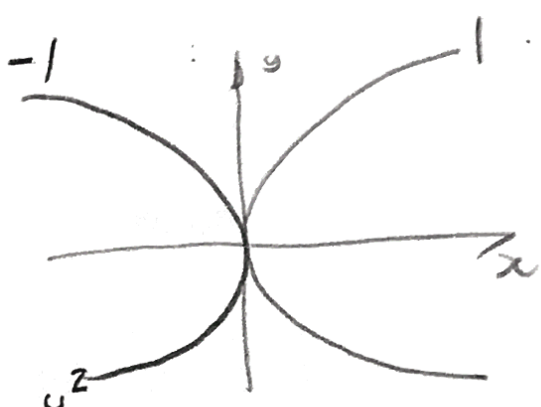
$cx^2 - 2xy^2 + cy^4 = 0$

$c=1 \rightarrow (x-y^2)^2 = 0 \rightarrow x = y^2$

$c=2 \rightarrow x^2 - xy^2 + y^4 = 0$

$c=-1 \rightarrow -x^2 - 2xy^2 - y^4 = 0 \rightarrow x = -y^2$

$c=0 \rightarrow xy^2 = 0 \rightarrow$ ou $x=0$ ou $y=0$ → 2 retas!



1.10.

a) $f(x,y) = x + 2y - 3$

$k = -2$

$-2 = x + 2y - 3 \rightarrow x = -2 + 3 - 2y = -2y + 1$

$\gamma(t) = (-2t+1, t)$ ~~ou $(t, \frac{1}{2} - \frac{t}{2})$~~

$\gamma'(t) = (-2, 1)$ ~~ou $(1, -1/2)$~~

$X = (1/2, 1/4) + \lambda(-2, 1)$ $\lambda \in \mathbb{R}$)

ou

b) $5 = x - \sqrt{1 - 2y^2}$ → $2y^2 < 1$

$(x-5)^2 = 1 - 2y^2$ $y < \pm\sqrt{\frac{1}{2}}$

$y < \pm \frac{\sqrt{2}}{2}$

$(x-5)^2 + (\sqrt{2}y)^2 = 1$ $\cos\theta < \pm 1$

ou

$\text{Sen}\theta = x - 5 \rightarrow x = \text{Sen}\theta + 5$

$\cos\theta = \sqrt{2}y \rightarrow y = \cos\theta/\sqrt{2}$

$\gamma(t) = (\text{Sen}\theta + 5, \cos\theta/\sqrt{2})$

$\text{Sen}\theta + 5 = 6$

$\text{Sen}\theta = 1$

$\theta = \pi/2$

$\gamma'(\theta) = \left(\cos\theta, -\frac{\text{Sen}\theta}{\sqrt{2}}\right)$

$\gamma'\theta = \left(\cos\frac{\pi}{2}, -\frac{\text{Sen}\,\pi/2}{\sqrt{2}}\right)$

$\gamma'\theta = (0, -\sqrt{2}/2)$

$X = (6, 0) + \lambda(0, -\sqrt{2}/2)$, $\lambda \in \mathbb{R}$

c) $f(x,y) = \frac{1}{x^2 - y^2}$

$x^2 - y^2 = 1$

$\sec^2\theta - \text{tg}^2\theta = 1$

$x = \sec\theta$

$y = \text{tg}\theta$ $\gamma(\theta) = (\sec\theta, \text{tg}\,\theta)$

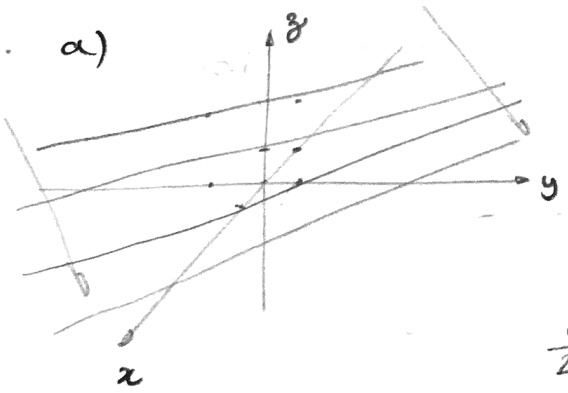
$\gamma'(\theta) = \left(\frac{\text{Sen}\theta}{\cos^2\theta}, \sec^2\theta\right)$

$\sec\theta = \sqrt{2}$, $\text{tg}\theta = 1$ $\Big\}$ $\theta = \pi/4$

$\gamma'(\pi/4) = \left(\frac{\sqrt{2}/2}{(\sqrt{2}/2)^2}, 2\right) = (\sqrt{2}, 2)$

$X = (\sqrt{2}, 1) + \lambda(\sqrt{2}, 2)$

1.11. a)



$z = 1 - x - y$ ~ dom $\mathbb{R}$

$y = 1 - x$ $z = 0$

$y = -x$ $z = 1$

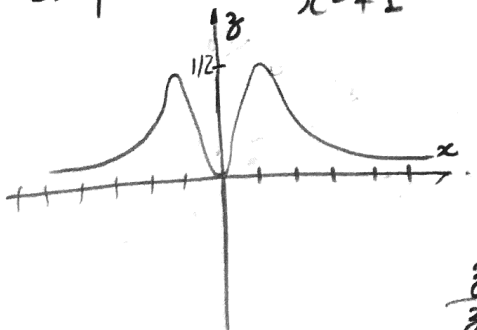
$y = -1 - x$ $z = 2$

$\frac{1}{2} \cdot \frac{1}{16+1} = \frac{1}{\quad}$

$\frac{1}{1/2+1} - \frac{z \cdot 1/x^2}{(1/2+1)^2}$

$\frac{2}{3} - \frac{4}{9} > 0$

b) $f(x,y) = \frac{x}{x^2+1}$



$z = \frac{x}{x^2+1}$ ~ dom: $\mathbb{R}$, $0 = \frac{x}{x^2+1} \rightarrow x = 0$

$z' = \frac{1}{x^2+1} + \frac{x}{-(x^2+1)^2} \cdot 2x = 0$

$2x^2 = x^2 + 1$

$x^2 = 1$

$x = \pm 1$

	-1		0		1	
z	↘		↗		↘	
z'	−	0	+	0	−	

$z'' = \frac{1}{(x^2+1)^2} \cdot 2x + \frac{4x}{-(x^2+1)^2} + \frac{2x^2}{2(x^2+1)^3} \cdot 2x$

$z'' = -2x/(x^2+1)^2 + x^3/(x^2+1)^3$

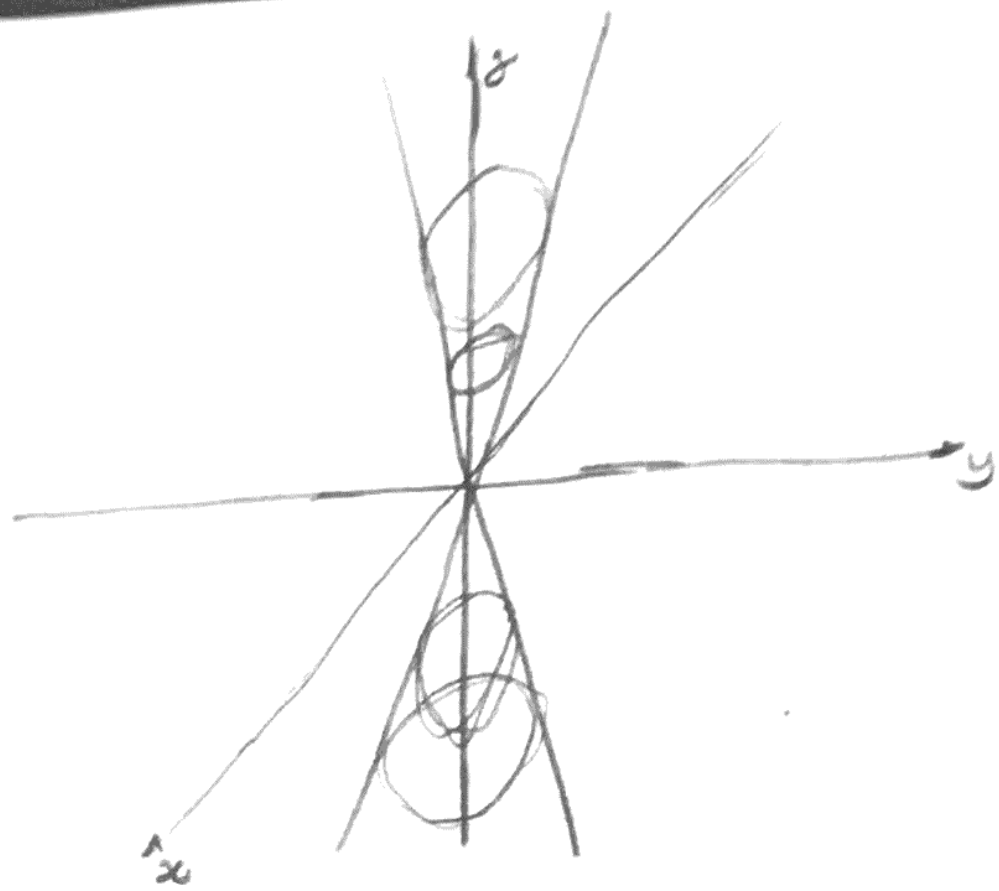
c) $f(x,y) = \sqrt{x^2 + 9y^2}$

$z^2 - x^2 - 9y^2 = 0$

hiperboloide de 2 folhas

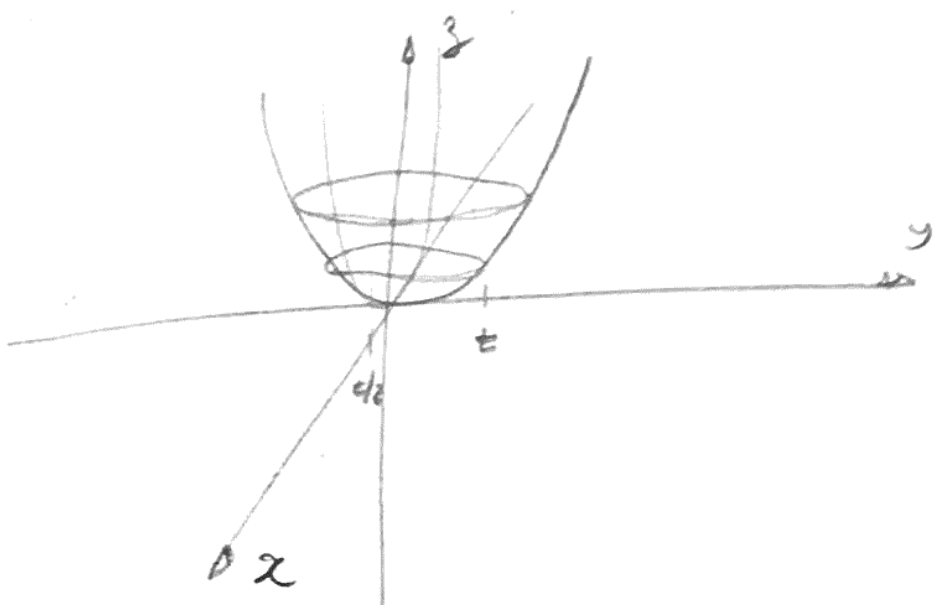
$1 = x^2 + 9y^2$

$y = 1/3 \quad x = 0$

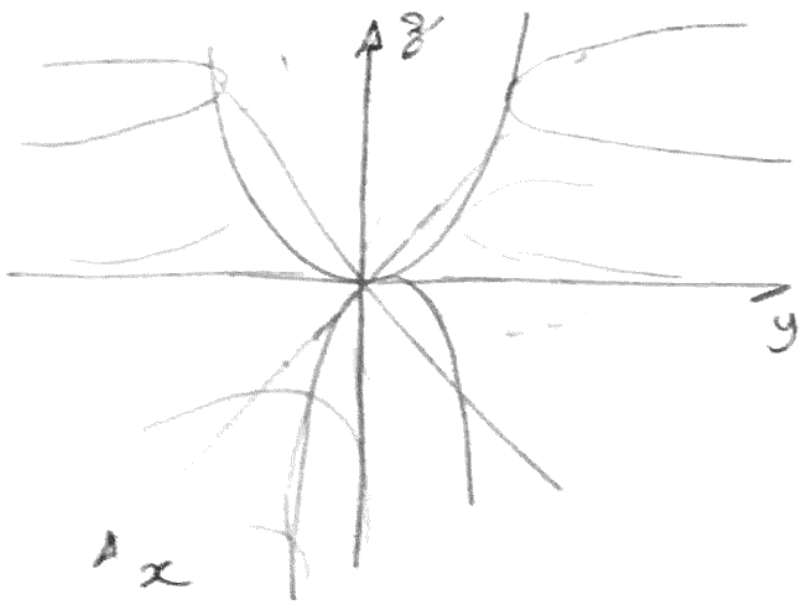


d) $f(x,y) = 4x^2 + y^2$

$z = (2x)^2 + (y)^2$

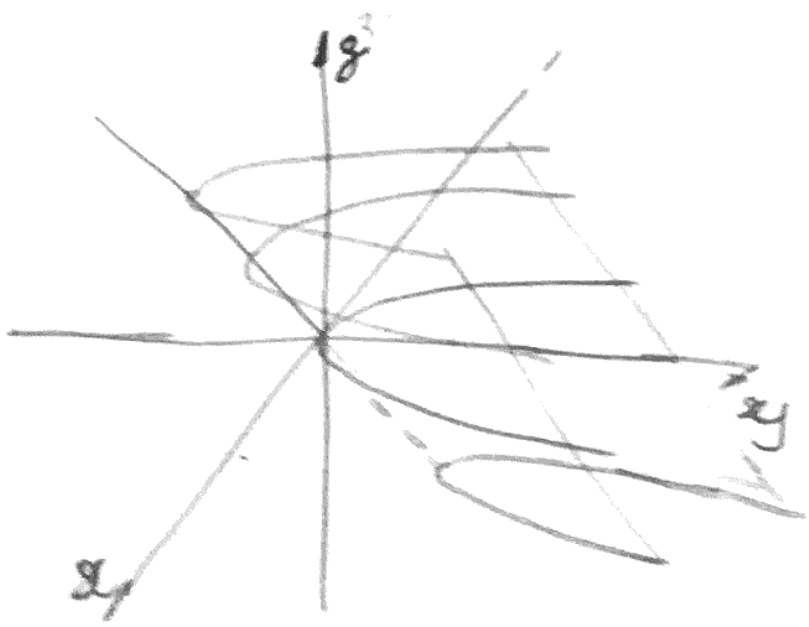


e) $f(x,y) = y^2 - x^2$

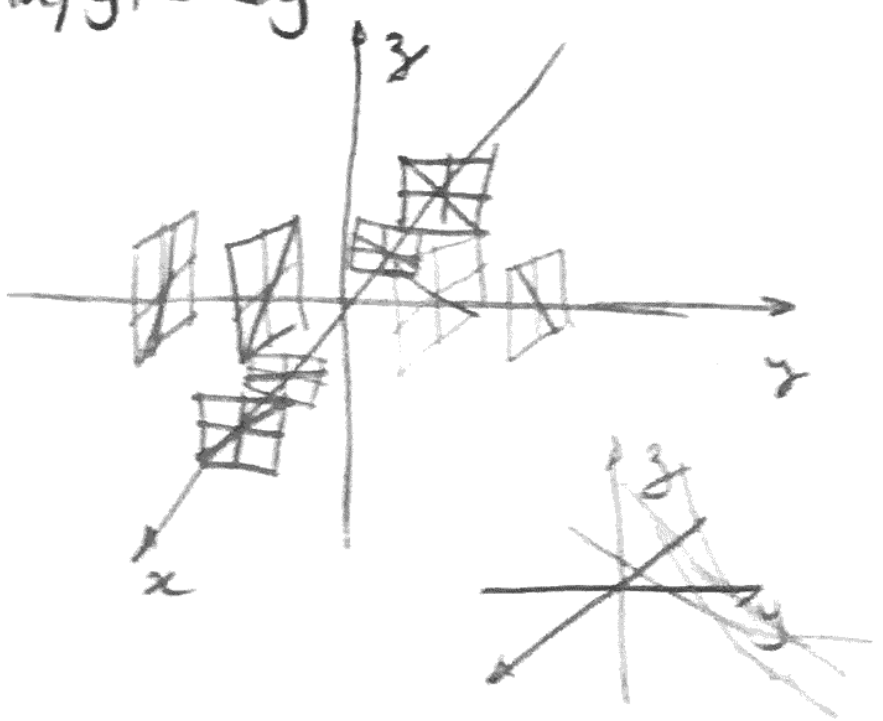


f) $f(x,y) = y^2 + 1$

g) $f(x,y) = y^2 + x$



h) $f(x,y) = xy$

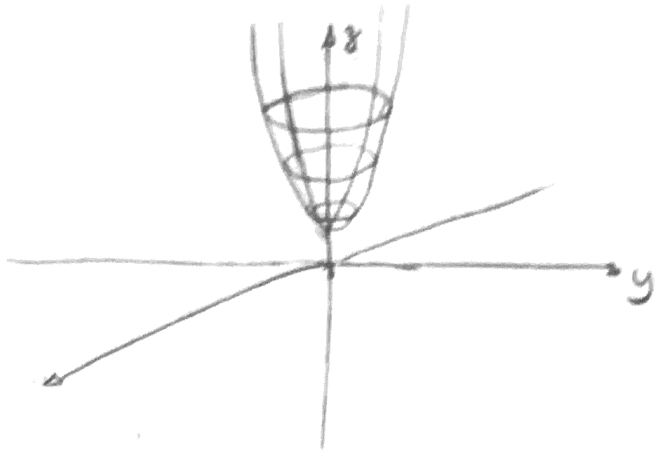


$f(x,y) = e^{\sqrt{x^2+y^2}}$

$\ln z = \sqrt{x^2+y^2}$

$\ln^2 z = x^2 + y^2$

$z > 0$



$\ln^2 z = 1$

$z = e$

$\ln^2 z = 2$

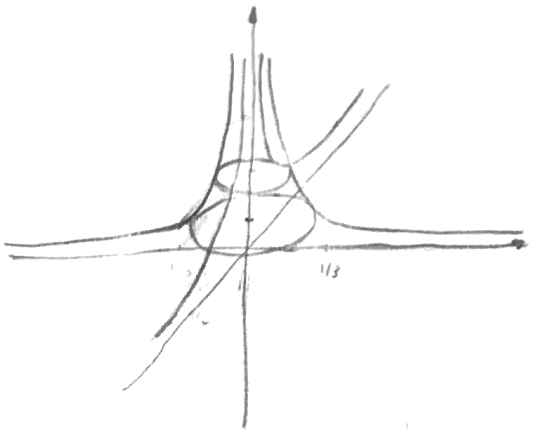
$e^{\sqrt{2}} = z \to 2$

$e^{\sqrt{3}} = z \to 3$

$e^{\sqrt{1/2}} = z \to 1/2$

j. $f(x,y) = \dfrac{1}{4x^2 + 9y^2}$ $\quad z > 0$

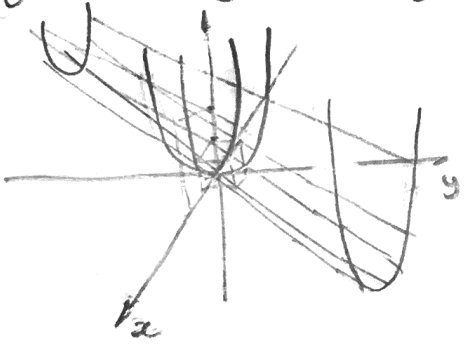
$4x^2 + 9y^2 = \dfrac{1}{z}$



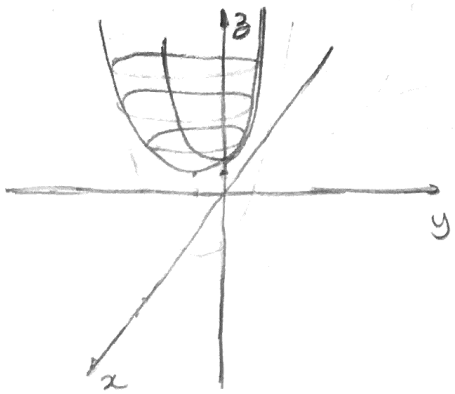
k. $f(x,y) = (x-y)^2$

$z = x^2 - 2xy + y^2 \to \pm\sqrt{z} = x - y$

$y = x \pm \sqrt{z}$



I. $f(x,y) = x^2 + y^2 + 2y + 3$



$\Delta y = 2^2 - 4 \cdot 1 \cdot (3 + x^2) =$

$4 - 12 - 4x^2 = -8 - 4x^2$

$x^2 + y^2 + 2y = -3 + z \to y^2 + 2y + 3 = z$

$x^2 + (y+1)^2 - 1 = -3 + z$

$x^2 + (y+1)^2 = -2 + z$

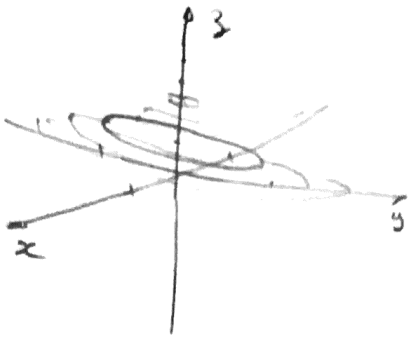
$\Delta: 4 - 4 \cdot 1 \cdot 3 < 0$

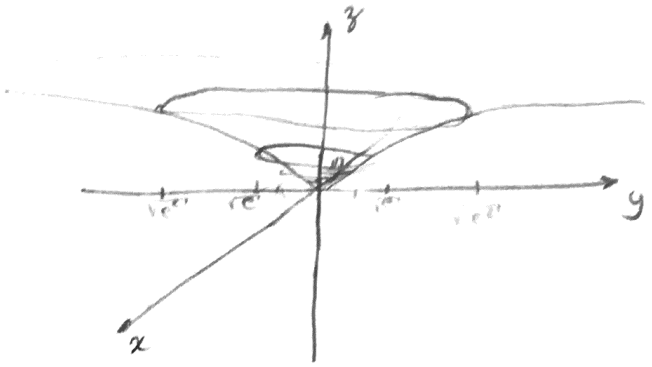
$-\frac{b}{2a} = -\frac{2}{2} = -1$

$-\frac{\Delta}{4a} = -\frac{(4 - 12)}{4} = 2$

$f(x,y) = \frac{1}{(x^2+2y^2)^2} \rightarrow \pm\sqrt{z} = \frac{1}{x^2+2y^2} \rightarrow \pm\frac{1}{\sqrt{z}} = x^2+2y^2$

$z = \frac{1}{x^4}$ $\quad z = \frac{1}{4y^4}$





m. $f(x,y) = \ln(9x^2+y^2)$

$e^z = 9x^2+y^2$

o. $f(x,y) = 2 - \sqrt[4]{x^2+4y^2}$

$x^2+4y^2 = (2-z)^4 \rightarrow z \leq 2$

$x^2+4y^2-(2-z)^4 = 0$

$4y^2 = (2-z)^4$

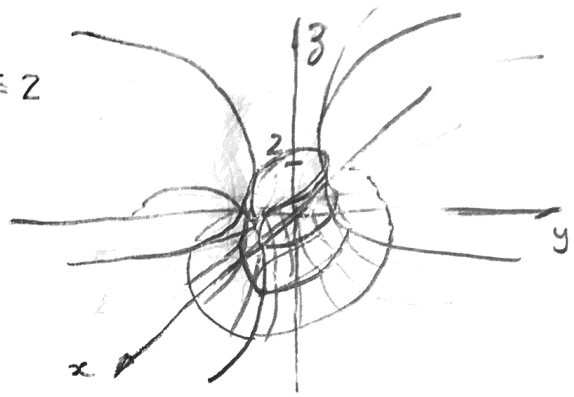
$4y = \pm(2-z)^2$

$4y = -4+4z-z^2$

$y = 1-z+\frac{z^2}{4}$ $\quad z_v = \frac{-(-1)}{1} = 1$

$x^2 = (2-z)^4$

$x = 4-4z+z^2$



$x^2+4y^2 = 2^4 = 16$

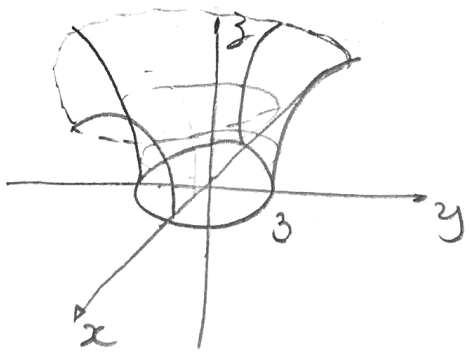
$x^2+(2y)^2 = 16$

$x^2+4y^2 = 1$

p) $f(x,y) = \sqrt{x^2+y^2-9}$

$z^2-x^2-y^2 = -9$

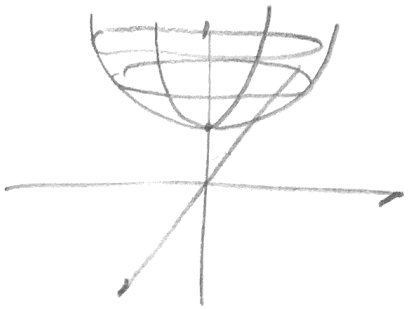
$-\frac{z^2}{3^2}+\frac{x^2}{3^2}+\frac{y^2}{3^2} = 1$ $\quad z \geq 0$



q) $f(x,y) = \sqrt{x^2+y^2+1}$

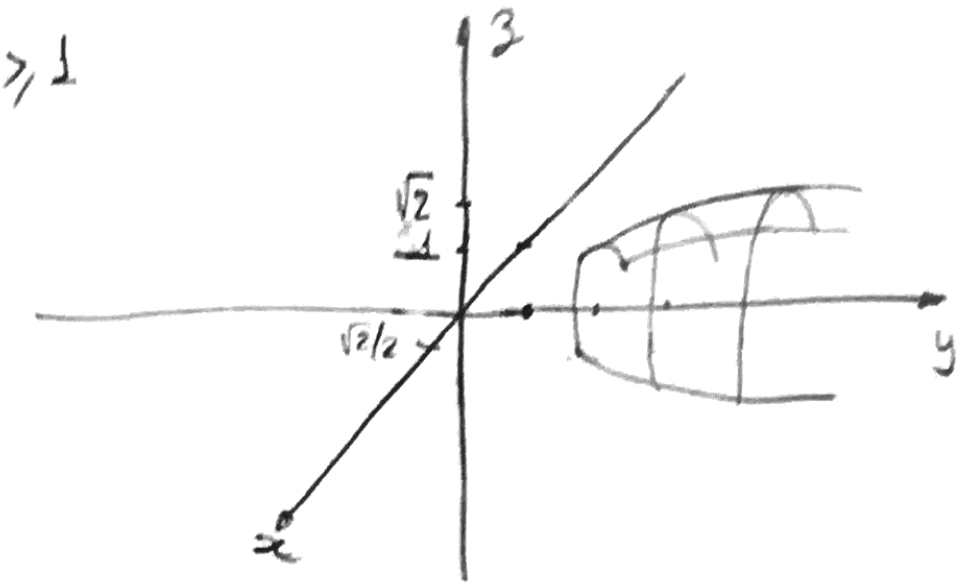
$z^2 = x^2+y^2+1$

$z^2-x^2-y^2 = 1$



a) $z^2 = y - 2x^2 - 1 \quad z \geq 0 \quad y \geq 1$

$z^2 + 2x^2 = y - 1$

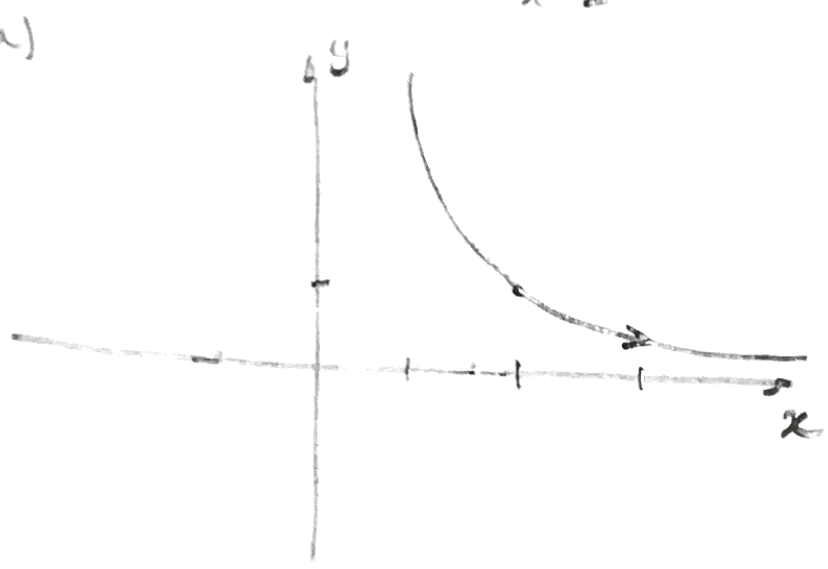


1.12.* $g_1(t) = (e^t + 1, e^{-t}) \rightarrow x \geq 1 \quad y \geq 1$

$x = e^t + 1 \rightarrow x - 1 = e^t \rightarrow e^{-t} = \frac{1}{e^t} = \frac{1}{x-1}$

$y = e^{-t} = \frac{1}{x-1}$

a)



$e^t \cdot t, \; -t(e)^{-2t} \cdot e$

$e^t t, \; -t e^{-2t+1}$

b) $f(x,y) = x^2y^2 - 2y - y^2 + 4$

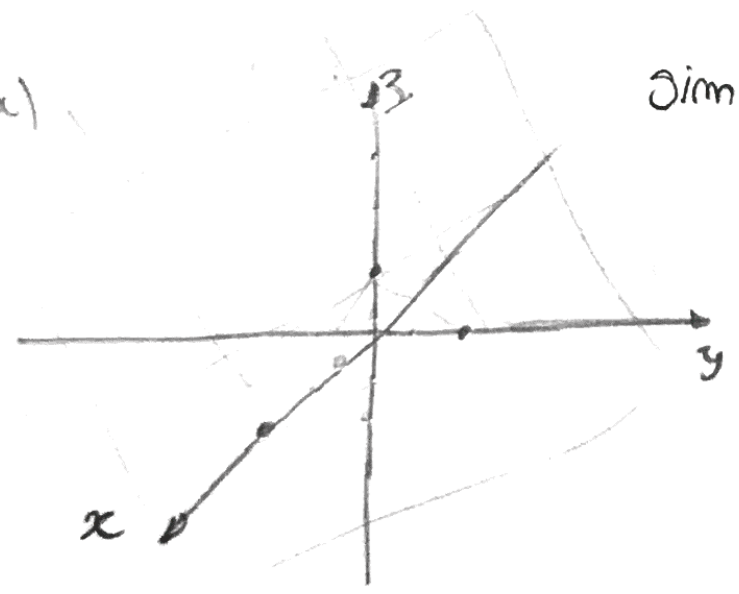
$f(r(t)) = (e^t+1)^2 \cdot (e^{-t})^2 - 2e^{-t} - e^{-2t} + 4$

$= (e^{2t} + 2e^t + 1) \cdot e^{-2t} - 2e^{-t} - e^{-2t} + 4$

$= 1 + 2e^{-t} + e^{-2t} - 2e^{-t} - e^{-2t} + 4 =$

5 constante

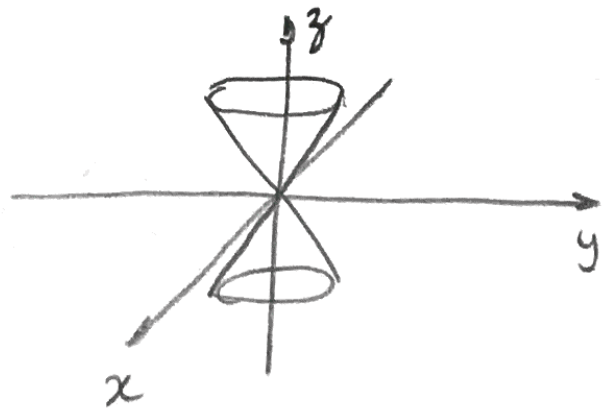
1.13 a)



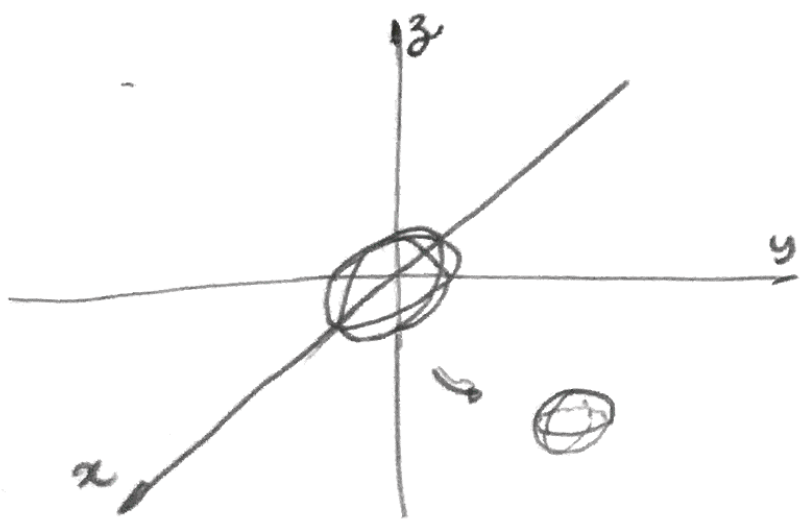
Sim, ogni k dà un plano

SIM

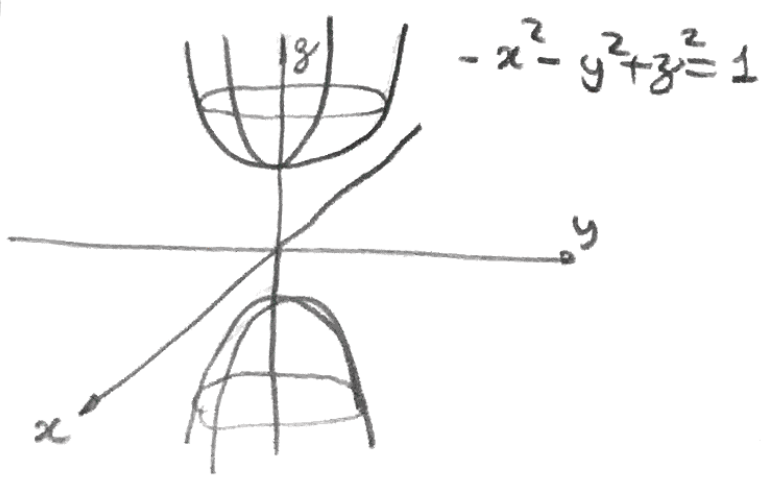
c) $x^2 + y^2 - z^2 = 0$



b) $x^2 + 2y^2 + 3z^2 = 1$

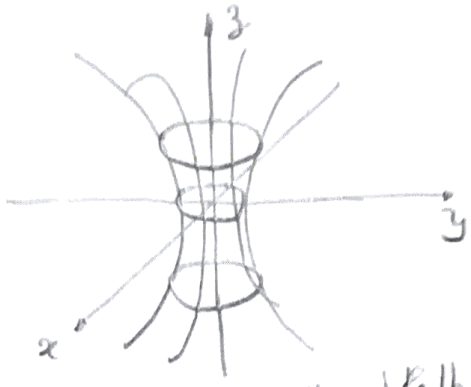


d)



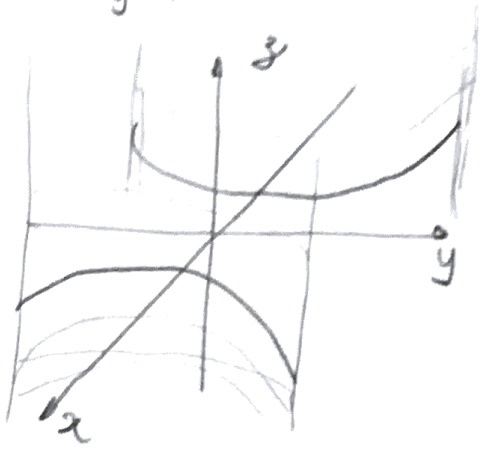
$-x^2 - y^2 + z^2 = 1$

a) $x^2+y^2-z^2=1$



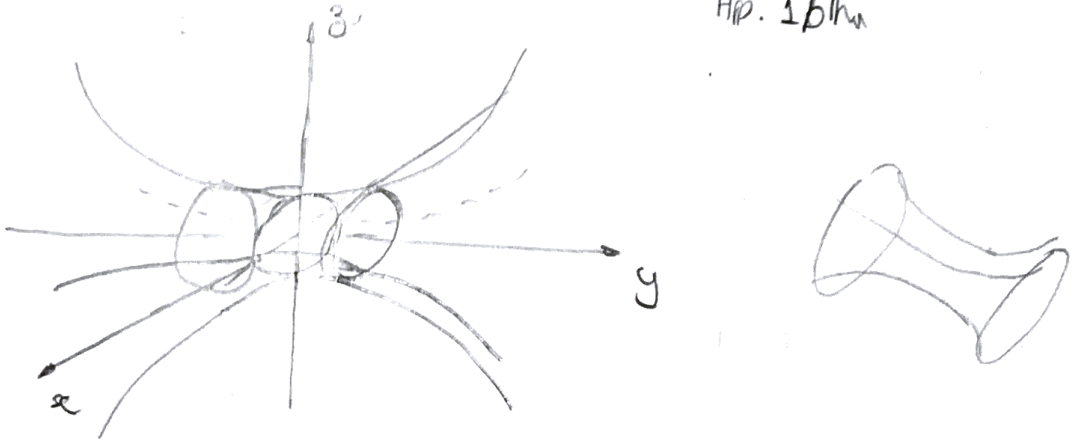
Hip. 2 folhas

b) $x^2-y^2=1$



g) $x^2-y^2+z^2=1$

Hip. 1 folha



1.14. a) $x^2+y^2+z^2=r^2(x,y,z)$

$\gamma(t)=(\cos t, \cos t, \sqrt{2}\sin t)$

$\cos^2 t+\cos^2 t+\left(\sqrt{2}\right)^2\sin^2 t=r^2(\gamma(t))$

$1-\sin^2 t+1-\sin^2 t+2\sin^2 t=r^2(\gamma(t))$

$r(\gamma(t))=\sqrt{2}$ sim

b) $\gamma(t)=(\sqrt{t^2+1}\cos t, \sqrt{t^2+1}\sin t, t)$, $t\in\mathbb{R}$ e s: $x^2+y^2-z^2=1$

$(t^2+1)\cos^2 t+(t^2+1)\sin^2 t-t^2=1$

$t^2+1-t^2=1$ sim

$0=0$

c) $\gamma(t)=(t\cos t, t\sin t, \sqrt{t^2+4})$, $t\geq 0$ o suporte gráfico de

$f(x,y)=\sqrt{x^2+y^2+4}$ $\rightarrow$ $\sqrt{t^2\cos^2 t+t^2\sin^2 t+4}=\sqrt{t^2+4}=z$ sim

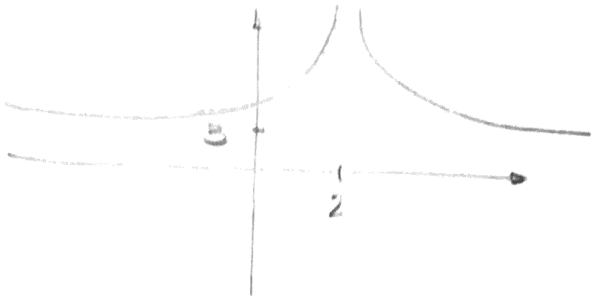
1.15 $\gamma(t) = (2\cdot\cos t, \sec^2 t + 3)$, $t \in [0, \pi/2[$

$f(x) = \left((x^2 - 2)^2 (y-3)\right)^{2/3} + 1$

Im $\gamma(t)$

$2 - K = x \to K = 2 - x$

$\frac{1}{K'} + 3 = y \to K' = \frac{1}{y-3} \to (2-x)^2 = \frac{1}{y-3} \to y = \frac{1}{(2-x)^2} + 3$



$f(x) = \left((2\cdot\cos t - 2)^2 (\sec^2 t + 3 - 3)\right)^{2/3} + 1$

$\left(\cos^2 t \cdot \frac{1}{\cos^2 t}\right)^{2/3} + 1 = 2 \rightsquigarrow$ Nivel 2

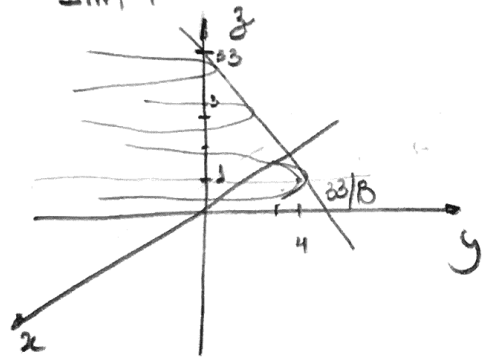
1.16 $g(x,y) = (x-2)^2 + (y-3)^2 + 1 \to$ Dom: $\mathbb{R}$; Im: $x \in \mathbb{R}$; $x \geq 1$

$T(t) = (2-t, 3+t, g(t)) = (2-t, 3+t, 2t^2+1) \to z \geq 1$

$g(x,y) = g(t)$

$g(t) = (2-t-2)^2 + (3+t-3)^2 + 1 = \boxed{2t^2+1}$

Im T



$x = 2 - t \Rightarrow x^2 = 4 - 4t + t^2$

$y = 3 + t \quad 4y = 12 + 4t$

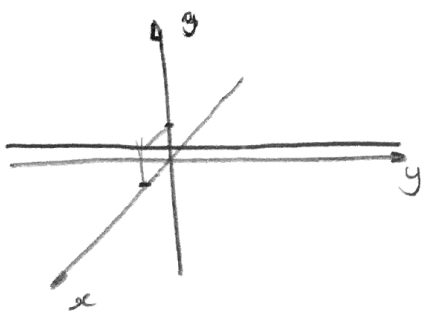
$z = 2t^2 + 1 \quad \frac{z}{2} = \frac{2t^2}{2} + \frac{1}{2}$

$x^2 + 4y + \frac{z}{2} = 4 + 12 + \frac{1}{2}$

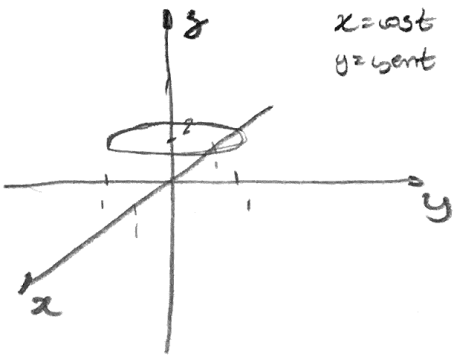
$2x^2 + 8y + z = 8 + 24 + 1 = 33$

$y = 4 - \frac{1}{4}x^2$

1.17 a) $\gamma(t) = (1, t, 1)$



b) $\gamma(t) = (\cos t, \sin t, 2)$



$x = \cos t \quad x^2 + y^2 = 1$

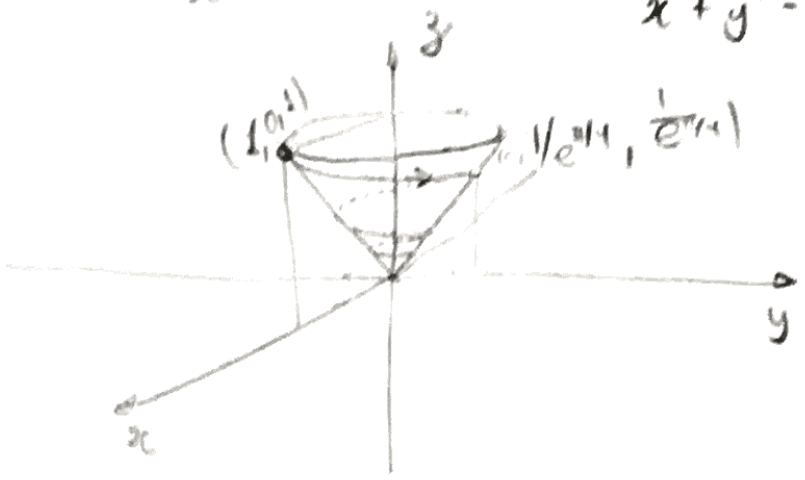
$y = \sin t$

c) $\gamma(t) = (e^{-t}\cos t, e^{-t}\sin t, e^{-t})$ $t \geq 0$

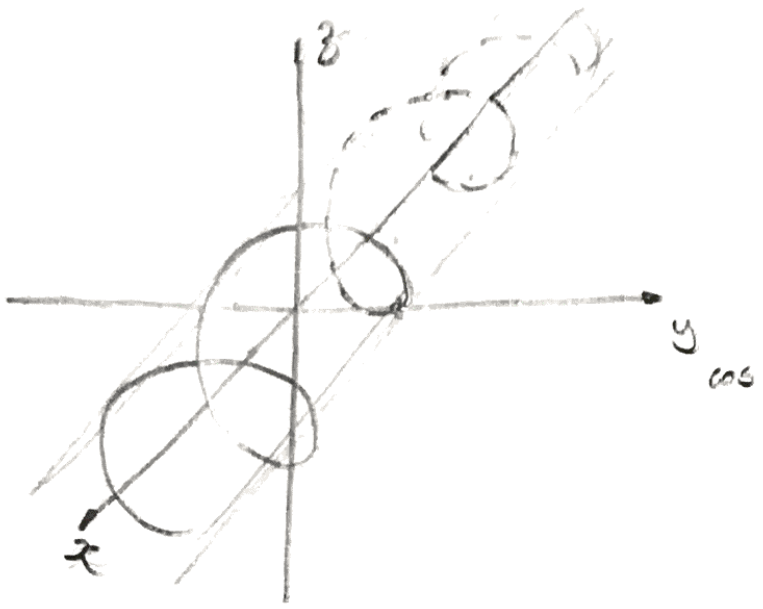
$y = e^{-t}\cos t$
$x = e^{-t}\sin t$

$x^2 + y^2 = e^{-2t} = z^2$

$x^2 + y^2 - z^2 = 0 \rightarrow z \geq 0$



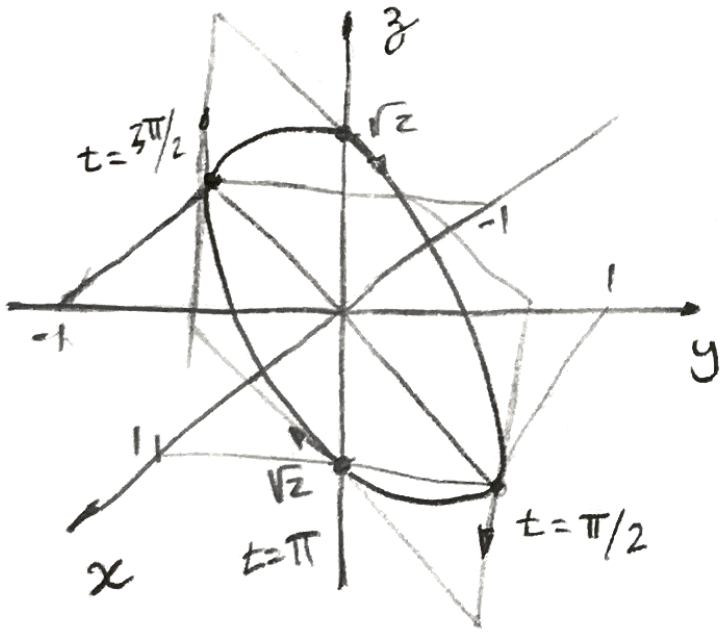
d) $\gamma(t) = (t, \cos t, \sin t)$, $t \geq 0$



$1 = y^2 + z^2$

$x = t$

e) $\gamma(t) = (\sin t, \sin t, \sqrt{2}\cos t)$ $0 \leq t \leq 2\pi$



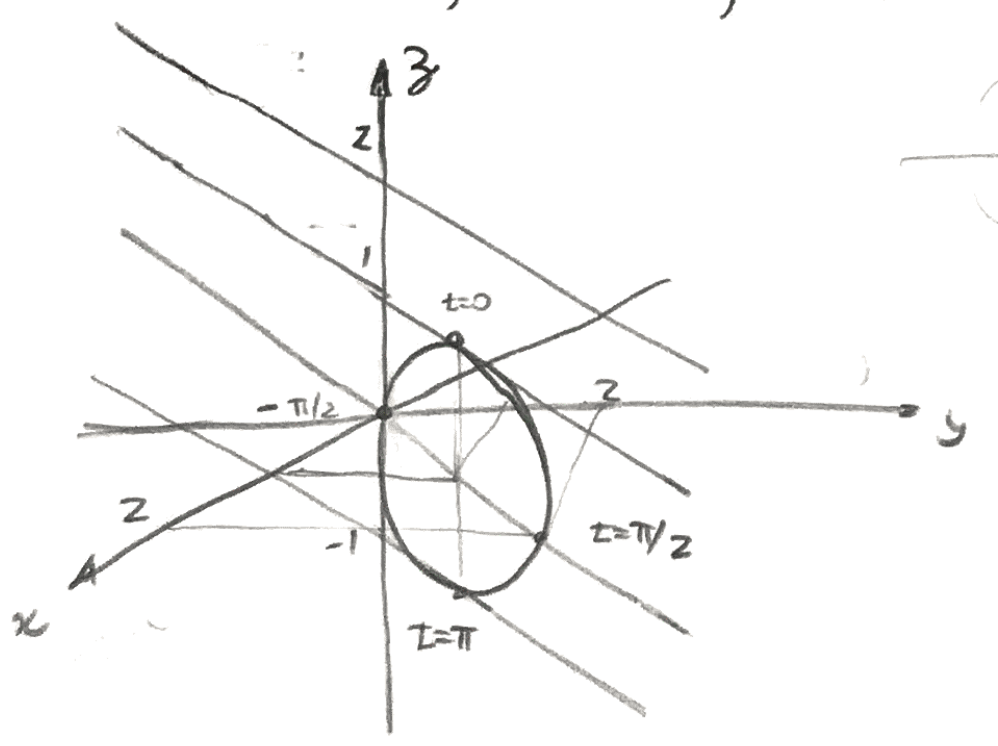
$x^2 + \frac{z^2}{2} = 1$

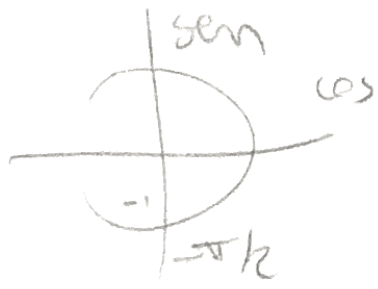
$\gamma'(t) = (\cos t, \cos t, -\sqrt{2}\sin t)$

$\gamma(\pi/2) = (\cos\pi/2, \cos\pi/2, -\sqrt{2}\sin\pi/2) = (0, 0, -\sqrt{2})$

$\gamma(0) = (\cos 0, \cos 0, -\sqrt{2}\sin 0) = (1, 1, 0)$

f) $\gamma(t) = (1 + \sin t, 1 + \sin t, \cos t)$





1.18.

a) $C = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2=1 \text{ e } z = x+1\}$ e $P = (-1/2, \sqrt{2}/2, 1/2)$

$\gamma'(t) = \left(\frac{\sin t - 1}{2}, \frac{\cos t}{\sqrt{2}}, \frac{\sin t + 1}{2}\right)$

$x^2 + x^2 + 2x + 1 + y^2 = 1$

$2x^2 + 2x + y^2 = 0$

$\left(\sqrt{2}\,x + \frac{1}{\sqrt{2}}\right)^2 + y^2 = \frac{1}{2}$

$(2x+1)^2 + (\sqrt{2}\,y)^2 = 1$

$\sin t = 2x + 1 \Rightarrow x = \frac{\sin t - 1}{2}$

$\cos t = \sqrt{2}\,y \rightarrow y = \frac{\cos t \sqrt{2}}{2}$

$z = \sin t + 1/2$

$\rightarrow \gamma'(t) = (\cos t/2, -\sin t/\sqrt{2}, \cos t/2)$

$T = (-1/2, \sqrt{2}/2, 1/2) + \gamma(\cos 0/2, -\frac{\sin 0}{\sqrt{2}}, \frac{\cos 0}{2}) = \gamma(1/2, 0, 1/2)$

↳ $t \Rightarrow \frac{\sin t - 1}{2} = -\frac{1}{2}$

$\sin t = 0$

$t = \pi, 0$

$\frac{\cos t}{\sqrt{2}} = \sqrt{2}/2 \Rightarrow \cos t = 1 \rightarrow t = 0$

b) $C = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 = 1 \text{ e } (x-1)^2 + y^2 + (z-1)^2 = 1\}$ $P = (\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2})$

$x^2 - (x-1)^2 + z^2 - (z-1)^2 = 0$

$x^2 - x^2 + 2x - 1 + z^2 - z^2 + 2z - 1 = 0$

$2x + 2z = 2$

$x + z = 1$

$x^2 + y^2 + (1-x)^2 = 1$

$x^2 + x^2 - 2x + y^2 = 0$

$+ \frac{1}{2} + \frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{2}y^2 =$

$(2x-1) + (\sqrt{2}y)^2 = 0$

$2x - 1 = \sin t \rightarrow x = \frac{\sin t + 1}{2}$

$\sqrt{2}y = \cos t$

$y = \frac{\cos t \sqrt{2}}{2}$

$z = \frac{1 - \sin t}{2}$

$C_{(t)} = \left(\frac{\sin t + 1}{2}, \cos t \frac{\sqrt{2}}{2}, \frac{1 - \sin t}{2}\right)$

$C'_{(t)} = \left\{\frac{\cos t}{2}, -\sin t \frac{\sqrt{2}}{2}, -\frac{\cos t}{2}\right)$

$\frac{\sin t + 1}{2} = 1/2 \rightarrow \sin t = 0 \rightarrow t = 0, \pi$

$\frac{\cos t}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 1 \rightarrow t = 0$

$C'_{(0)} = [1/2, 0, -1/2]$

$R = (1/2, \sqrt{2}/2, 1/2) + \gamma(1/2, 0, -1/2)$

c) $C = \{(x,y,z) \in \mathbb{R}^3 \mid z = y^2 - x^2 \text{ e } x^2 + y^2 = 1\}$ $P = (\sqrt{2}/2, \sqrt{2}/2, 0)$

$z = y^2 - (1 - y^2) = 2y^2 - 1$

$x = \text{sen}\,t \quad y = \cos t$

$z = 2\cos^2 t - 1$

$C'(t) = (\cos t, -\text{sen}\,t, -2\text{sen}\,t)$ ⟶ $R = (\sqrt{2}/2, \sqrt{2}/2, 0) + (\sqrt{2}/2, -\sqrt{2}/2, -\sqrt{2})$

- $P \to t = \pi/2$

d) $C = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 - 2z^2 = 1 \text{ e } y = 2z + 1\}$

$P = (-\sqrt{2}, -1, -1)$

$x^2 + (2z+1)^2 - 2z^2 = 1$

$x^2 + 4z^2 + 4z + 1 - 2z^2 = 1$

$x^2 + 2z^2 + 4z = 0$

$x^2 + (\sqrt{2}z + \sqrt{2})^2 = 2$

$\left(\frac{x}{\sqrt{2}}\right)^2 + (z+1)^2 = 1$

$x/\sqrt{2} = \text{sen}\,t \to x = \text{sen}\,t\sqrt{2}$

$z + 1 = \cos t \to z = \cos t - 1$

$y = 2(\cos t - 1) + 1 = 2\cos t - 1$

$C(t) = \{\text{sen}\,t\sqrt{2}, 2\cos t - 1, \cos t - 1\}$

$C'(t) = \{\cos t\sqrt{2}, -2\text{sen}\,t, -\text{sen}\,t\}$

$P \to t' = -\pi/2$

$C'(t') = \{0, +2, +1\}$

$R = (-\sqrt{2}, -1, -1) + \gamma(0, 2, 1)$

e) $x = z \quad x^2 + y^2 = z$

$P = 1/2, 1/2, 1/2$

$x^2 - x + y^2 = 1/4$

$(x - 1/2)^2 + y^2 = 1/4$

$2x - 1 = \text{sen}\,t$

$2y = \cos t$

$C(t) = \left(\frac{\text{sen}\,t + 1/2}{2}, \frac{\cos t}{2}, \frac{\text{sen}\,t + 1/2}{2}\right)$

$t = 0 \to \text{sen}\,t = 0, \cos t = 1 \} \; P = (1/2, 1/2, 1/2)$

$C'(t) = \left(+\frac{\cos t}{2}, \frac{\text{sen}\,t}{-2}, \frac{\cos t}{2}\right)$

$P' = (1/2, 0, 1/2)$

$R = (1/2, 1/2, 1/2) + \gamma(1/2, 0, 1/2)$

f) $z = \sqrt{4x^2 + y^2} \qquad z = 2x + 1 \qquad (2x+1)^2 = 4x^2 + y^2$

$P = (0, 1, 1)$

$4x^2 + 4x + 1 = 4x^2 + y^2$

$x = \frac{y^2 - 1}{4}$

$C(t) = \left(\frac{t^2 - 1}{4}, t, \frac{t^2 + 1}{2}\right)$

$z = \frac{y^2}{2} - \frac{1}{2} + 1 = \frac{y^2}{2} + \frac{1}{2}$

$C'(t) = (t/2, 1, t) \to C'(P) = (1/2, 1, 1)$

$R = (0, 1, 1) + \gamma(1/2, 1, 1)$

1.) $F(x,y) = \dfrac{2x^2+4y^2}{x^2+y^2+1}$

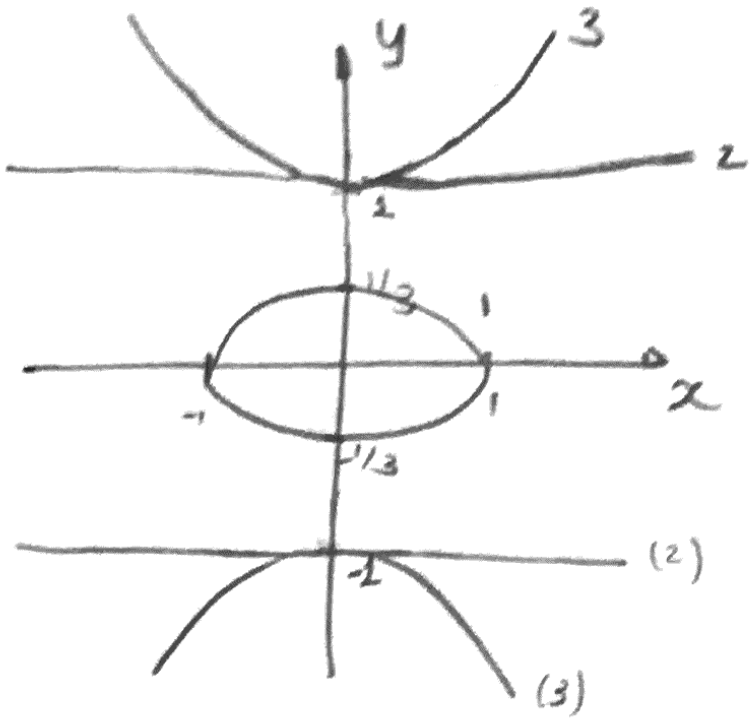
a) $c=1$ $\quad x^2+y^2+1 = 2x^2+4y^2$

$1 = x^2+3y^2$

$c=2$ $\quad 2 = 2x^2+4y^2$

$y^2 = 1$

$c=3$ $\quad 1 = -x^2+y^2$



b) $g(t) = (\text{sen}t, \cos t/\sqrt{3})$

$x^2+3y^2 = 1 \rightarrow \text{sen}^2t + 3\cdot\dfrac{\cos^2 t}{(\sqrt{3})^2} = 1 \rightarrow \text{sen}^2t+\cos^2 t = 1$ ok!

c) $g'(t) = (\cos t, -\text{sen}t/\sqrt{3})$

$g'(-\pi/2) = (0, \sqrt{3}/3)$

$(\text{sen}t, \cos t/\sqrt{3}) = (-1, 0)$

$\text{sen}t = -1 \quad \cos t = 0$

$t = -\pi/2$

d) $T(t) = (\text{sen}t, \cos t, z(t))$

$\text{Im} \in \text{graf } F$

$z(t) = \dfrac{2\cdot\text{sen}^2 t + 4\cos^2 t}{\text{sen}^2 t + \cos^2 t + 1} = \text{sen}^2 t + 2\cos^2 t = 1+\cos^2 t$

$T(t) = (\text{sen}t, \cos t, 1+\cos^2 t)$

$T'(t) = (\cos t, -\text{sen}t, 2\cos t\cdot(-\text{sen}t)) = (\cos t, -\text{sen}t, -2\cos t\,\text{sen}t)$

$T'(\pi/3) = (1/2, -\sqrt{3}/2, -2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}) = (1/2, -\sqrt{3}/2, -\sqrt{3}/2)$

$T(\pi/3) = (\sqrt{3}/2, 1/2, 1+1/4) = (\sqrt{3}/2, 1/2, 5/4)$

$R = (\sqrt{3}/2, 1/2, 5/4) + \eta(1/2, -\sqrt{3}/2, -\sqrt{3}/2)$

...ites de função de duas variáveis

R. 0, 1, ∞, −∞, N.E.

a) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

$(0,t) \to 0/t^2 = 0$

$(t,-t) \to -t^2/2t^2 = -\frac{1}{2}$ ⟹ N.E.

b) $\lim_{(x,y)\to(0,0)} \frac{x^2 y \cos(x^2+y^2)}{x^2+y^2} = \lim_{(x,y)\to(0,0)} y \cdot \cos(x^2+y^2) \cdot \frac{x^2}{x^2+y^2} = 0$

$y \to 0$; $\cos \leq 1$; $\frac{x^2}{x^2+y^2} \leq 1$

$x^2 \leq x^2+y^2$

$\frac{x^2}{x^2+y^2} \leq 1$

c) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} = \lim_{(x,y)\to(0,0)} x \frac{x^2}{x^2+y^2} + \lim_{(x,y)\to(0,0)} y \frac{y^2}{x^2+y^2} = 0$

$x \to 0$; $\frac{x^2}{x^2+y^2} \leq 1$; $y \to 0$; $\frac{y^2}{x^2+y^2} \leq 1$

d) $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{2x^4+x^2y+y^2} = \lim_{(x,y)\to(0,0)} \frac{x^2 y}{(\sqrt{2}x^2+y)^2+x^2y(1-2\sqrt{2})} =$

$\lim_{(x,y)\to 0,0} \frac{x^2 y}{(\sqrt{2}x^2+y)^2} + \frac{1}{(1-2\sqrt{2})}$

$\lim_{(0,t)\to(0,0)} \frac{x^2 y}{(\sqrt{2}x^2+y)^2} = \lim_{(0,t)\to(0,0)} \frac{0}{t^2} = 0 \to 0$ Limite

$\lim_{(t,t^2)} t^4/((1+\sqrt{2})t^2)^2 = \frac{1}{(1+\sqrt{2})^2} \cdot \frac{t^4}{(t^2)^2} \neq 0 \to$ NÃO EXISTE

e) $\lim_{(x,y)\to 0,0} \frac{2x^2+3xy+4y^2}{3x^2+5y^2}$

$\lim_{(t,-t)\to(0,0)} \frac{2t^2-3t^2+4t^2}{8t^2} = \frac{3}{8}$

$\lim_{(t,t)\to(0,0)} \frac{2t^2+3t^2+4t^2}{8t^2} = \frac{9}{8}$ ⟹ NE

f) $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2}$ ⇒

$\lim_{(t,t^2)\to(0,0)} \frac{t^4}{t^4+t^4} = \frac{1}{2}$

$\lim_{(t,2t^2)\to(0,0)} \frac{2t^4}{t^4+2t^4} = \frac{2}{3}$ ⟹ NE

g) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^3-y}$

Curvas de nível

$c=2 \quad \frac{xy}{x^3-y} = 2 \to y = \frac{2x^3}{x+2}$

$\gamma_2(t) = (t, 2t^3/t+2)$

$f(\gamma_2(t)) = \frac{t \cdot 2t^3/t+2}{t^3 - \frac{2t^3}{t+2}} = \frac{2t^4}{t^3(t+2)-2t^3} = \frac{2}{2}$

$c=1 \quad \frac{xy}{x^3-y} = 1 \to xy = x^3-y \to y = x^3/x+1$

$\gamma(t) = (t, t^3/t+1) \to \lim_{t\to(0,0)} f(\gamma(t)) = \frac{t^4}{t+1} \cdot \frac{1}{(t^3 - t^3/t+1)} =$

$\frac{t^4(t+1)}{(t+1)(t^3(t+1)-t^3)} = \frac{t^4}{t^3(t)} = \frac{1}{1}$ NE//

h) $\lim_{(x,y)\to(0,0)} \dfrac{x^4 \operatorname{sim}(x^2+y^2)}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \dfrac{x^4}{x^4+y^2} \operatorname{sim}(x^2+y^2) = 0$

↳ Limitada ↳ 0 pelos 2 lados

i) $\lim_{(x,y)\to(0,0)} \dfrac{(x+y)^3}{x^2+y^2} = \dfrac{(x^2+y^2+2xy)(x+y)}{x^2+y^2} = \dfrac{x^3+xy^2+2x^2y+x^2y+y^3+2xy^2}{x^2+y^2}$ (lim $(x,y)\to(0,0)$)

$x \cdot \dfrac{x^2}{x^2+y^2} + y \cdot \dfrac{y^2}{x^2+y^2} + 3y \dfrac{x^2}{x^2+y^2} + 3x \dfrac{y^2}{x^2+y^2} = \dfrac{x^3+y^3+3x^2y+3y^2x}{x^2+y^2}$

$x \to 0$ Ltd · Ltd; $y \to 0$ Ltd · Ltd; $3y \to 0$ Ltd; $3x \to 0$ Ltd $= 0$

j. $\lim_{(x,y)\to(0,0)} \dfrac{x^2}{x^2+y^2} \operatorname{sim}\left(\dfrac{xy}{\sqrt{x^2+y^2}}\right) = 0$

↳ limitada ↳ $\lim_{(x,y)\to(0,0)} \dfrac{x}{\sqrt{x^2+y^2}} y$; $y \to 0$

↳ Limitada

k. $\lim_{(x,y)\to(0,0)} \dfrac{x^3y+y^4+x^4}{x^3y-xy^3} =$

$\lim_{(t,t)\to(0,0)} \dfrac{2t^4+16t^4+t^4}{2t^4-8t^4} = \dfrac{-19}{6}$

$\lim_{(t,-t)\to(0,0)} \dfrac{-2t^4+16t^4+4t^4}{-2t^4+8t^4} = \dfrac{15}{6}$ NE

l. $\lim_{(x,y)\to(0,0)} \dfrac{x^3+\operatorname{sim}(x^2+y^2)}{y^4+\operatorname{sim}(x^2+y^2)}$

$f(x,y) = \dfrac{x^3+\operatorname{sim}(x^2+y^2)}{y^4+\operatorname{sim}(x^2+y^2)} = \dfrac{\frac{x^2}{x^2+y^2}\cdot x + \frac{\operatorname{sim}(x^2+y^2)}{x^2+y^2}}{\frac{y^2}{x^2+y^2}\cdot y + \frac{\operatorname{sim}(x^2+y^2)}{x^2+y^2}} = \dfrac{1}{1} = 1$

$\frac{x^2}{x^2+y^2}$: Ltda · $x \to 0$; $\frac{\operatorname{sim}(x^2+y^2)}{x^2+y^2}$: $\lim_{a\to 0} \frac{\operatorname{sim} a}{a} = 1$

$\frac{y^2}{x^2+y^2}$: Ltda · $y \to 0$; $\frac{\operatorname{sim}(x^2+y^2)}{x^2+y^2}$: $\lim_{a\to 0} \frac{\operatorname{sim} a}{a} = 1$

se $a = x^2+y^2$ e $x \to 0$ e $y \to 0$, $a \to 0$

m. $\lim_{(x,y)\to(0,0)} \dfrac{x^3y^4+x^5\sqrt[3]{y^4}}{x^6+y^8}$

$\gamma_1(t) = (t^4, t^3) \to \lim_{(t^4,t^3)\to(0,0)} \dfrac{t^{12}\cdot t^{12} + t^{20}\cdot t^4}{t^{24}+t^{24}} = \dfrac{t^{24}+t^{24}}{2t^{24}} = 1$

$\gamma_2(t) = (-t^4, t^3) \to \lim_{(t^4,t^3)\to(0,0)} \dfrac{-t^{12}\cdot t^{12} + (-t^{20})\cdot t^4}{t^{24}+t^{24}} = \dfrac{-2t^{24}}{t^{24}} = -1$

NE

n. $\lim_{(x,y)\to(0,0)} \frac{x^3(1-\cos(x^2+y^2))}{(x^2+y^2)^3} = \lim_{(x,y)\to(0,0)} \frac{x^3 \cdot \sin^2(x^2+y^2)}{(x^2+y^2)^3 \cdot (1-\cos(x^2+y^2))}$

$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2} \cdot \frac{x}{1+\cos(x^2+y^2)} \cdot \left(\frac{\sin(x^2+y^2)}{(x^2+y^2)}\right)^2$

- $\frac{x^2}{x^2+y^2}$: $0 \le x^2 \le x^2+y^2$, limitado
- x: tende a 0
- $\frac{1}{1+\cos(x^2+y^2)}$: tende a $\frac{1}{2}$
- $\left(\frac{\sin(x^2+y^2)}{(x^2+y^2)}\right)^2$: tende a 1, pois $\lim_{t\to 0} \frac{\sin t}{t} = 1$ e $\lim_{(x,y)\to(0,0)} x^2+y^2 = 0$

$\lim_{(x,y)\to(0,0)} x \left(\underbrace{\frac{x^2}{x^2+y^2}}_{\text{limitado}} \cdot \underbrace{\frac{1}{1+\cos(x^2+y^2)}}_{1/2} \cdot \underbrace{\left(\frac{\sin(x^2+y^2)}{x^2+y^2}\right)^2}_{1} \right) = 0$ (limitado)

2.2

a. $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$ → sim, pois $\lim_{t\to 0} \frac{\sin t}{t} = 1$ e $\lim_{(x,y)\to(0,0)} x^2+y^2 = 0$ → $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = 1$ ($x^2+y^2 \to 0$)

b) $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$ → $0 \cdot -\infty$ → ⊗ -1 e l'hopital

$\lim_{t\to 0} t\ln t = \lim_{t\to 0} \frac{\ln t}{1/t} = \lim_{t\to 0} \frac{-1/t}{-1/t^2} = \lim_{t\to 0} t = 0$ ($\ln t \to -\infty$, $1/t \to \infty$)

então $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) = 0$ // (x^2+y^2 tende a 0)

c) $\lim_{(x,y)\to(0,0)} x^2 \ln(3x^2+y^2) \operatorname{arctg}\left(\frac{1}{y^2-x^2}\right)$

$0 \le \operatorname{arctg} \frac{1}{y^2-x^2} \le 2\pi$

$\lim_{(x,y)\to(0,0)} \frac{x^2}{3x^2+y^2} \cdot \ln(3x^2+y^2) \cdot (3x^2+y^2) \cdot \operatorname{arctg}\left(\frac{1}{y^2-x^2}\right) = 0$ //

- $\frac{x^2}{3x^2+y^2}$: $0 \le x^2 \le 3x^2+y^2$, $0 \le \frac{x^2}{3x^2+y^2} \le 1$ → limitado
- $\ln(3x^2+y^2) \cdot (3x^2+y^2)$: $\to 0$
- $\operatorname{arctg}\left(\frac{1}{y^2-x^2}\right)$: $0 \le \operatorname{arctg} \frac{1}{y^2-x^2} \le 2\pi$

d) $\lim_{(x,y)\to(1,1)} x^2 \ln(3x^2+y^2) \operatorname{arctan}\left(\frac{1}{y^2-x^2}\right)$

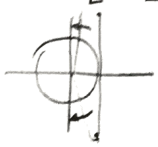
$f(x,y) = x^2 \ln(3x^2+y^2) \operatorname{arctan}(1/-x^2+y^2)$

$f(1,t) = \ln(3+t^2) \operatorname{arctan}\left(\frac{1}{t^2-1}\right)$

- $\lim_{(1,t)\to(1,1)^+} f(1,t) = \ln 4 \cdot \pi/2$

↳ $\operatorname{arctg}\frac{1}{t^2-1}$ com $t^2>1$ → $\frac{1}{t^2-1}$ → ∞ grande

- $\lim_{(1,t)\to(1,1)^-} f(1,t) = \ln 4 (-\pi/2)$



tende a $\pi/2$

tende a $-\pi/2$

2.2. Pontos de Continuidade ?

$$f(x,y) = \begin{cases} \dfrac{(x^2-y^2)(x-1)^2}{(x^2+y^2)((x-1)^2+(y-1)^2)} & \text{se } (x,y)\neq(0,0) \text{ e } (x,y)\neq(1,1) \\ 1 \text{ se } (x,y)=(0,0) \\ 0 \text{ se } (x,y)=(1,1) \end{cases}$$

- Dom $f(x,y) = \mathbb{R}$
- Contínua se $\lim_{(x,y)\to(z,w)} f(x,y) = f(z,w)$

→ $\lim_{(x,y)\to(0,0)} f(x,y) = 1$

Descontínua se $(x,y) = (0$

$$\lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)(x-1)^2}{(x^2+y^2)((x+1)^2+(y-1)^2)} = \frac{x^2}{(x^2+y^2)} \cdot \frac{(x-1)^2}{(x+1)^2+(y-1)^2}$$

↳ Limitada

$\gamma_1(t) = (0,t)$

$$\lim_{t\to 0} \frac{(-t^2)(-1)^2}{t^2(1^2+(t-1)^2)} = \frac{-t^2}{t^2(1+t^2-2t+1)} = \frac{-1}{t^2-2t+2} = -\frac{1}{2}$$

$\gamma(t) = (t,0)$

$$\lim_{t\to 0} \frac{(t^2-0)(t-1)^2}{(t^2)((t+1)^2+(-1)^2)} = \frac{t^2-2t+1}{t^2+2t+2} = \frac{1}{2}$$

N

→ Em $\dfrac{(x^2-y^2)(x-1)^2}{(x^2+y^2)((x-1)^2+(y-1)^2)}$

$$\lim_{(x,y)\to(1,1)} \frac{(x^2-y^2)(x-1)^2}{(x^2+y^2)((x-1)^2+(y-1)^2)} = 0$$ contínua

$0 \le (x-1)^2 \le (x-1)^2+(y-1)^2$

LTDA

↳ 0

2.4 Seja

$$f(x,y) = \begin{cases} \dfrac{x^4}{x^4+y^2}\,\operatorname{sim}\left(e^{-1/(x^2+y^2)}\right), & \text{se } (x,y)\neq(0,0) \\ L, & \text{se } (x,y)=(0,0) \end{cases}$$

Existe número L para $f(x,y)$ contínua em $(0,0)$? Justifique!

Para ser contínua $\rightarrow \lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) = L$

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}\cdot\frac{\operatorname{Sim}\left(e^{-1/(x^2+y^2)}\right)}{e^{-1/x^2+y^2}} \cdot e^{-1/x^2+y^2} = 0 =$$

↳ Limitada $\quad L = 1 \qquad L = 0$

$$\boxed{L=0}$$

Como $\lim_{t\to 0} e^{-1/t}$; $t>0 \rightarrow e^{\lim_{t\to 0^+} -1/t} \rightarrow e^{-\infty} \rightarrow 0$

se $(x,y)\to(0,0)$, $x^2+y^2 \to 0 \rightarrow \lim_{(x,y)\to(0,0)} e^{-1/x^2+y^2} = 0$

2.5 Seja $f(x,y) = \dfrac{3(x-1)^2+(y-1)^2}{x^2-y^2}$

a) Esboce curvas de nível de f nos níveis $k=1$ e $k=3$

$$1 = \frac{3(x-1)^2+(y-1)^2}{x^2-y^2} \rightarrow x^2-y^2 = 3x^2-6x+3+y^2-2y+1$$

$$-2x^2+6x-2y^2+2y = 4$$

$$\left(\sqrt{2}x-\frac{3\sqrt{2}}{2}\right)^2+\left(\sqrt{2}y-\frac{\sqrt{2}}{2}\right)^2 = -4+\left(\frac{3\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2$$

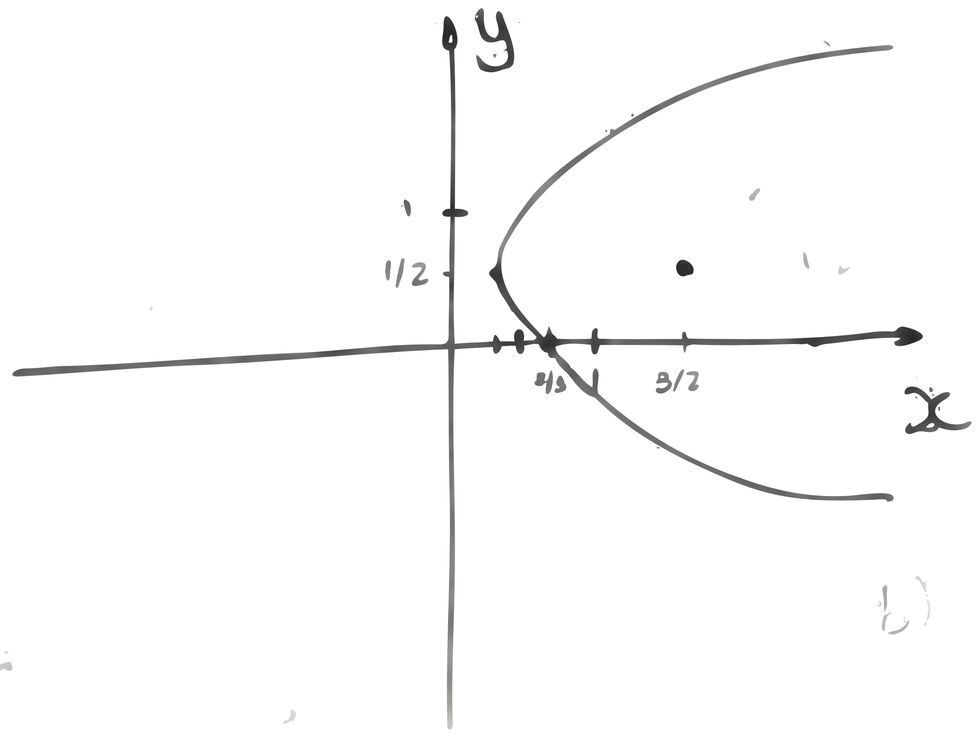
$$(x-3/2)^2+(y-1/2)^2 = -2+3/2+1/2 = 0$$

$$3 = \frac{3(x-1)^2+(y-1)^2}{x^2-y^2} \rightarrow \cancel{3x^2}-3y^2 = \cancel{3x^2}-6x+3+y^2-2y+1$$

$$-4y^2+2y-4 = -6x$$

$$x = \frac{2}{3}y^2-\frac{1}{3}y+\frac{2}{3} \rightarrow y_v = -\frac{b}{2a} = \frac{1}{2}$$

$$x_v = \frac{1}{3}\left(\frac{1}{2}-\frac{1}{2}+2\right) = \frac{2}{3}$$



b) Em $C \rightarrow \gamma(t) = (3/2, 1/2)$

$$\lim_{t\to(0,1)} f(\gamma(t)) = \frac{3(3/2-1)^2+(1/2-1)^2}{(3/2)^2-(1/2)^2} = \frac{2}{8/4} = 1$$

$\gamma(t) = (1,t)$

$$\lim_{t\to(1,1)} f(\gamma(t)) = \frac{(t-1)^2}{1-t^2} = \frac{\lim_{t\to 1}(t-1)^2}{\lim_{t\to 1} -t+1} = \frac{t-1}{t+1} = 0$$

NE